Nonlinear Static Analysis and Evaluation of an Existing Structure

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Abstract *— A seismic evaluation was performed to an existing structure in the city of Carolina, who is a municipality located on the northeast coast of Puerto Rico. It is a concrete structure of two levels with a lateral resistance system Special Reinforced Concrete Moment Frame in both directions. The structure was evaluated in different ways, the main evaluation is an iterative Non-linear Static Analysis. We performed this analysis using geometric nonlinearity "P-Δ" included in ETABS program and the materials nonlinearity using constructive equations of concrete and steel to create the curve moment versus curvature for each element. An evaluation of the seismic details requirements was performed by comparing the ACI 318-95 code designed with the current code ACI 318-14. We also evaluate using the ASCE 41 and FEMA 356 code for existing structures using a Static Nonlinear Analysis "Pushover". As a result, the structure meet the demand of the Static Nonlinear Analysis, but did not comply with the requirements especially seismic shear reinforcement and joints shear. ASCE 41 evaluation comply with the performance determined which is not necessary structural retrofitting and post-earthquake the structure is capable to resists gravity load in service.*

Key Terms *— Nonlinear Static Analysis, Pushover Analysis, Seismic Evaluation Procedure, Structural Design Codes.*

INTRODUCTION

In the evaluation by Linear Static Analysis, iterative Nonlinear Static Analysis and Nonlinear Static Analysis "Pushover" we can observe the behavior of the structure for lateral loads. It can be considered that the structure that we will evaluate it is a recent structure because was designed to resist lateral loads. The design codes were: ACI 318-95 and UBC-97. We evaluate the nonlinear behavior based on the geometric nonlinearity and materials nonlinearity. Also, how the load will be distributed on a structure that is designed to earthquake and what would be the performance. When perform a Nonlinear Static Analysis by iteration or when we do an evaluation of ASCE 41 using the "pushover" analysis we can see how the stiffness and the required seismic details affect the behavior of the structure. With this analysis we will know if the structure have deficiencies and need to retrofit elements to reach the performance level required.

DESCRIPTION OF STRUCTURE

The structure consists of two levels with exterior precast walls attached to the frames and bearing its own weight. The lateral resistant system used are Special Moment Frame in both directions, the frame in "X" direction support the post-tension slab and resist lateral loading, and the frames in "Y" direction are used for lateral resistant frame only. The post-tension concrete slab act like rigid diaphragm to transfer the load to the frames.

The height of the structure is 13'-0" each floor and the foundation are 3'-0" below the existing grade. The dimension are 192'-0"x36'-8" with an area of 7040 ft^2. The gravity load are supported with post-tension slab and two frames in one direction. The system to resist lateral load consist in two special moment resistant frame in "X" direction and three special moment frame in "Y" direction. The base support are mostly square isolated foundations.

The gravity loads at the second level are 10 lb/ft^2 superimposed and 65 lb/ft^2 live, at the roof 20 lb/ft^2 live and 100 lb/ft^2 in the corridors.

The properties of material used are the followings: for post-tension slab the concrete strength is 3500 psi and for the rest of the structure

3000 psi. The steel reinforcement ASTM 615 Grade 60 and the tendon are ½" diameter, 7 wire-270 grade and low relaxation.

STRUCTURAL ANALYSIS METHODOLOGY AND PARAMETERS OF LINEAR STATIC ANALYSIS

Before we perform a nonlinear static analysis, we need to make a Linear Static analysis to compare it with the final results. In both cases we need to calculate the loads of the structure by the static method using the ASCE 07-10 and IBC 2009 code [1]-[2].

In the following we show the calculations and parameters needed for earthquake loads following the code. These loads were used for the all analysis performed in this investigation except for the pushover analysis.

Table 1
Mass and Rigidity Properties

Level	Weight	XCM	YCM	XCR	YCR
	Kip	ft.	ft.	ft.	ft.
Roof	1329.80	96.36	21.16	37.23	24.16
Level 1	1777.00	95.42	19.71	43.15	23.90
Total	3106.80				

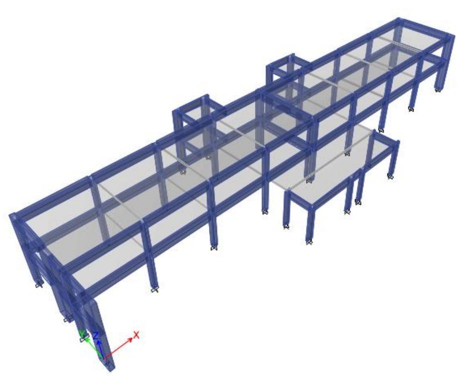


Figure 1
Analytic Model from ETABS

The parameter required by the ASCE 07-10 code to calculate the response spectrum and the static load are the followings [1]:

- The basic system is moment resistant frame-special moment frame of reinforced concrete,
- Response coefficient is R=8
- Factor of over-strength is Ω=3
- Amplification deflection is Cd=5.5
- The structure do not has height limits
- Soil type is D
- Location – Carolina
- Occupational Category – III
- Importance – 1.25
- Puerto Rico Building Code Spectral Response accelerations, Ss = 0.88 and S1=0.31

With these parameters and using the equation and tables of ASCE 07-10, we determine the site coefficients, Fa=1.15 and Fv=1.78, to calculate the design response acceleration parameters, Sds =0.67 and Sd1=0.552, and determine the seismic design category that is SDC-D. With these parameters and periods we can plot the following design response spectrum.

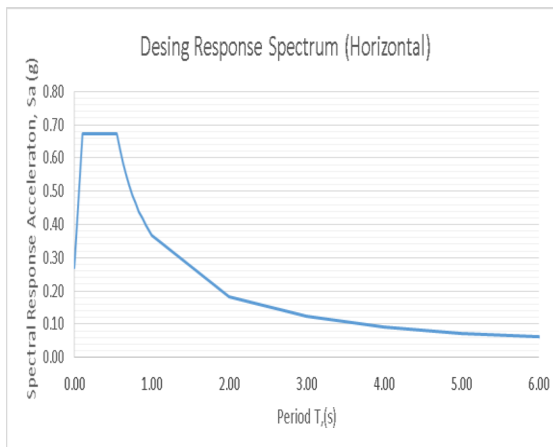


Figure 2
Design Response Spectrum

Finding the period of the structure and the design spectrum we can determine the acceleration and calculate the equivalent lateral load. In our case the period is Ta=0.464 and the equivalent load is V=326.9 kips.

Table 2
Vertical Distribution of Seismic Forces

Floor	H	Weight	wi*hi^k	Cvx	Fx
	ft	kip			kip
Roof	26.09	1329.8	34694	0.599	195.7
Level 1	13.09	1777.0	23261	0.401	131.2
Total		3106.8	57955		326.9

NONLINEAR STATIC ANALYSIS

The method of nonlinear static analysis used in this part of the project considers the differences in the rigidities and deformations of each an independent element. We will modify the inertia iteratively for each element depending on the load that absorb the element and the degradation of it inertia. The forces and moments in which the existing structure will be evaluated are earthquake loads in service.

Finally, we have not to iterate again because inertias of element do not reduce anymore, we use the latest effective inertia and evaluated the capacity of the element with ultimate loads combinations.

In order to calculate the effective inertia of the elements we need constructive equations that describe the behavior of concrete and steel. For the concrete we will use Todeschini; this only use one equation that is simpler than the accepted by ACI Hognestad that used two equations to describe the behavior of concrete, both are for unconfined concrete.

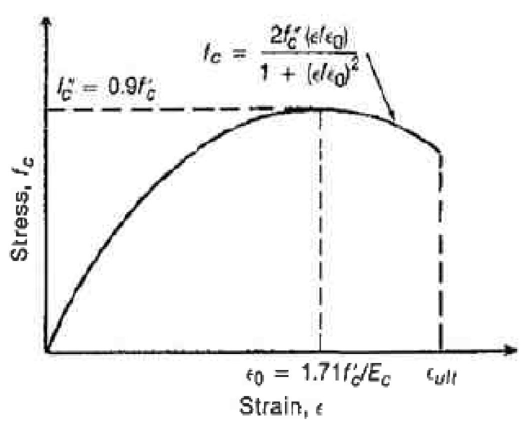


Figure 3
Stress-Strain of Concrete by Todeschini

In the case of steel we use the bi-linear equation and the equation by Park. They are equal until reach the yield point, the bi-lineal still constant and the park increase until failure. We were used in the program ETABS Section Designer the equation of Park and with a MCURV the bi-linear equation [3]-[4].

