CFD-Based Method for Validation of DATCOM and Potential Flow Stability and Control Derivatives

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Abstract *— DATCOM was use traditionally as the method to determine the aircraft downwash, and longitudinal stability and control derivatives for small unmanned air vehicles conceptual design at the Polytechnic University of Puerto Rico. Computational fluid dynamic can be used to validate the DATCOM and the new downwash gradient method of Dr. Mondhler for low aspect ratios and low Reynolds number unnamed air vehicles. The results showed that the DATCOM empiric downwash law produces significant errors in longitudinal stability, while the downwash method of Dr. Mondhler predicts well the downwash behaviour. A code in MATLAB was developed to compare the longitudinal stability, and generate an in-house code that accurately represent the computational fluids dynamics predictions for the conceptual design process of small unmanned air vehicles.*

Key Terms**—** *DATCOM, Downwash gradient, Longitudinal stability, Low Reynolds number.*

INTRODUCTION

Low Reynold's number and low aspect ratio unmanned air vehicles (UAV) complex aerodynamics makes downwash gradient difficult to predict based on full-scale aircraft methods. There is a challenge where in comes to design small unmanned air vehicles since the viscous effect its a dominant factor in drag, tail size, and therefore handling qualities. The current methods of UAV design are based on empiric data of full-scale aircraft which can results in more errors. A computational fluids dynamics (CFD) based method is ideal to better predict the downwash and handling qualities of an UAV during the conceptual design process. As part of the validation process a low aspect ratio UAV was design to develop a package of codes in MATLAB from empiric data to better predict aircraft stability during the conceptual design process, validate aerodynamics, downwash, and stability and control derivatives using ANSYS FLUENT 2021 R2. By developing an in-house code, the iteration time during conceptual design is decreased substantially, while increasing fidelity in the stability and handling qualities during this phase.

AIRCRAFT LAYOUT AND CONFIGURATION

A small unmanned air vehicle was designed (Figure 1) based on the SAE Aero Design 2022 Regular Class competition requirements. The airplane has an aspect ratio of 2.99, and a flying Reynolds number of 600,000. The airfoil S1223 was selected due to its high lift capacity at low Reynold's number.

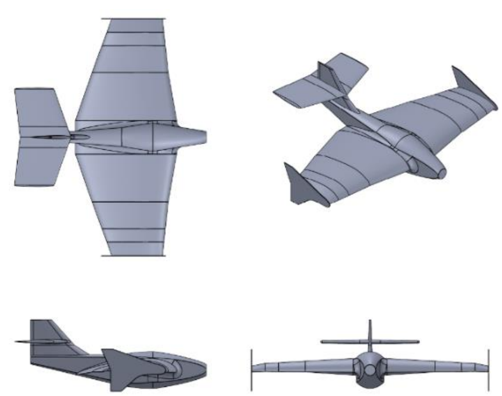

Figure 1
Aircraft Design Configuration

DOWNWASH GRADIENT METHODS

The downwash gradient has an impact in the aircraft aerodynamic center location and lift curve slope, therefore affecting the handling qualities of the aircraft. Two methods were analysed, the USAF-DATCOM empirical Law, and a new empirical law propose by Dr. Yahyaoui Mondher [1]. The USAF-DATCOM empirical downwash gradient law is given by (1).

$$\frac{d\varepsilon}{d\alpha} = 4.44\{K_{AR}K_{mr}K_{\lambda}\sqrt{\cos(\Lambda_{0.25})}\}^{1.19} \quad (1)$$

The constants $K_{AR}, K_{mr}, and\ K_{\lambda}$ are based on the aircraft geometry, aspect ratio (AR), wingspan (b), the distance between the wind root quarter chord and horizontal tail root quarter chord (Xwh), and the vertical distance between the wing root chord and the horizontal tail root chord (Zwh), as shown in Figure 2.

