Use of genetic algorithms in experimental design optimization and thermal property estimation

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Abstract

The goal of this paper is both to introduce the fields of experimental design optimization and thermal property estimation and to present the Genetic Algorithm (GA) optimization method to the faculty of PUPR. This research environment is part of an ongoing overall research effort conducted at the Virginia Polytechnic Institute and State University (Virginia Tech) with an aim to instrument a complex structure and acquire meaningful thermal property data. Some preliminary work demonstrates that the GA method is a powerful means for both design optimization and parameter estimation.

Uso de algoritmos genéticos para optimizar el diseño y estimar propiedades termales

Sinopsis

Este artículo busca presentar los campos de optimización de diseños experimentales, estimar las propiedades termales y presentar el método de optimización por algoritmos genéticos a la facultad de la Universidad Politécnica de Puerto Rico. Este medio investigativo es parte de un esfuerzo continuo que se conduce en Virginia Polytechnic Institute and State University (Virginia Tech) con el propósito de instrumentar una estructura compleja y adquirir datos significativos sobre propiedades termales. El trabajo preliminar demuestra que el método de algoritmos genéticos (GA) es una herramienta muy útil tanto para optimización de diseño como para estimar propiedades termales.

Introduction

Optimization of experimental designs is crucial to maximize the amount of information that can be obtained from the experiments. In the estimation of thermal properties, the accuracy of the estimates has been demonstrated to increase when the experiments are designed carefully (Beck and Arnold, 1977; Hanak, 1995). The optimal input conditions are typically found by maximizing a single criterion, the D-optimal criterion, which allows thermal property estimates to be obtained with minimum variances. Due to the complexity of an analytical scheme in most cases, the optimization technique typically applied is a stepwise parametric study. However, because it is an iterative process, this technique is tedious and time intensive and therefore restricts the researchers not only from expanding their work to a large number of design variables and more complex designs, but also to the incorporation of additional information or constraints or a multicriteria optimization. In addition, the parametric study does not guarantee the determination of global optima.

The optimal designs are then used in a parameter estimation procedure. In the utilization of new advanced materials, such as composite materials, reliable estimation of thermal properties is extremely important. Indeed, when the composite is subjected to a non-isothermal environment, knowledge of its thermal properties is required to accurately predict thermal stresses and thus prevent component failure. An effective technique for parameter estimation consists of the minimization of the least squares function. The modified Box-Kanemasu method is a predominant method that allows for the parameters to be estimated simultaneously. However, this procedure has encountered unstable behavior, resulting in non-convergence when the parameters are correlated. As correlation exists between the directional thermal conductivities of composite materials, these properties therefore cannot be estimated simultaneously.

The need to be able to instrument a complex structure and acquire meaningful property data has provided the motivation for exploring the use of genetic algorithms (GA) with real number coding in the development of both an optimal experimental design strategy and a simultaneous parameter estimation procedure. This research project is associated with a dual US-French doctoral program between the Heat Transfer Laboratory of the Department of Mechanical Engineering of Virginia Tech, Blacksburg, Virginia, and the *Laboratoire de Thermocinétique de l'ISITEM*, Nantes, France. Some preliminary work conducted at Virginia Tech demonstrates that GA are a powerful means for both design optimization and parameter estimation. Since the present paper focuses on presenting the fields of experimental design optimization and thermal property estimation and describing the GA optimization method, only a brief summary of the results of an application example will be provided. The readers are referred to the literature for detailed results and discussions (García and Scott, 1996, 1997; García et al., 1997).

Literature review

Optimization of experiments

The use of optimization for designing experiments is essential to provide the maximum amount of insight and information on the phenomena being analyzed (Scott and Haftka, 1995). Numerous studies on this topic have been published. Most of them deal with the field of statistical inference and data analysis (Brown et al., 1985); however, an increasing number of publications can be found over the past two decades in the field of engineering design.