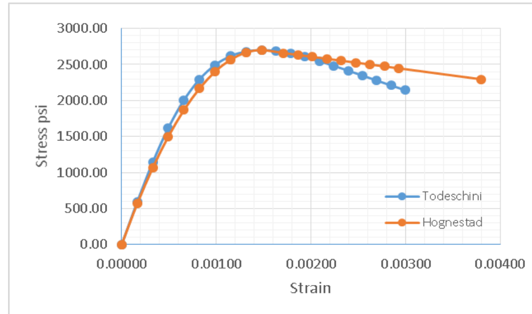


Figure 4
Comparison of Todeschini and Hognestad Stress vs Strain

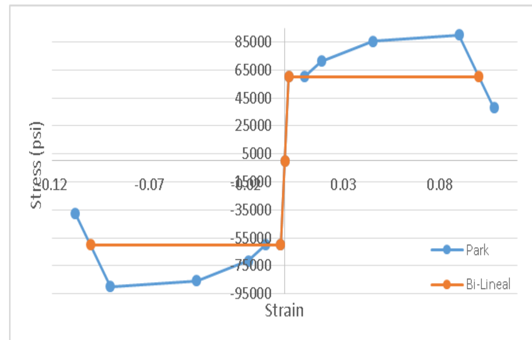


Figure 5
Comparison of Bi-lineal and Park

We need both equations mentions before of concrete and steel materials to generate a moment vs curvature for a specific structural section. Using the moment-curvature and the demand of moment we can calculate the effective inertia of element. We compare these results with ETABS program that used Todeschini and Park with a MCURV created by the author using Todeschini and bi-linear. It is important to know that if we used an equation of concrete that considered confinement the strength increase significantly.

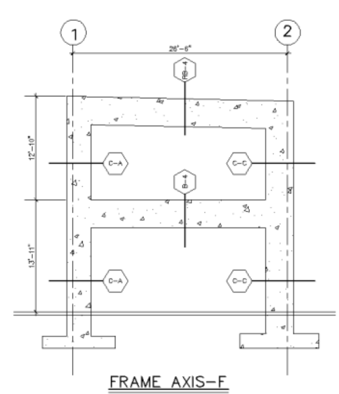


Figure 6
Frame for Beam and Column Example

We will take one frame for example and find the moments vs curvature of one beam and column in the analysis we has to made it for all section.

PROCEDURE OF NONLINEAR ANALYSIS

The Nonlinear Static Analysis performed consider the geometric nonlinearity and the material nonlinearity, to apply the material nonlinearity we started the analysis using the inertia gross in all elements. After we made the first analysis, we used the moments of curvature for beam in the ends using the negative moment and in the mid-span using the positive moment. For the example we used use the section beam B-4 for positive moment.

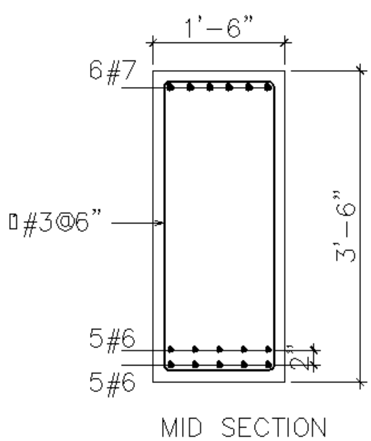


Figure 7
Section Beam B-4 at Mid-Span

In Figure 8 we can see the comparison of moment vs curvature between the following programs: ETABS and MCURV [3]-[4]. As we can see they are very similar until it reaches the area of yield, how we mention before they used the same concrete equation but the equation of steel are different, ETABS use Park and the other the bi-linear.

Using the moment of curvature of ETABS and the demand of moment in the analysis performed with service load, we solve for the effective Inertia using the following equations.

$$\varphi = Ms/EI \quad (1)$$

Solving for effective inertia;

$$Ie = Ms/E\varphi \quad (2)$$

$$fcr = 7.5\sqrt{fc} \quad (3)$$

$$Mcr = S \times fcr \quad (4)$$

With the table of moment vs curvature for the section we can interpolate for each element the curvature "φ" and with the maximum moment of the service combination "Ms" and using the equation (1) we can calculate the effective Inertia.

This is always correct only when the moment is greater than the cracking moment that we can calculate using the equations (3) and (4), if the moment is lesser than "Mcr", the inertia still full. To apply the inertia for each element we write in term of ratio Ie/Ig. That mean if the inertia is full the ratio is 1.

