Airfoil Design Using Blade Element Momentum Theory

Rodrigo Lobeto Sanfeliz
Master of Engineering in Mechanical Engineering
Bernardo Restrepo Torres, Ph.D.
Mechanical Engineering Department
Polytechnic University of Puerto Rico

Abstract *— A wind turbine is a device designed to extract kinetic energy from the wind and convert it to mechanical energy, which in turn can be equivalent to electric power. On this work we will focus on the aerodynamics of wind turbines and on the overall effect that aerodynamic variables have on the rotor's performance. For this purpose, test three (3) different airfoil configurations a rotor's blade to determine which design will optimize our power production at the same time it reduces manufacturing costs and efforts.*

Key Terms *— Betz Limit, Blade Element Momentum Theory, Blade Element Theory, Power Coefficient.*

INTRODUCTION

When predicting a rotor's performance based on aerodynamic variables and on the fluid's behavior it is important to take into consideration several design/analytical issues as shown in [1] and [2]. The most important of these issues concerns the type of analysis that must be performed on the rotor-air system: we might analyze it as an open system (better known as control volume) because there is a continuous flow of air across the rotor. This last assumption is the key to the **momentum theory** concerning the analysis of power extracting devices. This theory assumes the following conditions: (1) Homogeneous, incompressible and steady state fluid flow, (2) no frictional drag, (3) infinite number of blades, (4) uniform thrust over the disk or rotor area, (5) non-rotating wake after the rotor and (6) static pressures far upstream and far downstream the rotor are equal to the ambient static pressure.

From the momentum theory and from energy concerns we can determine the maximum amount of energy that may be extracted from the fluid relative to the total amount of energy on the fluid itself under ideal conditions, which is better known as the **power coefficient**. The fact that this power coefficient is not 100% even under ideal conditions is given by the **Betz Limit** which sets the power coefficient's limits up to 59.3%. Although Betz Limit is a variable of great importance, it's important to know that the momentum theory introduces huge limitations into our design. This is because control volumes only consider flow quantities at the system's inlet/outlet and we have no control over the design variables that are of great importance and whose location is at the rotor inter stages. This is the case of the airfoil shape, which is the most important parameter when it comes to determining the rotor performance based on blade-fluid interaction. There is also some fluid rotation after the rotor stage that is induced by the blade's rotational motion and that reduces the amount of energy that is extracted by the rotor. This phenomenon is known as wake rotation.

In order to address these issues, we need to develop an analysis that can both consider blade design from the airfoil's shape and overall performance based on momentum theory along with the additional effects that the local wind field might cause on the rotor's performance. The results obtained from the airfoil's design iterations must be physically consistent with the principles derived from the momentum theory. Particular blade airfoils performance is analyzed using the **blade element theory** and when these results are combined with the momentum theory, we have the **blade element momentum theory**.

Theory: Design Algorithm for a Rotor with Blade Element Momentum Theory

We first begin by determining ideal blade shape as suggested on [3]. First Obtain and examine empirical curves for the aerodynamic properties of the airfoil at each section. It is important to look for the following parameters: lift coefficient vs angle of attack curves, drag coefficient vs angle of attack curves and angle of attack at which the drag to lift coefficient ratio is a minimum. This last statement simulates a condition for which drag coefficient approaches zero.

The blade is then divided into N elements (10-20) and use optimum rotor theory. First it is important to estimate ideal blade shape (this is the shape of the blade with midpoint radius r_i). Some important parameters to look for are:

- Local tip speed ratio:

$$\lambda_{r,i} = \lambda\left(\frac{r_i}{R}\right) \quad (1)$$

- Angle of relative wind:

$$\varphi_i = \left(\frac{2}{3}\right)\tan^{-1}\left(\frac{1}{\lambda_{r,i}}\right) \quad (2)$$