When the purpose of the experiment is to estimate parameters, the objective is to design an experiment in which there is minimum correlation between the estimated properties, as well as maximum sensitivity of the measured experimental variables to changes in the properties being estimated (Beck and Arnold, 1977). The selected design variables are sized to provide the best estimates of the desired parameters by maximizing or minimizing an objective function, or optimality criterion, subjected to constraint functions. The optimality criterion is therefore a measure of the goodness of the design. Although its establishment should not be codified in terms of a single recipe, the optimality criterion is usually associated with the Fisher information matrix of the design (Kiefer, 1975a). The Fisher information matrix is defined by $X^T X$, where X is the sensitivity matrix. The sensitivity coefficients are the derivatives of the experimental process variables, such as temperature, with respect to the unknown parameters, for example, the thermal conductivity (Scott and Haftka, 1995). The main optimality criteria include maximization of the determinant of $X^T X$ (D-optimality), maximization of the minimum eigenvalue of $X^T X$ (E-optimality) and maximization of the trace of $X^T X$ (A-optimality). The first criterion is the most common one. The effect of D-optimality is to minimize the volume of the usual confidence ellipsoids of the estimated values, providing the minimum variance estimators (Kiefer, 1981). D-optimality proponents have also pointed out that this criterion is invariant under linear transformation of the estimated vector. However, this criterion has been found to sacrifice much in the values of all but one or two diagonal elements of the Fisher information matrix given by $X^T X$ (Kiefer, 1975b). For instance, in the thermal characterization of honeycomb core structures, Copenhaver (1996) pointed out that the D-optimality criterion would increase the accuracy of some parameters at the expense of creating a large error in others.

Following the selection of the optimality criterion, a mathematical optimization procedure is needed to determine the optimal experimental parameters which satisfy the selected criterion. In the optimization of experimental designs used to estimate parameters, typically two choices are considered: an analytical analysis or a parametric study (Beck and Arnold, 1977). The first method consists of maximizing the objective function by differentiating it with respect to each of the design variables and then solving the resulting set of equations simultaneously for the optimal values of the design variables. Because of the complexity of the equations involved, this method can be very tedious and time intensive. The second method is an iterative approach; it is characterized by first the determination of the practical range of the optimal values, and second by the reduction of this range to obtain the optimal design variables more precisely. As presumed, this methodology can get messy and time intensive for a large number of design variables. However, for only a few design variables to optimize, the parametric study has been found to be efficient with the D-optimality criterion. This technique was applied by Beck (1966) to determine the optimal conditions for the simultaneous estimation of the thermal conductivity and specific heat, and to determine the optimum transient experiment for estimating the thermal contact conductance (Beck, 1969). Taktak et al. (1991) used this procedure to estimate the thermal properties of isotropic composite materials by optimizing the number of sensors, sensor placement, and the duration of an imposed heat flux. Two-dimensional D-optimum experimental designs have also been developed by Moncman et al. (1995) using a parametric study for the simultaneous estimation of thermal properties of anisotropic composite materials. It is relevant to mention at this point that a third method, the GA method, has recently been proven to be highly efficient and well-suited in designing optimal experiments for the estimation of thermal properties. Indeed, the work by García and Scott (1996 and 1997) shows that GA outperform the parametric study. The appraisal of the use of the GA method in the field of experimental design optimization is engaging as genetic agorithms could allow the optimization of both experiments with a large number of design variables (e.g.>3) and more complex designs.

The present state of knowledge should be concluded with the importance for the optimal designs to be verified. This ensures that the best possible estimates have been obtained and allows for the validation of not only the optimization procedure but also the mathematical model used to describe the process. Hanak (1995) demonstrated that the optimal design provided the most accurate combined thermal property estimates by testing the optimal design along with two non-optimal designs. The non-optimal experimental parameters were chosen so that they did not satisfy the D-optimal criterion used in the optimization technique. Hanak's results showed that an individual property might be estimated with greater accuracy at a non-optimal setting but the combination of properties reached a higher accuracy at the optimal setting. In the same manner, Knight et al. (1992) stressed the performance of an optimal air cooled aluminum fin. The two non-optimal designs analyzed maintained fewer and greater fins. Also, verification of an optimal structure against buckling allowed Thompson and Supple (1973) to show that the optimization produced larger imperfection sensitivity. This was the consequence of the optimization of a limited analytical model. Indeed, the analytical model was found not to account for the unavoidable imperfections in the material due to manufacturing.