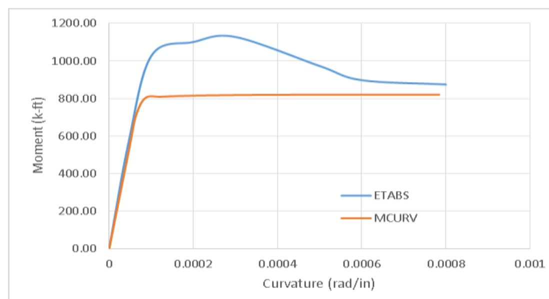


Figure 8
Comparison of Moment-Curvature for B-4 at Mid-Span

For a typical beam we can calculate the effective inertia in three different location two at the top with negative moment and one at mid span with positive moment. The ACI 318-14 at chapter 24 permit the use of average of the effective inertia [5]. For our study we will take result of less inertia.

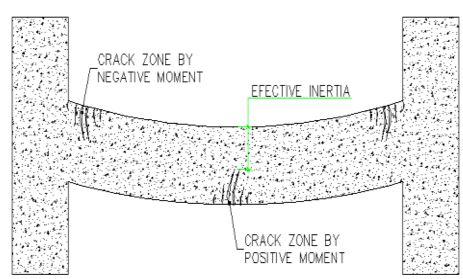


Figure 9
Location of Cracking in a Typical Beam

Before we have the ratio of all element we introduce in the ETABS program and run again, when we change the rigidity of the elements will occur a redistribution of forces and the elements that maybe reduce the inertia absorb less moments and the elements that no affect the inertia receive more moment in the next iteration. We made the iteration four time until any beam or column reduced the effective inertia between the last two analyses.

Below we show the results of beam before all iteration, if one ration increase we do not change the ratio in the model because the element cannot recover.

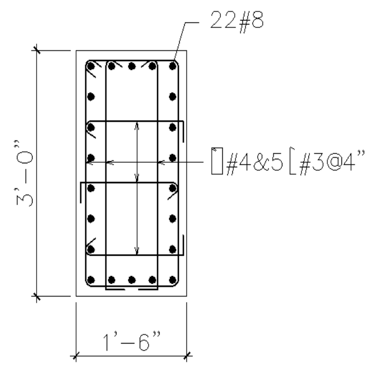


Figure 10
Section of Column C-A

The procedure for the column was the same, the only difference is the location that we took the moment to evaluate the effective inertia. We took at bottom and top, but we have two directions and we reduced the inertia of each direction separately. We selected the minor effective inertia between bottom and top for each direction. Below we will show the moments of curvature for the column C-A, with the assumption that the axial load was equal to cero. This assumption is more conservative and the curve of moment vs curvature between the two programs was very similar.

Table 3
Ratio of Ie/Ig for Each Iteration at Level 1

Beam	1	2	3	4
B1	0.25	0.25	0.25	0.25
B2	0.25	0.25	0.25	0.25
B3	0.25	0.25	0.25	0.25
B4	0.25	0.25	0.25	0.25
B5	0.25	0.25	0.25	0.25
B6	0.25	0.25	0.25	0.25
B7	0.25	0.25	0.25	0.25
B8	0.25	0.25	0.25	0.25
B10	0.20	0.20	0.20	0.20
B11	0.20	0.20	0.20	0.20
B12	0.25	0.25	0.25	0.25
B13	0.25	0.25	0.25	0.25
B14	0.20	0.20	0.20	0.20
B15	0.20	0.20	0.20	0.20
B16	0.20	0.20	0.20	0.20
B9	0.20	0.20	0.20	0.20
B18	0.15	0.15	0.15	0.15
B19	0.57	0.57	0.42	0.42
B23	1.00	1.00	1.00	1.00
B24	1.00	1.00	1.00	1.00
B21	1.00	1.00	1.00	1.00

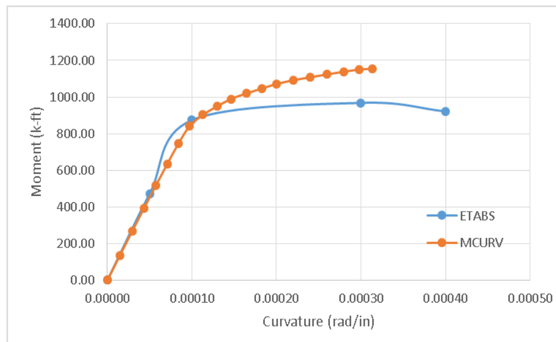


Figure 11
Comparison of Moment-Curvature for Column C-A.

Results of Nonlinear Analysis

After we made all iteration we compare the moments and shear for all element with linear static analysis, in general we have difference moment in the element if we review the values of the beams in the first level we see that the more rigid element increase the final moment in this case in the beam B21 as we can see in Table 3 that element maintain it inertia full in al iterations. Then we design the element they still having the capacity for the new demand. We realize the one of the reason is because the structure is overdesign.