- Chord length:

$$c_i = \frac{8\pi r_i}{BC_{l,design,i}}(1 - \cos\varphi_i) \quad (3)$$

- Twist angle:

$$\theta_{T,i} = \theta_{p,i} - \theta_{p,0} \quad (4)$$

- Twist, pitch and angle of relative wind relation:

$$\varphi_i = \theta_{p,i} + \alpha_{design,i} \quad (5)$$

Now by using the optimum blade shape as a guide, it is possible to determine the blade shape that promises to be a good approximation. For ease of fabrication, choose linear variations of chord, thickness and twist angle. Suppose that a_1, a_1 and a_1 are coefficients for the chosen chord and twist distributions, we have that thee following quantities can be expressed as:

- Chord length:

$$c_i = a_i r_i + b_1 \quad (6)$$

- Twist angle:

$$\theta_{T,i} = a_2(R - r_i) \quad (7)$$

After some preliminary ideal shape is chosen, both the blade shape as well as its performance must be modified. First solve for lift coefficient and angle of attack. This means we have to find actual angle of attack and lift coefficient for the center of each element. We will use the following empirical equation along with airfoil data curves:

- Lift coefficient:

$$C_l = 4F_i \sin\varphi_i \frac{(\cos\varphi_i - \lambda_{r,i}\sin\varphi_i)}{\sigma_i'(\sin\varphi_i + \lambda_{r,i}\cos\varphi_i)} \quad (8)$$

- The solidity is defined as:

$$\sigma_i' = \frac{Bc_i}{2\pi r_i} \quad (9)$$

- Angle of attack is:

$$\varphi_i = \alpha_i + \theta_{T,i} + \theta_{P,0} \quad (10)$$

Note that this process is about finding the angle of attack α that makes this lift coefficient equal to the one obtained from the empirical airfoil curves considering tip losses associated with boundary layers behavior at the airfoil's tip.

- Tip speed loss:

$$F_i = \frac{2}{\pi}\cos^{-1}\left[e^{-\left(\left\{\frac{\left(\frac{B}{2}\right)\left[1-\left(\frac{r_i}{R}\right)\right]}{\left(\frac{r_i}{R}\right)\sin\varphi_i}\right\}\right)}\right] \quad (11)$$

Axial induction factor can now be determined:

- Given by the expression:

$$a_i = \frac{1}{\left[1 + \frac{4\sin^2\varphi_i}{\sigma_i' C_{l,i}\cos\varphi_i}\right]} \quad (12)$$

If axial induction factor is greater than 0.4 consider using another method. Remember that

Betz Theory does not apply for axial induction factors greater than 0.5. This analysis must include the following parameters: chord length, angle of attack, tip loss, lift coefficient, drag coefficient, power coefficient and axial induction factor.

A design procedure may look like this:

- Determine the ideal airfoil shape.
- Obtain and examine empirical curves for the aerodynamic properties of the airfoil at each section.
- Divide the blade into N elements (10-20) and use optimum rotor theory.
- Using the optimum blade shape as a guide, determine the blade shape that promises to be a good approximation.
- Modify blade shape.
- Solve for lift coefficient and angle of attack.
- Find angle of attack α that makes this this lift coefficient equal to the one obtained from the empirical airfoil curves.
- Determine the axial induction factor a.

Rotor Design

NREL's suggestions to design a 25 m rotor will be used (S airfoil families). The paper's proposal is to analyze the aerodynamic performance of a 50 m rotor using airfoils typically used for rotors half the size to the one we are considering. Each rotor's blade must contain the following sections: Root airfoil (0~75 % span), primary airfoil (75 ~ 95 % span) and tip airfoil (95 ~ 100 % span). Suggested airfoil families for root, primary and tip airfoils respectively are: S811-S809-S810, S814-S812-S813 and S815-S812-S813. The rotor consists of three blades at a pitch angle of -2.0° and at a local wind field of speed 2.5 m/s with λ=7. Air density and speed are assumed to be 1.23 $\frac{kg}{m^3}$.

RESULTS

This section presents the results obtained for the different combinations of airfoil families based on the design procedure stated on [3] and [4].