$$K_{AR} = \frac{1}{AR} - \frac{1}{1+AR^{1.7}} \quad (2)$$

$$K_{\lambda} = \frac{10-3\lambda}{7} \quad (3)$$

$$K_{mr} = \frac{1-\frac{m}{2}}{r^{1/3}} \quad (4)$$

$$m = \frac{2Z_{wh}}{b} \quad (5)$$

$$r = \frac{2X_{wh}}{b} \quad (6)$$

Figure 2
Wing Tail Geometry Relationships

Dr. Yahyaoui Mondher new law for unswept tapered wing is:

$$\frac{d\varepsilon}{d\alpha} = \frac{1+c_1 m}{c_2 + c_3 m} \quad (7)$$

For swept tapered wings.

$$\frac{d\varepsilon}{d\alpha} = \frac{1+c_1 m}{\acute{c}_2 + \acute{c}_3 m} \quad (8)$$

Where:

$$\acute{c}_2 = c_2 + 1.8\Lambda_{c/4} \quad (9)$$

$$\acute{c}_3 = c_3 + \left[(C_{A\lambda} - C_{\lambda}m)\sin\Lambda_{c/4} + (\sin\Lambda_{c/4})^{3.5}\right]\Lambda_{c/4} \quad (10)$$

$$C_{A\lambda} = 1.1AR + 12\lambda - 8.15 \quad (11)$$

$$C_{\lambda} = 35\lambda - 7.5 \quad (12)$$

Constant $c_1, c_2, and\ c_3$ are given in a table in his paper and can be interpolated.

LONGITUDINAL STABILITY AND CONTROL DERIVATIVES

The determination of the longitudinal stability is essential to have a flyable aircraft. The aircraft must have a proper tail size and position of the center of gravity with respect of the aerodynamic center (static margin), to produce enough pitching moment for take-off, landing, various flaps configurations, and trim conditions. Without this requirement the aircraft will not be able fly without the use of robust flight computer system.

To determine the stability and control derivatives we must take in consideration the shifts in the aerodynamic center due to the fuselage, by using Munk's theory (13).

$$\Delta\bar{X}_{AC_B} = -\frac{\sum_{i=1}^{N} {w_B}_i^{\,2}(\frac{d\bar{\varepsilon}}{d\alpha})\Delta x_i}{2.92(S\bar{c})} \quad (13)$$

The empiric relation (14) is used to determine the wing, and wing-body aerodynamic

$$\bar{x}_{AC_{WB}} = \bar{x}_{AC_W} + \bar{x}_{AC_B} \quad (14)$$

Where;

$$\bar{x}_{AC_W} = K_1(\frac{\acute{x}_{AC}}{c_R} - K_2) \quad (15)$$

The value of $\frac{\acute{x}_{AC}}{c_R}$,and the coefficients K_1 and K_2 are determine by the interpolation of curves provided in Napolitano's book [2]. The longitudinal stability derivatives that are going to be determined are steady state derivatives, static longitudinal control derivatives, small perturbations control derivatives, and longitudinal manoeuvring stability derivatives.

Steady state longitudinal stability derivatives are C_{L_α}, C_{D_0}, C_{L_0}, C_{m_0}, C_{L_1}, C_{M_1}, and C_{D_1}.;where C_{L_0} is the lift coefficient at zero AOA, C_{D_0} is the aircraft parasite drag, C_{m_0}, is the aircraft pitching moment at zero AOA, and $C_{L_\alpha}$is the aircraft lift curve slope (16).

$$C_{L_\alpha} = C_{L_{\alpha W}} + C_{L_{\alpha H}} + \eta_H \frac{S_H}{S} C_{L_{\alpha H}} \quad (16)$$

$$C_{L_1} = C_{L_0} + C_{L_\alpha}\alpha + C_{L_{\delta\varepsilon}}\delta_\varepsilon + C_{L_{iH}} i_H \quad (17)$$

$$C_{D_1} = C_{D_0} + C_{D_\alpha}\alpha + C_{D_{\delta\varepsilon}}\delta_\varepsilon + C_{D_{iH}} i_H \quad (18)$$

$$C_{M_1} = C_{M_0} + C_{M_\alpha}\alpha + C_{M_{\delta\varepsilon}}\delta_\varepsilon + C_{M_{iH}} i_H \quad (19)$$