Evaluation of Seismic Category

The seismic is base with the seismic chapter of ACI “American Concrete Institute” The project was designed with the ACI 318-95, we compared the requirements with the latest code ACI 318-14.

Some change between both codes is that the new code was reorganized and the seismic chapter in the ACI 318-95 was in the chapter 21 and in the new code is in the chapter 18.

Component resisting earthquake effect, unless otherwise noted	Seismic Design Category			
	A (None)	B (21.1.1.4)	C (21.1.1.5)	D, E, F (21.1.1.6)
Analysis and design requirements		21.1.2	21.1.2	21.1.2, 21.1.3
Materials		None	None	21.1.4 - 21.1.7
Frame members		21.2	21.3	21.5, 21.6, 21.7, 21.8
Structural walls and coupling beams		None	None	21.9
Precast structural walls	None	None	21.4	21.4,† 21.10
Structural diaphragms and trusses		None	None	21.11
Foundations		None	None	21.12
Frame members not proportioned to resist forces induced by earthquake motions		None	None	21.13
Anchors		None	21.1.8	21.1.8

*In addition to requirements of Chapters 1 through 19, except as modified by Chapter 21. Section 22.10 also applies in SDC D, E, and F.

†As permitted by the legally adopted general building code of which this Code forms a part.

Figure 12
Seismic Requirements ACI 318-14

TABLE R21.2.1—SECTIONS OF CHAPTER 21 TO BE SATISFIED*

Earthquake risk level†	High	Moderate
Frame members resisting earthquake effects	2,3,4,5	8
Walls, diaphragms, and trusses resisting earthquake effects	2,6	None
Frame members not resisting earthquake effects	7	None

* In addition to requirements of Chapters 1-17 in regions of high risk and Chapters 1-18 in regions of moderate risk,

† The terms refer to regions with earthquake risk identified in building codes such as American Society of Civil Engineers standard "Minimum Design Loads for Buildings and Other Structures," ASCE 7-88 (formerly ANSI A58.1) and Uniform Building Code.[21.8] Regions of high earthquake risk correspond approximately to Zones 3 and 4, and regions of moderate earthquake risk correspond approximately to Zone 2 in both documents.

Figure 13
Seismic Requirements ACI 318-95

If we observe Figure 13 for the code ACI 318-95 they has only two risk level which are high and moderate, it has less elements that has to comply with the requirement in the chapter [6]. In difference for new code that divide in four seismic design categories A, B, C,D,E and F. The lasts has more element to evaluate compared with the old one.

For the code ACI 318-95 the code to determinate the seismic zone and category was the Uniform Building Code [6], were in Puerto Rico we are zone 3, if we read the note below Figure 13 we are in the high risk category and in the code ACI 318-14 we determine the seismic design category base in the International Building Code and ASCE 07, for Puerto Rico the seismic design can varies between the categories C and D [5].

In ACI 318-14 the seismic design category A, do not has seismic requirement, just only design from chapter 1 to 17 [5]. The seismic design categories B and C are classified as Ordinary and Intermediate Moments frame respectively and they are equivalent to moderate risk level in the ACI 318-95. Seismic design category D, E and F they are classified as Special Moment Frame and is the equivalent to High risk level in the ACI 318-95.

Other difference that we have is the combinations and reduction factors.

Combinations ACI 318-95		**Combinations ACI 318-14**	
U= 1.4D+ 1.7L	(9-1)	U= 1.4D	(5.3.1a)
U = 1.05D + 1.28L + 1.40E	(9-2)	U=1.2D+1.6L+.5Lr	(5.3.1b)
U = 0.90D + 1.43E	(9-3)	U=1.2D+E+L	(5.3.1e)
		U=0.9D+E	(5.3.1g)

Reduction Factors (ϕ)	**ACI 318-95**	**ACI 318-14**
Moment, Axial Force or combination	0.9 a 0.7	0.9 a 0.65
Shear and Torsion	0.85	0.75
Bearing	0.70	0.65

Figure 14
Comparison of Combinations and Reduction Factors

For the evaluation we select a typical span of the frame in "1"axis and the column C-A. In summary for the beam satisfy all requirement in the code ACI 318-95, except that has two continue bar in all the span developed [6]. Compared with the new code the requirements are almost the same except that they change the spacing near the joint in the beam, before the minimum spacing was less of d/4, 8 times diameter of small longitudinal bar and 6" in the new code they change the second one 6 time the diameter of bar. For this code, the minimum spacing is 4.5" and we have 6". Either satisfy the demand of shear in special moments frame by the induced shear, because the combination factor in the ACI 318-95 are greater for gravity and the induced shear do not represent more than the half of the shear and we can considered the concrete resistance and in the new we have to neglect.