1. Analysis for the S811-S809-S810 airfoil family

Results for the S811-S809-S810 airfoil family are presented on this section. This family contains good power coefficient values but its manufacturability is poor as shown on the next figures.

Chord length distribution:

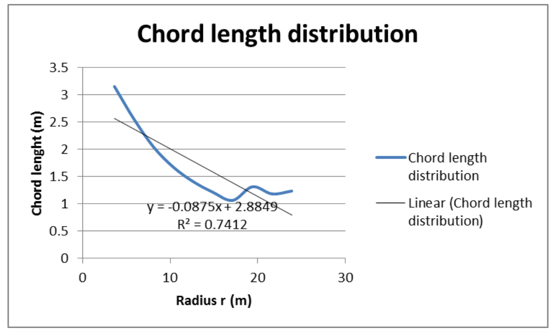

Figure 1
Chord Length Distribution for S811-S809-S810 Airfoil Family

Twist angle distribution:

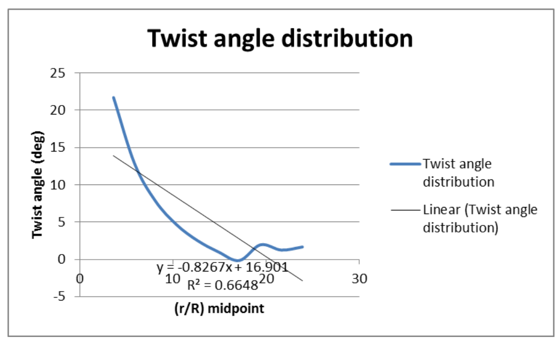

Figure 2
Twist Angle Distribution for S811-S809-S810 Airfoil Family

Angle of attack (AOA) distribution:

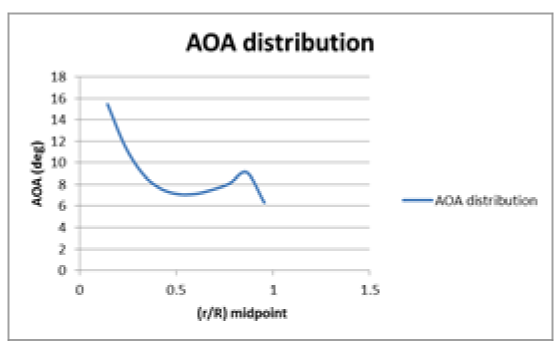

Figure 3
Angle of Attack (AOA) Distribution for S811-S809-S810 Airfoil Family

Table 1
Computed data for S811-S809-S810

N	r/R midpoint	r	$\lambda_{r,i}$	C (linear approx.)	Twist (linear)	Optimal Cp	Cl (actual)	AOA	F	Cp (tip loss)	a
1	0.145	3.625	1.015	2.5677125	13.9042125	0.012904213	1.684417	15.44646	1	0.01290421	0.374723
2	0.235	5.875	1.645	2.3708375	12.0441375	0.024691938	1.306649	11.54389	0.999999	0.02469191	0.301838
3	0.325	8.125	2.275	2.1739625	10.1840625	0.034843074	1.060789	9.004016	0.999983	0.03484249	0.270415
4	0.415	10.375	2.905	1.9770875	8.3239875	0.044227937	0.931064	7.663878	0.999899	0.04422347	0.260209
5	0.505	12.625	3.535	1.7802125	6.4639125	0.053742913	0.878456	7.120413	0.999579	0.0537203	0.264272
6	0.595	14.875	4.165	1.5833375	4.6038375	0.063662173	0.878087	7.116598	0.998495	0.06356634	0.278366
7	0.685	17.125	4.795	1.3864625	2.7437625	0.073713616	0.915884	7.507067	0.994795	0.07332995	0.298412
8	0.775	19.375	5.425	1.1895875	0.8836875	0.08014831	1.00358	8.097556	0.982303	0.07872994	0.330737
9	0.865	21.625	6.055	0.9927125	-0.9763875	0.083963674	1.106431	9.102947	0.928854	0.07799003	0.346081
10	0.955	23.875	6.685	0.7958375	-2.8364625	0.055927549	0.800758	6.317744	0.958626	0.0536136	0.82658
						0.527825397				0.51761224	