The static longitudinal control derivatives are: $C_{L_{\delta\varepsilon}}$ elevator deflection stability derivative (20), $C_{L_{iH}}$ is the horizontal stabilizer deflection stability derivative (21), and the pitching moment derivative with AOA C_{M_α} (22), C_{L_α}aircraft lift slope (23), $C_{M_{iH}}$ stabilator moment coefficient (24), and the drag derivative with angle of attack C_{D_α}(25).

$$C_{L_{\delta E}} = \eta_H \frac{S_H}{S} C_{L_{\alpha H}} \tau_E \quad (20)$$

$$C_{L_{iH}} = \eta_H \frac{S_H}{S} C_{L_{\alpha H}} \quad (21)$$

$$C_{M_\alpha} = C_{L_{\alpha W}}(\bar{x}_{CG} - \bar{x}_{AC}) - C_{L_{\alpha H}}\, \eta_H \frac{S_H}{S}\left(1 - \frac{d\varepsilon}{d\alpha}\right)(\bar{x}_{AC_H} - \bar{x}_{CG}) \quad (22)$$

$$C_{L_\alpha} = C_{L_{\alpha W}} + \eta_H \frac{S_H}{S} C_{L_{\alpha H}}\left(1 - \frac{d\varepsilon}{d\alpha}\right) \quad (23)$$

$$C_{M_{iH}} = -C_{L_{\alpha H}}\eta_H \frac{S_H}{S}\ (\bar{x}_{AC_H} - \bar{x}_{CG}) \quad (24)$$

$$C_{D_\alpha} \approx \left(\frac{2C_{L_1}}{\pi A R e}\right) C_{L_\alpha} \quad (25)$$

Small perturbation longitudinal stability derivatives capture the change in the aircraft forward velocity [3]. Since the small UAV in flying in very low Mach number (less than 0.05) these longitudinal stability derivatives are assumed to be zero. The longitudinal manoeuvring stability derivatives capture the rate or sensitivity of normal acceleration of the aircraft [4]. This rate of change in angle of attack due to pitch rate is given by the stability and control derivatives $C_{L_{\dot\alpha}}$ (26), $C_{M_{\dot\alpha}}$(27), C_{L_q}(28), and C_{M_q}.(32) [4].

$$C_{L_{\dot\alpha}} \approx 2C_{L_{\alpha H}}\eta_H \frac{S_H}{S}(\bar{x}_{AC_H} - \bar{x}_{CG})\frac{d\varepsilon}{d\alpha} \quad (26)$$

$$C_{M_{\dot\alpha}} \approx -2C_{L_{\alpha H}}\eta_H \frac{S_H}{S}\left(\bar{x}_{AC_H} - \bar{x}_{CG}\right)^2 \frac{d\varepsilon}{d\alpha} \quad (27)$$

$$C_{L_q} \approx C_{L_{qW}} + C_{L_{qH}} \quad (28)$$

Where:

$$C_{L_{qW}} = \left[\frac{AR + 2\cos\Lambda_{0.25}}{AR\sqrt{1 - Mach^2(\cos\Lambda_{0.25})^2}}\right] C_{L_{qW}}\ |Mach_{=0} \quad (29)$$

$$C_{L_{qW}}\ |Mach_{=0} = \left(\frac{1}{2} + 2|\bar{x}_{CG} - \bar{x}_{ACW}|\right) C_{L_{\alpha W}}\ |Mach_{=0} \quad (30)$$

$$C_{L_{qH}} \approx 2C_{L_{\alpha H}}\eta_H \frac{S_H}{S}(\bar{x}_{AC_H} - \bar{x}_{CG}) \quad (31)$$

$$C_{M_q} = C_{M_{qW}} + C_{M_{qH}} \approx C_{M_{qH}} \quad (32)$$

$$C_{M_{qH}} = -2C_{L_{\alpha H}}\eta_H \frac{S_H}{S}\left(\bar{x}_{AC_H} - \bar{x}_{CG}\right)^2 \quad (33)$$