<u>Estimation of parameters</u>

Once the optimization of the critical experimental parameters has been conducted, the optimal designs are employed in the development of an estimation procedure to determine the desired parameters. In the absence of correlation between the parameters, the modified Box-Kanemasu method has proven to be an effective routine to simultaneously estimate thermal properties. This method is a modification of the Gauss method, which is a first order unconstrained descent method. It is based on the minimization of an objective function containing calculated and measured temperatures (Beck and Arnold, 1977). Scott and Beck (1992a,b) implemented this approach for the estimation of the thermal properties of composite materials during and after curing as functions of temperature and fiber orientation, and for the development of an estimation methodology for thermoset composite materials during curing. Jurkowski et al. (1992) also used this method to simultaneously estimate the thermal conductivity and thermal contact resistance without internal temperature measurements.

The modified Box-Kanemasu method, however has the drawback of being inefficient for use in models which have correlated parameters or a nearly flat sum of squares function, as it may not converge. Indeed, using the two-dimensional optimal designs developed by Moncman et al. (1995), Hanak (1995) could not perform the simultaneous estimation of the effective thermal conductivity perpendicular and parallel to the composite fibers and the effective volumetric heat capacity with the modified Box-Kanemasu method. Note that the investigation of correlation between the parameters was actually considered in the optimization process. The experimental designs were then re-optimized for the restricted simultaneous estimation of the effective thermal conductivity parallel to the fibers and volumetric heat capacity.

This correlation problem has been addressed by different approaches. One approach is to modify the experimental design. For example, Loh and Beck (1991) were able to simultaneously estimate both effective thermal conductivities of anisotropic thermoset carbon composites through the use of nine thermocouples embedded at various locations within the samples. The number and location of the sensors were a priori fixed by the authors, but the potential correlation problem was never detected. This work shows that the use of multiple sensors could be a solution to overcome potential correlation between parameters as more information is provided to the estimation procedure. Nevertheless, modifications to the experimental design, such as the use of internal sensors, are not always feasible, especially when non-destructive testing is required. Another approach is to modify the minimization method. For example, Copenhaver (1996) used a constrained parameter estimation procedure based on a penalty function method with limited success to simultaneously estimate three nearly correlated properties of a honeycomb sandwich structure. Indeed, parameters could be simultaneously estimated for only a specific combination of sensor locations, with the use of a maximum of two sensors, and boundary conditions. An alternative approach is to apply a robust non-gradient optimization technique such as the GA method in the minimization procedure. The potential of this optimization method has indeed been recently illustrated by García et al. (1997), who were able to perform the simultaneous estimation problem unsolved by Hanak (1995) and mentioned earlier. Their work, which includes several case studies, demonstrates that GA is an effective strategy for overcoming correlation between thermal properties and is an effective means of simultaneously estimating thermal properties.

Genetic algorithms

Developed by Holland (1975), these algorithms are based on techniques derived from biology, as their name implies, and belong to the class of computational techniques based on artificial intelligence. They imitate genetic and selection mechanism of nature. Even though they are based on the law of coincidence, they show a steep gradient with regards to improvement, and ensure to a high degree of probability that the global optimum will be found (Krottmaier, 1993). Easily programmed, they require no prerequisites and are not bound to assumptions regarding continuity in the search area, which makes them an excellent probabilistic search tool. These features explain why GA is a better means of optimizing complex problems (defined by several local optima but only one global optimum) than both purely numerical and analytical methods. Indeed, in these latter, the examination of very small, limited areas require enormous efforts; furthermore, they do not guarantee reaching the global optimum. Excellent descriptions of the features of GA is detailed by Goldberg (1989) and Mitchell (1995). In addition, useful information can be found on the Internet (Heitkoetter and Beasley, 1995).