Power coefficient (Cp) distribution:

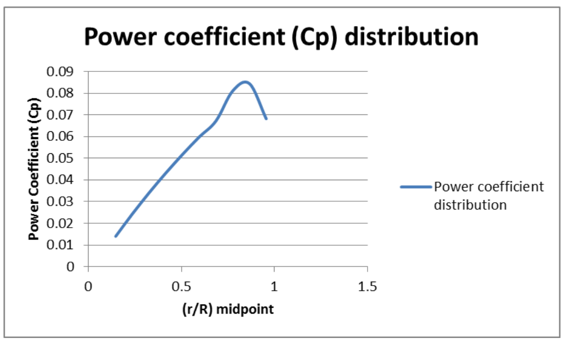

Figure 4
Power Coefficient Distribution for S811-S809-S810 Airfoil Family

Lift coefficient (Cl) distribution:

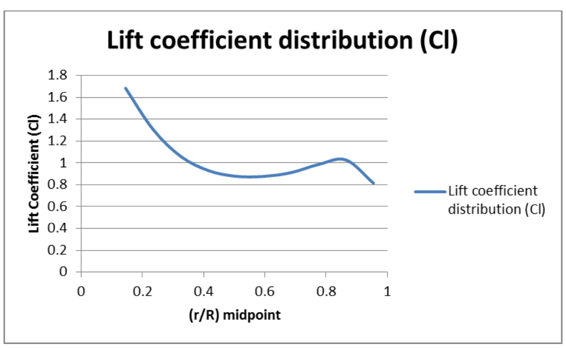

Figure 5
Lift Coefficient (Cl) Distribution for S811-S809-S810 Airfoil Family

2. Analysis for the S814_S812_S813 airfoil family:

Results for the S814-S812-S813 airfoil family are presented on this section. This family contains both good power coefficient values and manufacturability as shown on the next figures.

Chord length distribution:

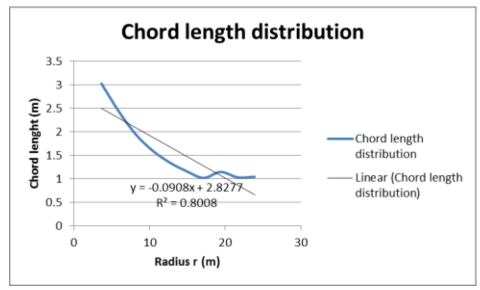

Figure 6
Chord Length Distribution for the S814_S812_S813 Airfoil Family

Twist angle distribution:

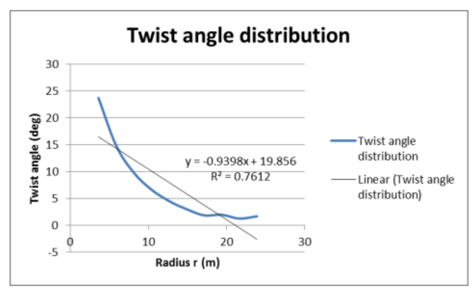

Figure 7
Twist Angle Distribution for the S814_S812_S813 Airfoil Family

Angle of attack (AOA) distribution:

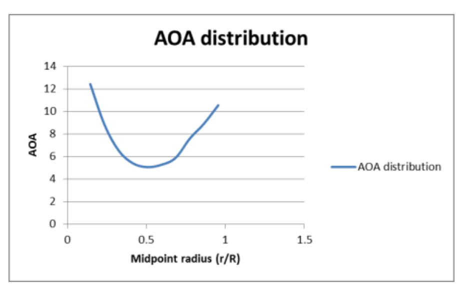

Figure 8
Angle of Attack (AOA) Distribution for the S814_S812_S813 Airfoil Family