ANSYS FLUENT COMPUTATIONAL FLUIDS DYNAMICS METHOD

To evaluate the external aerodynamics of the aircraft ANSYS FLUENT was used as the CFD method to validate the downwash and the longitudinal stability derivatives. FLUENT has been a proven method is the past at the Polytechnic University of Puerto Rico to determine the aerodynamics of small Reynolds number, when wind tunnel testing is not available. The mesh grids were refined by dividing the airplane in section increasing the number of elements at the leading edge, trailing edge, and wing-body connection surface (Figure 3).

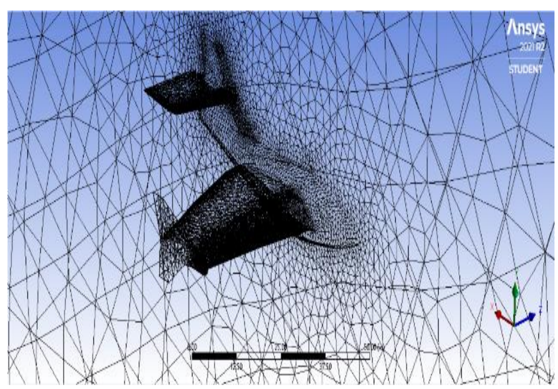

Figure 3
ANSYS FLUENT Mesh

Spalart-Allmaras model was used to run the analysis at a reference velocity of 40 feet per second and angles of attacks (AOA) from -8 degrees to 21 degrees, with three boundary conditions as inlet velocity (blue surfaces) and one symmetric boundary condition (yellow surface), as seen in Figure 4. The downwash was determined by utilizing the FLUENT velocity vectors to calculate the downwash angle (13), and downwash gradient (14).

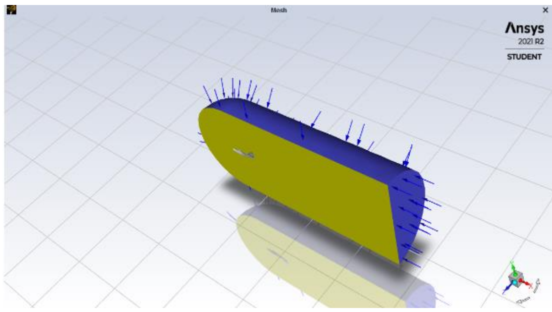

Figure 4
Boundary Conditions

$$\varepsilon = \tan^{-1}\left[\frac{-V_y \cos\alpha + V_x \sin\alpha}{\vec{V}}\right] \quad (34)$$

$$\frac{d\varepsilon}{d\alpha} = \frac{\varepsilon_2 - \varepsilon_1}{\alpha_2 - \alpha_1} \quad (35)$$

Where α is the aircraft angle of attack, and ε is the downwash angle.

MATLAB Code

A MATLAB code was created to facilitate the calculations of the longitudinal stability and control derivatives during the conceptual design, as shown in Figure 5. To mitigate wrongly reading charts coefficients and times loss during interpolation, a program was used to digitalize the charts and convert them as MATLAB function to automize the aircraft configuration process.

Downwash Gradient Results

The horizontal tail downwash was calculated from computational fluids dynamics by plotting the downwash angle versus angle of attack to determine the downwash gradient, as shown in Figure 6.