Because of the complexity associated with the traditional binary coding generally used to code design variables, research on GA has been slow to spread from computer science to engineering. The recent demonstrations of the effectiveness of these algorithms with both integer and real number coding should lead to their future utilization for any applications. For instance, as stated earlier, GA with real number coding have been applied with success in the present fields of interest, the optimization of experimental designs for thermal property estimation and the following estimation of the thermal properties. Note that to the author's knowledge, the work by García and Scott (1996 and 1997) and García et al. (1997) is the first known application of the GA method in these fields. In structural optimization, Furuya and Haftka (1993) formulated the problem of optimal locations of actuators on large space structures using GA with integer coding. They showed that the performance of the algorithms with integer coding was at least as good or better than the performance with binary coding. Doyle (1995) also illustrated GA with real number coding to efficiently locate the size and location of a crack in a frame structure. More recently, GA have been applied to fields such as biotechnology, physics, financial forecasting, and art and music, among other applications. The *Handbook of Genetic Algorithms* (Davis, 1991) is a striking demonstration of the possible real world GA applications in industry. In the second part of this book, 13 case studies written by various authors include the use of GA for:

1. Large scale optimizations (such as call routing in a US West telecommunication network, path planning for robot-arm

motion,...etc.)

2. Production systems (evolving strategies for aircraft missile avoidance for instance)
3. Scheduling (for activities in a laboratory)

The first part consists in a tutorial written by Davis in which emphasis is placed on tailoring the GA to the problem at hand. The author also recommends the use of current encoding. Davis's approach is to incorporate domain knowledge into the GA as much as possible, and to hybridize the GA with other optimization methods that work well. A comparable approach, although much simplified, as it does not deal with a hybrid GA, was conducted by Wright (1996). This latter applied a combination of the gradient-based Gauss method and a GA to determine the specific acoustic admittance of the inlet and outlet ports of a combustion chamber. By exploiting the advantages of both techniques, Wright was able to arrive at accurate estimates of the acoustic boundary conditions for nearly any candidate systems.

Another relevant feature of genetic algorithms pertains to their flexibility in the optimality criterion approach. Indeed, the effectiveness of these algorithms in optimizing multiple objectives has recently been demonstrated by Belegundu et al. (1994).

Design optimization and parameter estimation formulations

Design optimization formulation

The objective of design optimization is to select the values of the design variables x_i $(i=1,...,n)$ in such a way that an objective function $f=f(x)$ attains an extreme value within the various constraints. This can be expressed in the abbreviated form

$$\min_{x\in\Re^n}\left\{f(x): h(x)=0,\, g(x)\le 0,\, x^L \le x \le x^U\right\}, \quad (1)$$

where $\Re^n$ is the set of real numbers, f an objective function, $x \in \Re^n$ a vector of n design variables, h a vector of q equality constraints, g a vector of p inequality constraints, and x^L and x^U the respective lower and upper bounds on the values of the design variables. The set of design points that satisfy all the constraints correspond to the feasible domain (Arora, 1989).

The specific case of the optimization of experimental designs used for thermal property estimation is an unconstrained optimization, in which only the bounds x^L and x^U need be specified. The objective function is associated with the accuracy of the estimation. The D-optimality criterion is recommended by Beck and Arnold (1977) because it has the effect of minimizing the hypervolume of the confidence region.

Parameter estimation formulation

Once the optimal experimental parameters have been determined, the optimal designs are used to estimate the desired thermal properties. The estimation problem can be seen as an optimization in which the objective function to minimize is the least squares function S expressed mathematically as

$$S = [Y - T(\underline{\beta})]^T \, [Y - T(\underline{\beta})] \quad \textbf{(2)}$$

where Y is the measured temperature vector, $T(\underline{\beta})$ is the calculated temperature vector, and $\underline{\beta}$ is the exact parameter vector that contains the exact parameter values.

Note that the estimation methodology includes an analysis of the parameter sensitivity coefficients prior to the estimation procedure in order not only to provide insight into the experimental design (Moncman et al., 1995), but also to indicate whether enough information is supplied to accurately estimate the thermal properties.