Table 2

Computed Data for S814-S812-S813

N	r/R midpoint	r	$\lambda_{r,i}$	C (linear approx.)	Twist (linear)	Optimal Cp	Cl (actual)	AOA	F	Cp (tip loss)	a
1	0.145	3.625	1.015	2.49855	16.449225	0.013570666	1.752368484	12.44695611	0.999999998	0.013570666	0.385864218
2	0.235	5.875	1.645	2.29425	14.334675	0.025013746	1.383135997	8.841172244	0.999999142	0.025013724	0.315410347
3	0.325	8.125	2.275	2.08995	12.220125	0.03523091	1.147695346	6.541945024	0.999987044	0.035230454	0.288598693
4	0.415	10.375	2.905	1.88565	10.105575	0.045060222	1.032556927	5.417544116	0.999924698	0.045056829	0.284854523
5	0.505	12.625	3.535	1.68135	7.991025	0.054968005	0.998133201	5.081370987	0.999698704	0.054951443	0.296860679
6	0.595	14.875	4.165	1.47705	5.876475	0.064735526	1.019140581	5.286524369	0.998954429	0.06466784	0.319855057
7	0.685	17.125	4.795	1.27275	3.761925	0.073678842	1.082026119	5.900643141	0.996392446	0.073413041	0.348570674
8	0.775	19.375	5.425	1.06845	1.647375	0.077640987	1.067455211	7.615630787	0.979677744	0.076063147	0.303669028
9	0.865	21.625	6.055	0.86415	-0.467175	0.079248256	1.199261736	8.940320924	0.91998081	0.072906875	0.309016427
10	0.955	23.875	6.685	0.65985	-2.581725	0.078059662	1.391679357	10.56927073	0.660887462	0.051588652	0.295595538
						0.547206822				0.512462672	

Power coefficient (Cp) distribution:

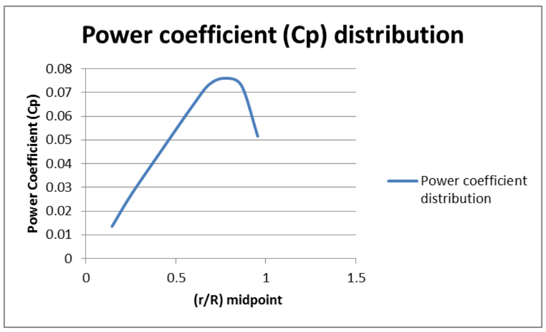

Figure 9
Power coefficient (Cp) distribution for the S814_S812_S813 airfoil family

Lift coefficient (Cl) distribution:

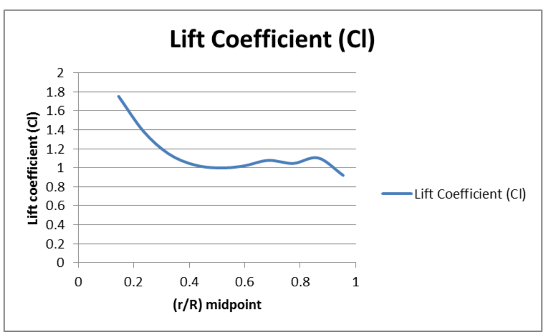

Figure 10
Lift Coefficient (Cl) Distribution for the S814_S812_S813 Airfoil Family

3. Analysis for the S815_S812_S813 airfoil family

Results for the S815-S812-S813 airfoil family are presented on this section. This family contains good power coefficient values but its manufacturability is poor as shown on the next figures.

Chord length distribution:

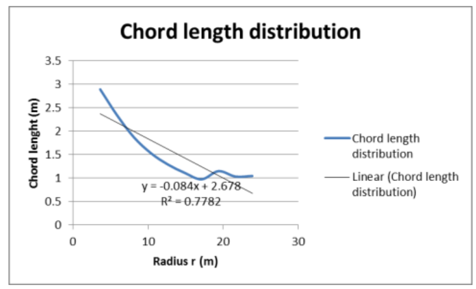

Figure 11
Chord length Distribution for the S815_S812_S813 Airfoil Family

Twist angle distribution:

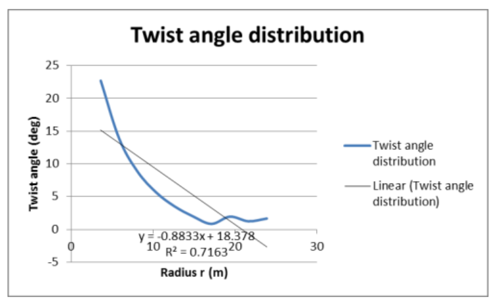

Figure 12
Twist Angle Distribution for the S815_S812_S813 Airfoil Family

Angle of attack (AOA) distribution:

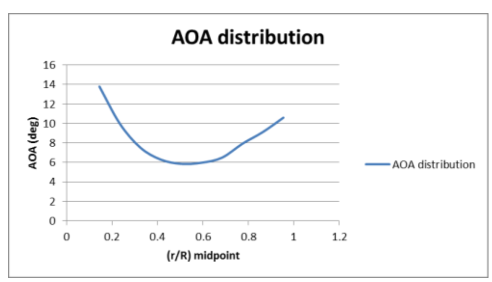

Figure 13
Angle of Attack (AOA) Distribution for the S815_S812_S813 Airfoil Family

Table 3

Computed Data for S815-S812-S13

N	r/R midpoint	r	$\lambda_{r,i}$	C (linear approx.)	Twist (linear)	Optimal Cp	Cl (actual)	AOA	F	Cp (tip loss)	a
1	0.145	3.625	1.015	2.891822679	15.1760375	0.013434136	1.841242074	13.79195953	0.999999998	0.013434136	0.384102
2	0.235	5.875	1.645	2.33687516	13.1886125	0.025157042	1.453368796	9.978062476	0.999999147	0.02515702	0.315713
3	0.325	8.125	2.275	1.866665922	11.2011875	0.035620285	1.204815044	7.534070714	0.99998726	0.035619832	0.289745
4	0.415	10.375	2.905	1.530998456	9.2137625	0.045610759	1.079130234	6.298230266	0.999925246	0.045607349	0.285445
5	0.505	12.625	3.535	1.289849142	7.2263375	0.055681497	1.035083897	5.865123951	0.999694763	0.055664501	0.295646
6	0.595	14.875	4.165	1.111113935	5.2389125	0.065787585	1.046623277	5.978593424	0.998914379	0.065716164	0.315795
7	0.685	17.125	4.795	0.974370864	3.2514875	0.075467497	1.099592331	6.499425168	0.996174688	0.07517881	0.341033
8	0.775	19.375	5.425	1.147713986	1.2640625	0.076758112	1.098757736	7.930227772	0.980332246	0.075248452	0.310269
9	0.865	21.625	6.055	1.033031133	-0.7233625	0.078094399	1.218078316	9.129431751	0.921703471	0.071979879	0.316188
10	0.955	23.875	6.685	1.045046502	-2.7107875	0.07741426	1.395028272	10.60203771	0.664998504	0.051480367	0.306936
						0.549025572				0.515086511	

Power coefficient (Cp) distribution:

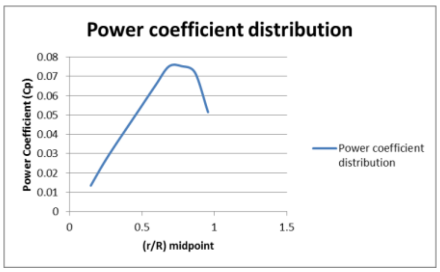

Figure 14
Power Coefficient (Cp) Distribution for the S815_S812_S813 Airfoil Family

Lift coefficient (Cl) distribution:

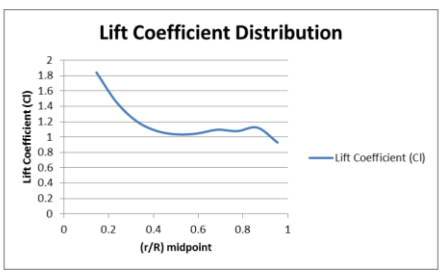

Figure 15
Lift coefficient (Cl) distribution for the S815_S812_S813 airfoil family

4. Selected Airfoils Plots:

S809:

S809 airfoil shape and parameter distributions along the x and y directions.