A comparison of downwash gradient from computational fluid dynamics, USAF-DATCOM law, and Dr. Mondhler new law is shown if Figure 7. The same small unmanned vehicle with different horizontal tail location was used to investigate the model that better estimate when rapid prototyping is needed during the small UAV conceptual design. By using the CFD model as reference, the comparison shows that for a "m" (2Zwh/b) between 0 and 0.2 the USAF-DATCOM law percentage of difference is substantial for low aspect ratio aircraft, and the new law accurately represent the downwash effects with a maximum difference of 3.5%. Extrapolation was done for "m" greater than 0.2 in the new law coefficients table to estimate the downwash gradient. The comparison shows that linear extrapolation is not the best method to calculate the new law coefficients, resulting up to 73% of difference, this maybe since tables are for AR between 4 to 12, so the linear extrapolation for an AR of 2.99 can increase the errors for values of "m" larger than 0.2.

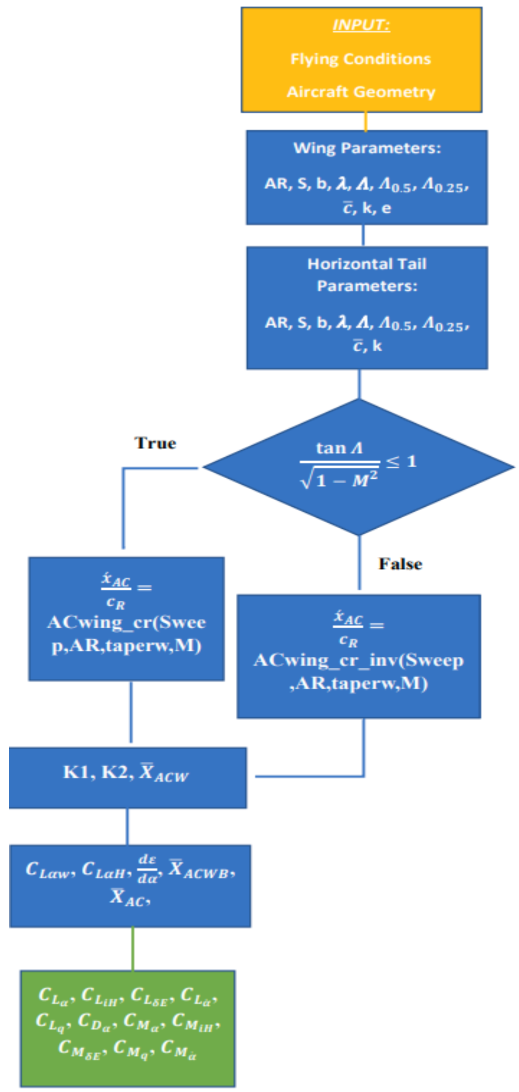

Figure 5
MATLAB Code Flowchart

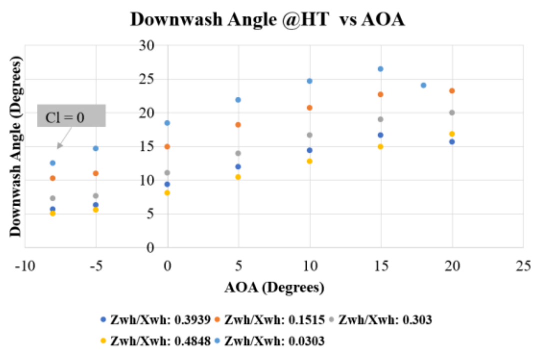

Figure 6
Downwash Angle

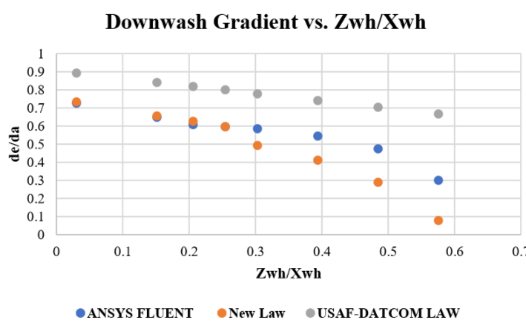

Figure 7
Downwash Gradient

LONGITUDINAL STABILITY RESULTS

Modeling the stability and control derivatives of the UAV was limited to the longitudinal analysis, because the ANSYS FLUENT student version limits the elements quantity up to 512,000, because of this half of the UAV was used with a symmetry boundary condition. Since the number of elements was limited, the full UAV aircraft could not be modelled to determine the lateral directional stability and control derivatives. The angle of attack range was from -8 to 21 degrees, using computational fluids dynamics and potential flow equations to determine the longitudinal stability and control derivatives.