Application example: Optimization of a one- and two-dimensional experimental design for the estimation of the thermal properties of an anisotropic composite material.

This application example deals with two optimization and estimation problems previously solved in the literature.

Optimization test problems

The experiments investigated include one- and two-dimensional optimal designs performed by Moncman (1994) and Hanak (1995), respectively, for the estimation of thermal properties of composite materials. They both used a parametric study with the D-optimal criterion which implied the maximization of the determinant D^+ of the Fisher information matrix given by $X^T X$. Note that their analysis was performed in nondimensional terms so their results could be applicable for any material. Their experiments were subjected to the constraints of a fixed large number of observations n with uniform spacing in time and the maximum temperature being T^+_{max} reached at steady state. Only a brief summary of their analysis is provided here.

In the one-dimensional analysis, the sides of the composite were insulated while an imposed heat flux was applied across the entire top surface and the bottom surface was held at constant temperature (fig. 1). Moncman sought to optimize three critical parameters which were the sensor location x_s^+, the duration of the heat flux t_h^+ and the overall experimental time t_n^+ for the simultaneous estimation of two properties, the effective thermal conductivity perpendicular to the fibers and volumetric heat capacity (product of density and specific heat). In this case, $D^+_{1\text{-}D}$ was a 2x2 matrix and was given by

$$D^+_{1\text{-}D} = \begin{vmatrix} d_{11}^+ & d_{12}^+ \\ d_{21}^+ & d_{22}^+ \end{vmatrix} \quad (3)$$

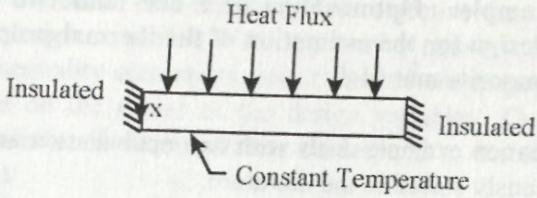

Figure 1. One-dimensional boundary conditions.

In the two-dimensional analysis, Hanak investigated two different experimental configurations (fig. 2). Both had a uniform heat flux applied over a portion of one boundary, with the remainder of the boundary insulated. In addition, Configuration 1 had constant temperatures at the remaining three boundaries, while Configuration 2 had a constant temperature boundary opposite to the heat flux, with the remaining two boundaries insulated. The five critical parameters optimized were the sensor locations perpendicular and parallel to the fibers, x_s^+ and y_s^+ respectively, the duration of the heat flux t_h^+, the heating area L_p^+ and the overall experimental time t_n^+. The objective was to simultaneously estimate three properties which were the effective thermal conductivities along perpendicular planes and volumetric heat capacity. In this case, $D^+_{2\text{-}D}$ was a 3x3 matrix and was expressed by

$$D^+_{2-D} = \begin{vmatrix} d_{11}^+ & d_{12}^+ & d_{13}^+ \\ d_{21}^+ & d_{22}^+ & d_{23}^+ \\ d_{31}^+ & d_{32}^+ & d_{33}^+ \end{vmatrix} \tag{4}$$

For both analysis, the one- and two-dimensional cases, the d_{ij}^+ were found from Beck and Arnold, 1977

$$d_{ij}^+ = \left[\frac{1}{T_{\max}^{+2}}\right]\left[\frac{1}{t_N^+}\right]\int_0^{t_N^+} X_i^+(t^+)X_j^+(t^+)\,dt^+ , \quad i = 1,n , \tag{5}$$

where X_i^+ are the dimensionless sensitivity coefficients associated with the thermal properties being estimated, β_i, and they are calculated from

$$X_i^+(t^+) = \beta_i \frac{\partial T^+(t^+)}{\partial \beta_i} \qquad (6)$$

It is relevant to point out that the optimal overall experimental time of each specific design was determined by evaluating the determinant D^+_m without time-averaging, using the optimal values of the other critical parameters that were obtained applying the parametric study. The optimal overall experimental time corresponded to the time when no significant information about D^+_m was provided. Consequently, in the one-dimensional analysis, the actual number of design variables was two (x_s^+ and t_h^+), and in the two-dimensional analysis, there were actually four design variables (x_s^+, y_s^+, L_p^+ and t_h^+).