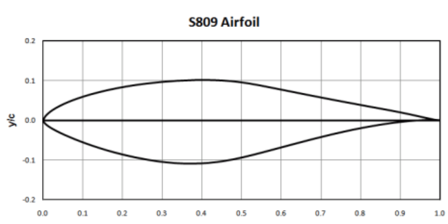

Figure 16
S809 Airfoil Shape

S810:

S810 airfoil shape and parameter distributions along the x and y directions.

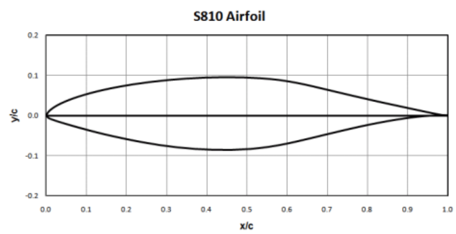

Figure 17
S810 Airfoil Shape

S811:

S811 airfoil shape and parameter distributions along the x and y directions.

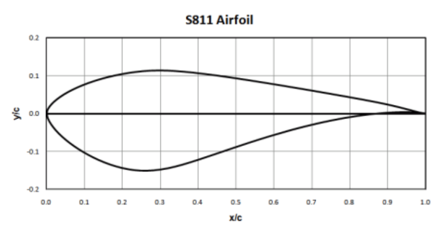

Figure 18
S811 Airfoil Shape

S812:

S812 airfoil shape and parameter distributions along the x and y directions.

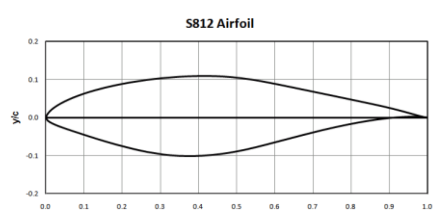

Figure 19
S812 Airfoil Shape

S813:

S813 airfoil shape and parameter distributions along the x and y directions.

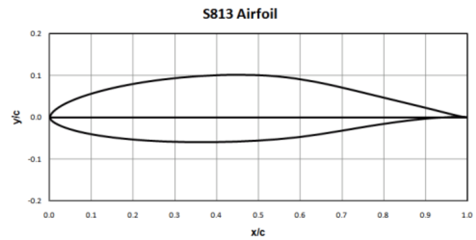

Figure 20
S813 Airfoil Shape

S814:

S814 airfoil shape and parameter distributions along the x and y directions.

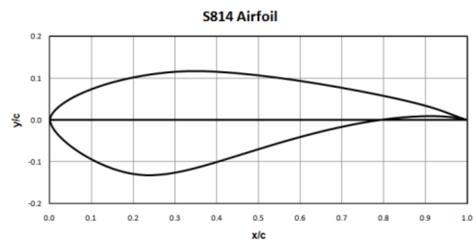

Figure 21
S814 Airfoil Shape

S815:

S815 airfoil shape and parameter distributions along the x and y directions.

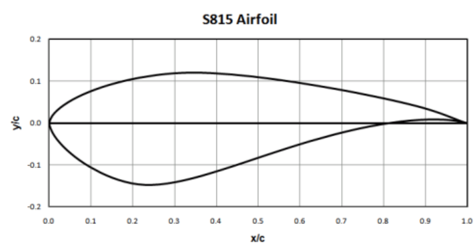

Figure 22
S815 Airfoil Shape

ANALYSIS OF RESULTS AND CONCLUSION

Analysis of results for this particular design takes into consideration chord length, twist angle, angle of attack and power coefficient and lift coefficient distributions. The first three of these parameters (chord length, twist angle and angle of attack) are considering for manufacturing purposes. Both the chord length and twist angle were linearized, since their manufacturing process can run into serious issues if the ideal distributions are used instead. In terms of chord length distribution, the **S814_S812_S813** was our best linear fit from the ideal profile since R squared was about 0.80 as we can see in Figure 6. If we consider the twist angle distribution from Figure 7, the **S814_S812_S813** was also the most reliable airfoil family since R-squared resulted in about 0.76. When considering angle of attack as seen from Figures 8 and 13, both the **S814_S812_S813** and the **S815_S812_S813** airfoil configurations show uniform AOA profiles, which make both of these families acceptable. Up to this point, we have just considered airfoil reliability in terms of manufacturing procedures and we conclude that the **S814_S812_S813** configuration will effectively fit our expectations in terms of manufacturability.