The lift coefficient and lift curve slope (Figure 8), and moment coefficient around the center of gravity (Figure 9) was determine for various horizontal stabilizer positions to capture the effect of downwash in the aircraft aerodynamic enter, and CL_{max}.

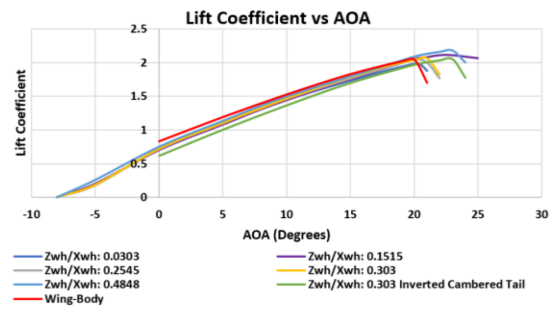

Figure 8
Lift Coefficient vs AOA

Due to the high lift capacity of the airfoil, there is a significant amount of nose down pitching moment, meaning that to trim the aircraft, more force or a larger horizontal stabilizer is needed or high elevator deflection, reducing the CL_{max} and increasing drag. A special case was run using a NACA 6412 inverted cambered airfoil at the tail to investigate how efficient is this configuration to reduce the nose down pitching moment, making the aircraft easier to trim. As shown in figure 9, this configuration reduces the Cm_0 from -0.16 to -0.04.

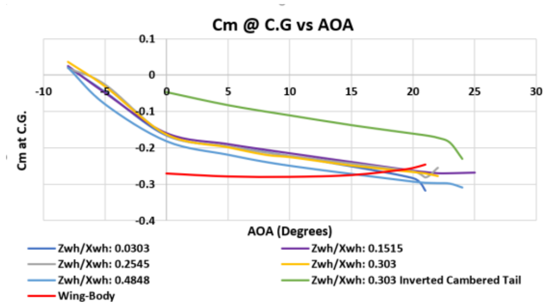

Figure 9
Moment Coefficient vs AOA

Is imperative to determine the aircraft aerodynamic center to calculate the aircraft longitudinal stability. Equation (36) was used to calculate the aircraft aerodynamic center (AC) for USAF-DATCOM and new method of downwash, with respect of the wing mean aerodynamic chord ($\bar{c}$). Equation (37) was used to calculate the aircraft AC from the CFG method.

$$\bar{x}_{ac} = \frac{\bar{x}_{ACWB} + \frac{C_{L\alpha H}}{C_{L\alpha W}} \frac{S_H}{S} \eta_H \left(1 - \frac{d\varepsilon}{d\alpha}\right) \bar{x}_{ACH}}{1 + \frac{C_{L\alpha H}}{C_{L\alpha W}} \frac{S_H}{S} \eta_H \left(1 - \frac{d\varepsilon}{d\alpha}\right)} \quad (36)$$

$$\bar{x}_{ac} = \bar{x}_{CG} - \frac{\frac{dC_{M_{CG}}}{d\alpha}}{C_{L\alpha}} \quad (37)$$

The aerodynamic center prediction (Figure 10) comparison shows that the USAF-DATCOM predicts the AC by a minimum 17.2% and maximum 23.76% of difference with respect of the CFD method. The new method of downwash predicts the AC by a minimum and maximum difference of 1.26% and 11.7% respectively. A special case combining XFRL5 and new downwash law method, to calculate the AC and longitudinal stability. This hybrid method and shows an AC difference between 1.98% and 6.25%. That hybrid method consists in utilizing the $\acute{x}_{AC/cr}$ of XFLR5 and use the new method of downwash in the in-house MATLAB code to determine the aircraft aerodynamic center and longitudinal stability. This method tends to be more accurate since XFLR5 uses the airfoil shape to calculate the aerodynamics, and therefore reduces the need to generate a 3D CAD model of the aircraft and running ANSYS FLUENT to determine the aerodynamics of the aircraft, which can be time consuming during the conceptual design iteration process.