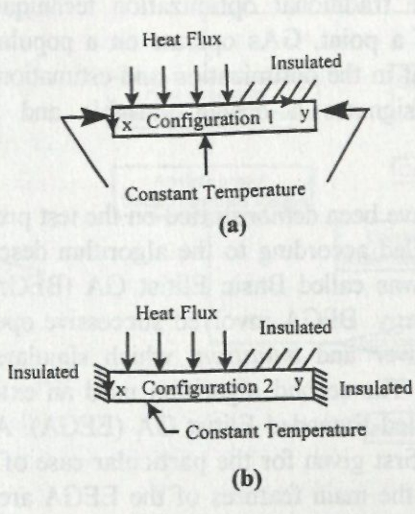

Figure 2. Two-dimensional boundary conditions (a) configuration 1 and (b) configuration 2.

Estimation test problems

The estimation test problems studied include the simultaneous estimation of two and three thermal properties of a composite material using the optimal designs of the one-dimensional and configuration 1 of the two-dimensional experiments described in the previous section, respectively. These problems were investigated by Hanak (1995), who used the modified Box-Kanemasu method for both estimations. In the one-dimensional analysis, the effective thermal conductivity perpendicular to the fibers and the volumetric heat capacity were simultaneously estimated. However, in the two-dimensional analysis, the simultaneous estimation of the effective thermal conductivities along perpendicular planes and volumetric heat capacity could not be performed because of the non-convergence of the estimation technique due to correlation between both thermal conductivities.

<u>The genetic algorithms</u>

Unlike some traditional optimization techniques that work in the neighborhood of a point, GAs operate on a population of chromosomal strings. Note that in the optimization and estimation procedures, a single chromosome designates a design variable and a thermal property, respectively.

Two GAs have been demonstrated on the test problems. The first one, which was modeled according to the algorithm described by Furuya and Haftka (1993), was called Basic Elitist GA (BEGA) because it used a basic elitist strategy. BEGA involved successive operations consisting of *selection*, *crossover* and *mutation*, which simulated the mechanics of natural genetics. The second algorithm used an extended elitist strategy and was thus called Extended Elitist GA (EEGA). A detailed description of the BEGA is first given for the particular case of the optimization test problems. Then, the main features of the EEGA are outlined. Simplified flowcharts of both algorithms are shown in figure 3.

<u>Description of the Basic Elitist GA</u>
Design coding

In the optimization of the one-dimensional experiment, a genetic string describing a particular design contained two chromosomes for the sensor location x_s^+ and the heating time t_h^+. In the same logic, in the optimization of the two-dimensional experiments, each string contained four chromosomes for the design variables x_s^+, y_s^+, L_p^+ and t_h^+. Because the design variables were continuous, BEGA used real string representation. The ranges of these real numbers depended on the lower and upper bounds of the design variables which were specified by the experiments. For instance, the chromosome x_s^+ (dimensionless sensor location perpendicular to the fibers) ranged from zero to one.