By looking at the lift coefficient curves seen in Figures 5, 10 and 15, it is possible to see that the lift coefficient follows the same trend as the angle of attack up to a span of 90 %. From this span up to the blade's tip the effects of tip losses become more evident. There is a tip loss as we get closer to the tip because the pressure gradient immediately becomes zero (0) at the blade's tip.

Now we consider airfoil performance asides from ease of manufacture. The parameter that will be used for this purpose is the power coefficient including F (the effect of tip losses due to boundary layer pressure gradient at the blade's tip). From power coefficient values shown in Figures 4, 9 and 14, it is seen that power coefficient is about 0.51 for all airfoil families. If tip losses were disregarded, the power coefficient values are 0.53, 0.55 and 0.55 for **S811-S809-S810**, **S814_S812_S813** and **S815_S812_S813** airfoil families respectively. This suggests that tip losses are less for the **S814_S812_S813** configuration. By considering all this information, the most efficient design in terms of both manufacturability and performance would be the one containing the **S814_S812_S813** airfoil family.

CONCLUSION

As it has been seen, the analysis of aerodynamic conditions revealed the **S814_S812_S813** airfoil family to be better suited to work under the current conditions. According to NREL data, the **S814_S812_S813** airfoil family would be more convenient for a rotor about half the size in diameter to the one that is used on this paper. However, we can conclude that this may be due to some other concerns that the ones related to aerodynamic conditions such as design or structural. The most important conclusion that can be drawn from this work is the one related to the influence of tip loss on the overall rotor performance.

Tip loss is due to a sudden drop on the pressure difference between the airfoil's suction and pressure sides which is responsible for the formation of vortices near the tip. This behavior contributes to the reduction on lift coefficient across the airfoil sections from 95% -100% of total span. However, a good design approach definitely helps on obtaining better results. Choosing an appropriate angle of attach is the key towards reducing tip loss effects. Also, the angle of attack distribution helps the designer on obtaining a better overall lift coefficient for the airfoil geometry. It is important to consider that the most efficient blades are the ones made up of several airfoil geometries because they allow the designer to obtain better flow profiles precisely because he/she can pick the proper airfoil geometry for each blade section.

Now that we have discussed how the angle of attack has an effect over the rotor's performance, it is time to state that blade manufacturability plays an important role. This is why it is important to use linearized chord length and twist angle patterns because they allow a smoother transition between the different airfoils that add up to form the blade. As a general conclusion, this paper encourages the designer to use the method outlined on this pages to determine which chord length, twist angle and angle of attack profiles can help optimize both overall power and lift coefficients under ambient aerodynamic conditions. A more in depth analysis of flow conditions over each airfoil would require the use of complex and time consuming CFD methods that may not be necessary at the first stage of the design process.

REFERENCES

[1] J. F. Manwell, J. G. Mcgowan and A. L Rogers, *Wind Energy Explained: Theory, Design and Application*, John Wiley & Sons vol. 1, 2nd ed., 2009, pp. 91-153.

[2] Intech. (2011). *Aerodynamics of Wind Turbines* [Online] Available: http://cdn.intechopen.com/pdfs-wm/16241.pdf html.

[3] NREL. (2012, July 6). *NREL Airfoil Family Data* [Online] Available: https://wind.nrel.gov/airfoils/AirfoilFamilies. html.

[4] R. Lobeto and J. Vázquez, "Design of an Optimum Blade," PUPR, Hato Rey Puerto Rico, February 2011.