For the evaluation of columns we satisfy almost every in the ACI 318-95 except the splice of column to the second floor, they are made in the face of joint and in the ACI 318-14 the requirements of shear by the special moment frame is not satisfied because the reduction factor of the shear is more conservative.

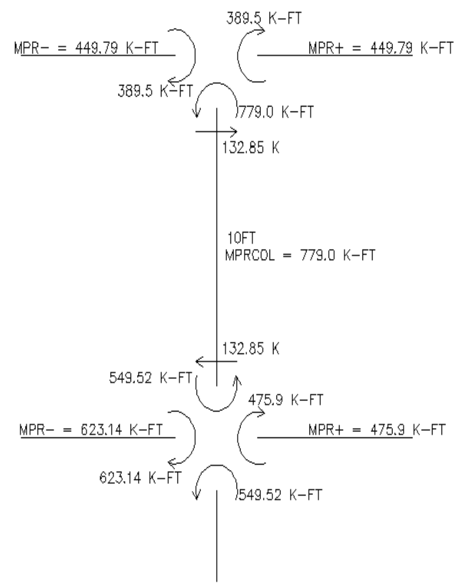


Figure 15
Joint Evaluation for Column C-A in Axis F

We evaluate it in the same frame the column C-A in "F"axis. The evaluation in this location was for joint requirements. In benefit that we made the preview free body of forces we can determinate that the frame do not satisfy strong column and weak beam in the roof joint and the capacity of the joint in the second floor cannot satisfy the demand.

The following computes are from the joint of the second floor and the free body of the diagram is the Figure 16.

$$Vx - x = T2 + C1 - Ve = 293.15kip \quad (5)$$

$$\varphi Vn = \varphi 15\lambda\sqrt{fc}Aj = 199.65kip \quad (6)$$

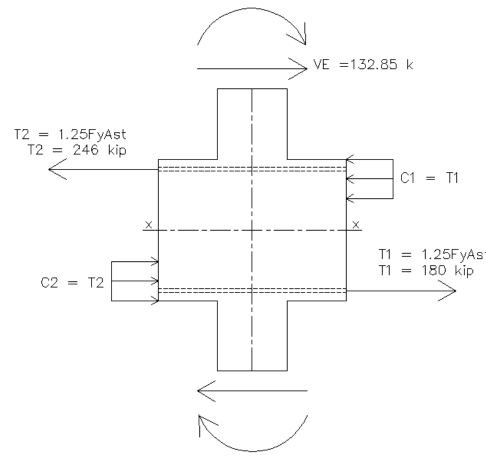


Figure 16
Free Body Diagram of Forces at Joint

EVALUATION OF EXISTING STRUCTURE WITH ASCE 41 AND FEMA 356

To perform the evaluation and pushover analysis, we started with an effective inertia using the following service load combination: D + 0.6L. We compute the inertias using the equations provided by ACI 318 code:

$$Ie = (\frac{Mcr}{Ma})^3 Ig + [1 - (\frac{Mcr}{Ma})^3]Icr \quad (7)$$

$$Ie = (0.8 + 25\frac{Ast}{Ag})(1 - \frac{Mu}{Puh} - 0.5\frac{Pu}{Po})Ig \quad (8)$$

To calculate the effective inertia for a beam we used the equation (7) and for a column the equation (8).

We follow a provided flow chart from ASCE 41 and FEMA 356 to accomplish the process of evaluation step by step [7]-[8]. We first selected the perform objective, this selection was decided by the architect, engineer or own. The probability of exceedance selected is 5% in 50 year "BSE-2E" Basic Safety Earthquake-2 for existing structure, the hazard of this earthquake can be greater than "BSE-2N" which is for a news structures design equivalent for maximum considered earthquake used by ASEC 07 [1].

The tier is a process of evaluations, tier 1 and tier 2 consist in visual evaluation and check list if it necessary through the evaluation is required an analysis, but if the building has high level of seismicity we shall evaluate the structure by tier 3 or the engineer can choose directly as conservative.

The Basic Performance Objective for the Building (BPOB) is Limited Safety 4-D and we do not has to considered nonstructural elements performance. For this level we shall comply with the damage control and building performance level between the targets Life Safety and Collapse Preventions.