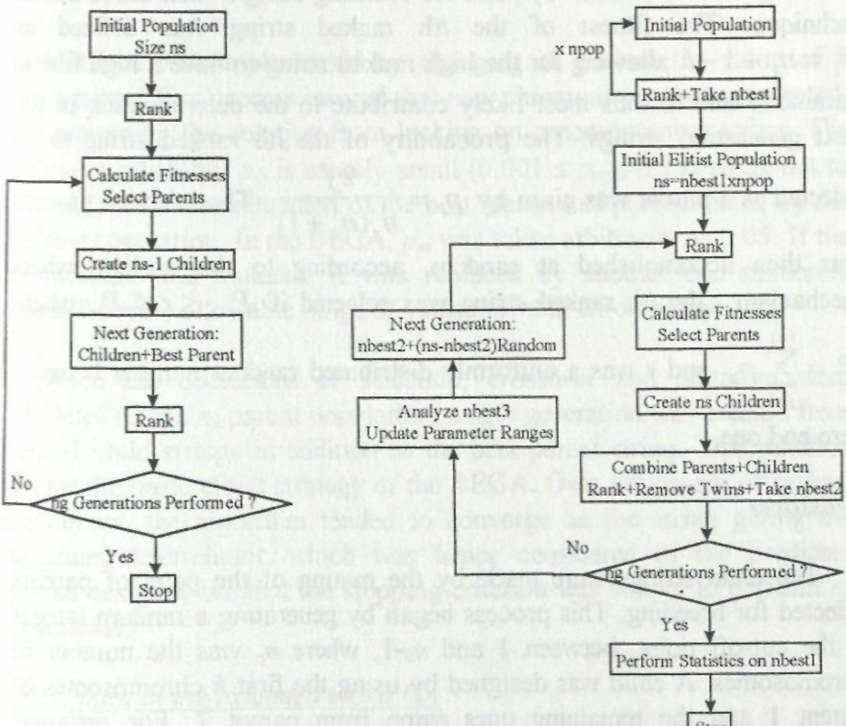

Figure 3. Flow charts of BEGA and EEGA.

Initial population

The optimization algorithm started by generating the initial parent population of n_s candidate strings (designs). Each string was created by randomly selecting n_c chromosome values (design variable) from the design space. The strings were then ranked in terms of the value of the nondimensional determinant D^+ using the D-criterion. Obviously, the best string had the highest D^+.

Selection

Parents were selected by pairs for breeding using a rank-based fitness technique. The fitness of the *i*th ranked string was defined as $f_i = n_s + 1 - i$ allowing for the high ranked string to have a high fitness parameter and to thus most likely contribute to the determination of the next generation strings. The probability of the *i*th ranked string to be selected as a parent was given by $p_i = \dfrac{2f}{n_s(n_s+1)}$. The selection process was then accomplished at random, according to the roulette wheel mechanism: the *i*th ranked string was selected if $P_{i-1} \le r \le P_i$, where $P_i = \sum_{j=1}^{i-1} p_j$ and r was a uniformly distributed random number between zero and one.

Crossover

The child strings were made by the mating of the pairs of parents selected for breeding. This process began by generating a random integer *k*, the cut-off point, between 1 and n_c-1, where n_c was the number of chromosomes. A child was designed by using the first *k* chromosomes of parent 1 and the remaining ones came from parent 2. For instance, consider in the one-dimensional analysis the strings with x_s^+=0.5, t_h^+=1.0 and x_s^+=0.7, t_h^+=1.5 as parents 1 and 2, respectively. As n_c=2 (recall that there are two design variables in the one-dimensional analysis), the only possible child string could be x_s^+=0.5, t_h^+=1.5. This process is the simplest crossover, the so called single-point crossover. Note that there exist more elaborated variants of this operation which are used with efficiency when the genes are all of the same kind [for instance, they represent location of actuators, see Furuya and Haktka (1993)]. However, these variants may not be adequate when the genes represent different parameters (such as the location of sensors, the heating time, the total experimental time, ...). Furthermore, because the primary goal in the study of these test problems was to test the effectiveness of the genetic algorithm method comparatively to the parametric study in the optimization of experimental designs, the simplest crossover operation was to be used.

Mutation

Mutation was implemented by changing at random the value of a chromosome. This process insured that new chromosomes were generated, thus preventing the solution from locking on a non-optimum value. The mutation probability p_m is usually small ($0.001 \leq p_m \leq 0.15$) so as not to interfere with the combination of the best features of parents made by the crossover operation. In the BEGA, p_m was taken arbitrarily as 0.05. If the chromosome was mutated, it was replaced by another one randomly chosen from the allowable range of values for that chromosome.