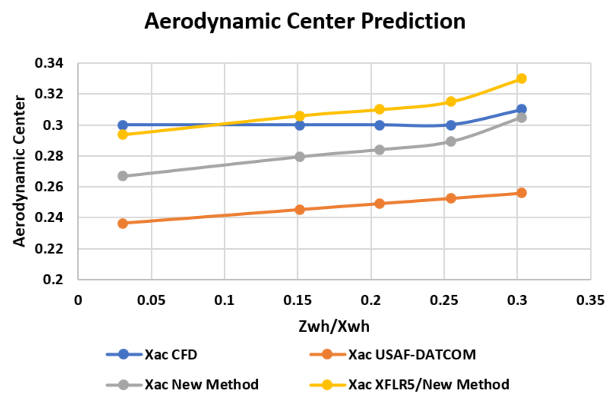

Figure 10
Aircraft Aerodynamic Center

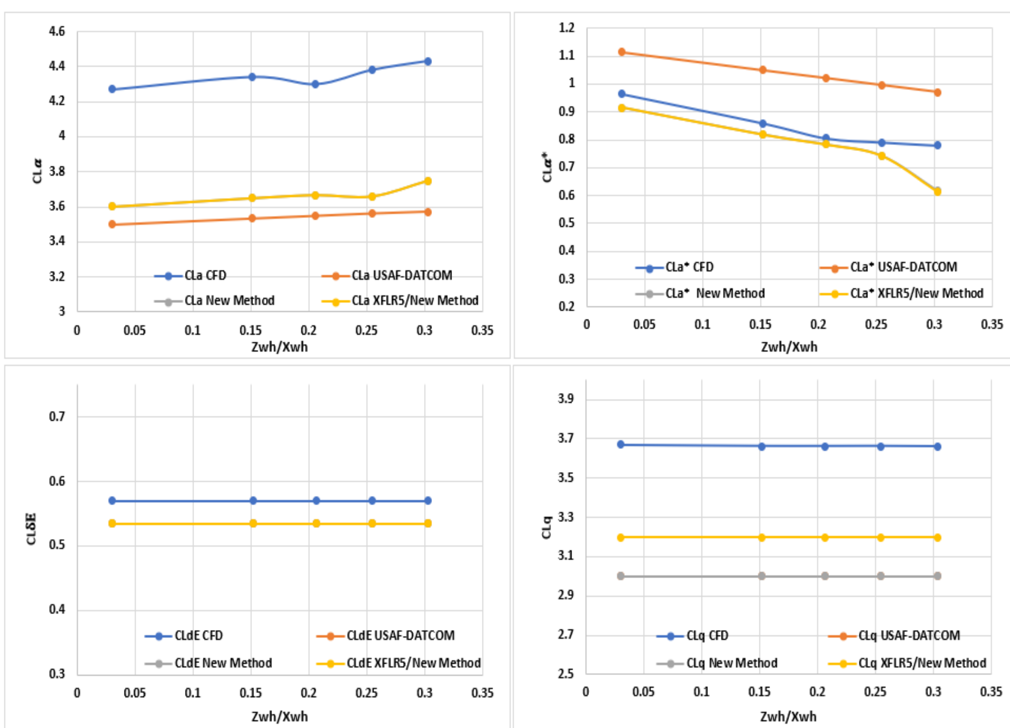

Figure 11
CL Stability Derivatives

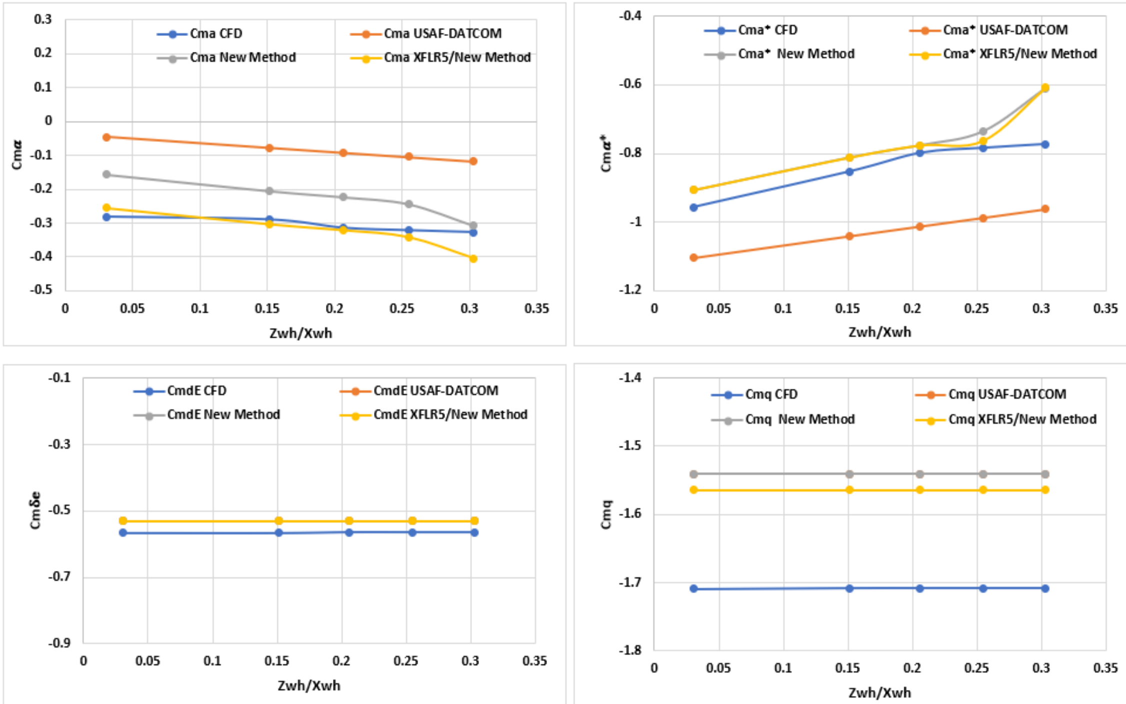

Figure 12
Cm Stability Derivatives

The MATLAB code was run for up to Zwh/Xwh 0.303 since the downwash prediction was not accurate after this value for an AR of lower than four. The best two methods that produce closer results to CFD methods are the XFLR5/New Method with an average difference of 9% and the method of downwash with an average difference of 13%. The longitudinal stability derivatives results

The aircraft lift (Figure 11) and moment (Figure 12) stability derivatives for each horizontal tail configuration using the same four methods for the aircraft AC. CFD is the best method to determine the stall angle, $C_{L_{max}}$, and lift curve slope since the other methods tends to overpredict where the $C_{L_{max}}$ occurs.

Conclusion

Four methods (CFD, DATCOM, Dr. Mondhler downwash, and XFLR5 hybrid) were used to determine the aircraft aerodynamic center and longitudinal stability derivatives of a low aspect ration aircraft flying in low Reynolds number, based on the Aero Design 2022 competition requirements. Dr. Mondhler dowwash method successfully predicts the downwash gradient, with a maximum difference of 3.5% for "m" between 0 to 0.2.

The aircraft aerodynamic center is best to use the XFLR5 hybrid method with a maximum of 5% difference overpredicting stability when the Zwh/Xwh is larger than 0.3. The new method of downwash produces an average 10% of difference underpredicting stability, conservatism can be applied by running both methods and stay between their predictions.

Further work can be done by implementing the propwash in the computational fluid dynamics method to determine how the downwash and tail effectiveness is affected, and the best configuration for the airfoil S1223 to reduce the nose down pitching moment.

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