To define the seismic hazard and the level of seismicity we search the acceleration response spectrum from United State Geological Survey which have a program [9]. We input the coordinate and specified the report of ACSE 41-13 Retrofit Standard, BSE-2E and provide the acceleration to full report by the code, choosing the minor accelerations between BSE-2N and BSE-2E.

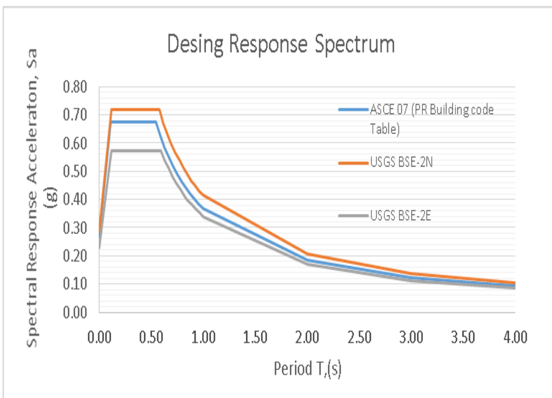
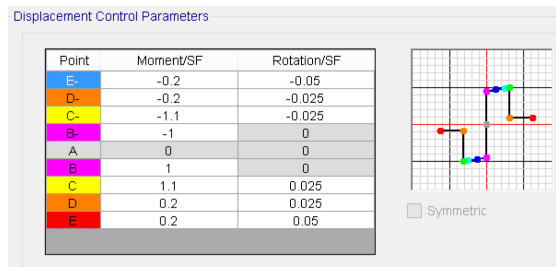


Figure 17
Comparison of Design Response Spectrum

In the Figure 17 we can see that BSE-2E acceleration are lowers than BSE-2N and the curve of ASCE 07 is the same of Figure 2 calculated for static load. The difference between the BSE-2N and ASCE 07 is the source of acceleration. The seismicity level determined is high equivalent to the seismic design category D in ASCE 07 [1].

PUSHOVER ANALYSIS

The pushover is a Nonlinear Static Analysis that consider the geometrical and material nonlinearity, to perform this analysis we have to create a hinge in each extreme of element. The parameter are based in the rotation capacity and moment of section, the ASCE 41 provide tables from chapter 8 to 12 depending of material [7]. We used the advantage that ETABS have internally the tables and using last run of design and analysis can created the hinges for each element, as shown in Figure 18 [3].


Displacement Control Parameters

Point	Moment/SF	Rotation/SF
E-	-0.2	-0.05
D-	-0.2	-0.025
C-	-1.1	-0.025
B-	-1	0
A	0	0
B	1	0
C	1.1	0.025
D	0.2	0.025
E	0.2	0.05

Figure 18
Moment vs Rotation Parameters for a Beam

The nonlinear static procedure or pushover is approximation of the response a structure will undergo when subjected to dynamic earthquake loading. The static approximation consists of applying a vertical distribution of lateral loads to a model which captures the material nonlinearities of an existing or previously designed structure, and monotonically increasing those loads until the peak response of the structure is obtained on a base shear vs. roof displacement as shown in Figure 19.

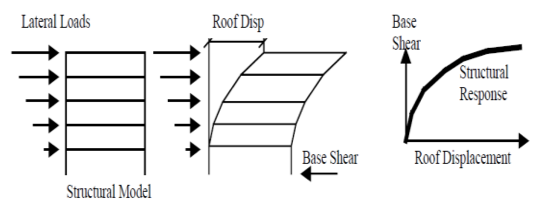


Figure 19
Static Approximation Used in the Pushover Analysis

From the response spectrum and base shear vs. roof displacement plot, the target displacement was determined. The target displacement represents the maximum displacement the structure will undergo during the design event.

In Figure 20 we shown the result of shear base vs displacement in “Y” direction of the building represented by the green line. The red line is the idealized force displacement developed using an iterative graphical procedure to balance the areas below the actual and idealized curves up to the displacement of maximum shear or target displacement. To calculate the target displacement we use the equation (9), and parameter of Table 4.

$$\delta d = C_o C_1 C_2 S_a \frac{T_e^2}{4\Pi^2} g \tag{9}$$

Table 4
Calculated Parameter

C0	1.25	Sa	0.859 g
C1	1.17	α	0.077
C2	1.03	R	3.63
Ti	0.48 sec	Dy	0.57 in
Te	0.49 sec	Vy	681.20 kip
Ki	1260.70 kip/in	Weight	2882.82 kip
Ke	1194.86 kip/in	Cm	1