When the operations of selection, crossover and mutation were completed on the n_s parent population, a new generation was created from the n_s-1 child strings in addition to the best parent string. This addition denotes the basic elitist strategy of the BEGA. Over the course of several generations, the algorithm tended to converge on the string giving the maximum determinant, which was hence considered as the predicted optimal design. Note that the stopping criterion was simply to perform n_g generations.

<u>Description of the extended elitist GA</u>

This algorithm was developed to improve the efficiency of the BEGA. Following is an outline of the five main differences between the EEGA and the BEGA:

1. A pure random search is initially performed to obtain good starting conditions for the lower and upper bounds of each chromosome (parameter to be optimized/estimated). The purpose of this seeding is therefore to help direct the genetic algorithm search. Note that this initial search is run separately from the EEGA run.

2. The EEGA starts by randomly searching a number of n_{pop} initial population in which only the n_{best1} first ranked strings are kept. This produces an initial elitist population of size $n_s = n_{pop} \times n_{best1}$.

3. n_s children are created and then combined to the parent population. After ranking this combined population of size $2n_s$ the "twins" are removed and only the n_{best2} first ranked strings are kept, where $n_{best2} < n_s$.

4. The parameter ranges are updated from the analysis of the n_{best3} first ranked strings, where $n_{best3} \leq n_{best2}$. The next generation comprises, therefore, the $n_{best2} + (n_s - n_{best2})$ random strings generated from the just updated parameter ranges. This addition allows some "new blood" in the population and prevents the EEGA from premature convergence on a non-optimal string.

5. When n_g generations have been performed, some statistics are performed on the n_{best1} first ranked strings. This enables the computation of the means and their 95% confidence intervals for each chromosome of these n_{best1} strings. The 95% confidence intervals computed are a good evaluation of the convergence of the EEGA.

When EEGA is used as an estimation procedure, the output for a particular experimental data set includes the means of each chromosome (thermal property) and the 95% confidence intervals representing the ranges of values which the actual properties lie within for that particular experiment.

<u>Summary of results</u>

In the optimization of the experimental designs the results from these two test problems indicated that the computational efficiency of the EEGA was much higher and the results were slightly better than those of either the BEGA or the parametric study. Therefore, by keeping the best information generated throughout the search process, the EEGA proved to be well-suited for optimizing experiments designed for thermal property estimation. In the estimation of the thermal properties, the results demonstrated that the EEGA is also an effective strategy for the non-linear simultaneous estimation of correlated properties. The readers should consult the work of both García and Scott (1997) and García et al. (1997) for detailed results and discussions. Note that in these papers, it is pointed out that the excellent results obtained by the EEGA in these specific optimization and estimation test problems do not imply that this algorithm would also excel for every other application. Indeed, the scheme used in a GA could by no means be quantified in terms of a single recipe and the performance of a GA can be viewed as application-specific. Therefore, there are no a priori best GAs but good GAs that are tailored to the problem at hand (Davis, 1991).

Conclusions

The focus of this paper was on both presenting the fields of experimental design optimization and thermal property estimation and describing the genetic algorithm (GA) optimization method. One example of the application of the GA method to these two fields was supplied and the results were briefly stated. This example dealt with two optimization and estimation problems previously solved in the literature. The study of these test problems allowed assessment of the use of genetic algorithms (GAs) as a procedure for both the optimization of experiments designed for thermal property estimation and the following simultaneous estimation of the thermal properties. The methodology developed exploits both the elitist feature and the non-gradient nature of the GA method to overcome the main limitations of the conventional optimization and parameter estimation techniques (García and Scott, 1997 and García et al., 1997).

The enhanced elitist GA described in this paper has also been demonstrated in other case studies to be a flexible, powerful and easy-to-apply, all-purpose algorithm (García et al., 1997).

This research, which is associated with a dual US-French Ph.D. program between the Heat Transfer Laboratory of the Department of Mechanical Engineering Department of Virginia Tech, Blacksburg, Virginia, and the *Laboratoire de Thermocinétique de l'ISITEM*, Nantes,

France, is currently directed toward the enhancement of this innovative procedure based on GAs. Future research is projected on its application to the thermal characterization of composite materials during curing.

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