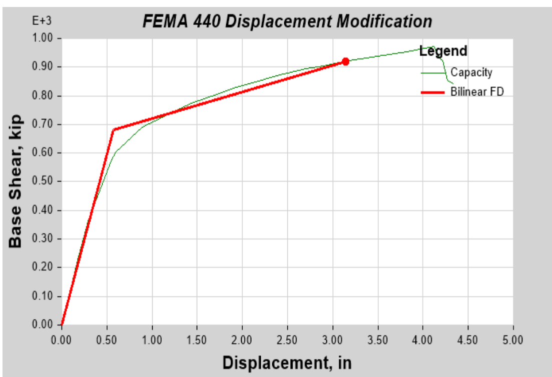


Figure 20
Displacement Modification in Y direction

The structure should satisfy the range of acceptance criteria in the hinge for the selected performance at the target displacement. The hinge will change the color for each step of displacement in ETABS that represent the status of the hinge in the acceptance criteria. Those status are represent in the normalized force vs deformation of element hinge. For ETABS at point B is purple, blue for IO immediate occupancy, cyan LS life safety, green CP collapse prevention, yellow at point C, orange at D and red for E [2].

In Figure 22, the step of pushover is the next greater most close to the target displacement. The worse status hinge are in the frame at "J" axis that have 3 hinge in cyan color that represent Life Safety and the performance objective of building is limited life safety. In the "X" direction of the worse hinge status was Immediate Occupancy.

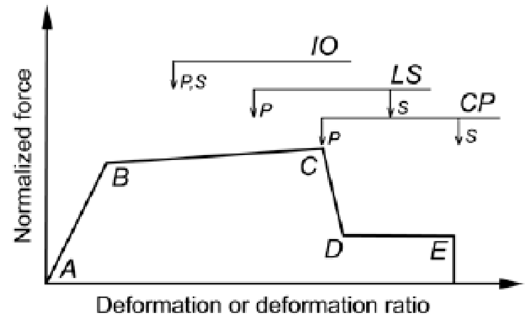


Figure 21
Element Deformation Acceptance Criteria

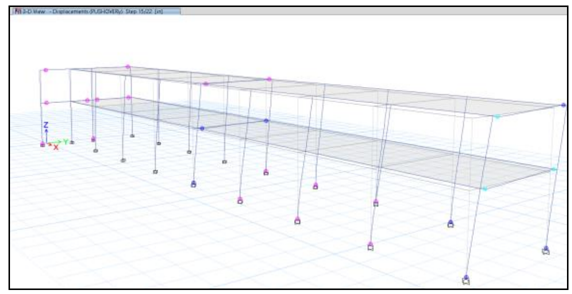


Figure 22
Pushover Hinge Results in Y direction

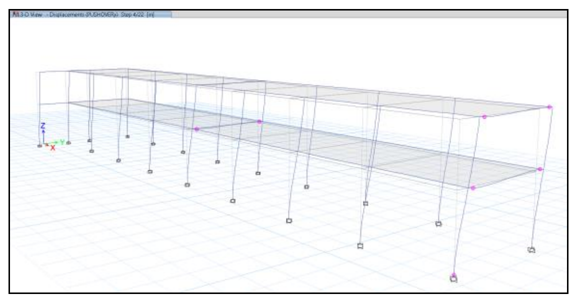


Figure 23
Hinge Formed at Δy Displacement

We also perform an evaluation post-earthquake at the displacement of Δy as shown in Figure 23, where the hinge that pass the yield limit in pushover analysis loses the rigidity. Then analyzed the structured with gravity load at service only and all member have the capacity to resist the moments and the structure have the stability to do not collapse.

CONCLUSION

After performing the following evaluations: Linear Static Analysis, Nonlinear Static Analysis taking into account the geometric nonlinearity and material analysis, Pushover Analysis and seismic detailing requirements by the code, we can conclude that the overall structure has the ability to resist the demand of gravitational loads and static earthquake load for lineal and nonlinear analysis. We can see that the structure is overdesigned based on it has a greater capacity in most of the elements. The structure comply with most seismic details except for the spacing of the hoops in the beam because the latest code is more demanding at near joint, and do not comply with the shear capacity at beam-column joint, and neither with a strong column and weak beam.

In the evaluation of existing structures using the Nonlinear Static Analysis "Pushover" the structure complies with the performance (4-D) in all hinges in the maximum displacement determined by modifying curve force versus displacement. It was also determined that the structure is capable of supporting gravity loads in service after an earthquake with Δy displacement from the curve force vs displacement.

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