

# Design of an automatically guided mechanical manipulator

*Eduardo J. Veras*

Professor

Department of Mechanical Engineering, UPPR

## Abstract

An automatically guided mechanical manipulator is an electromechanical device used for transportation and material handling. A combination of electronics control and mechanics (mechatronics) design and analysis is needed during the conceptualization of the design and preliminary design phases. The prototype constructed is capable of carrying a load of two pounds, transporting it from one work station to another and placing it on a shelf. The electronic controller developed regulates all aspects of the dynamics of the manipulator. It produces a smooth startup, constant motion speed, and smooth stop. This process is repeated as required and programmed by the designer.

## Sinopsis

### Manipulador mecánico guiado automáticamente

Un manipulador mecánico guiado automáticamente es un dispositivo electromecánico que se usa para transportar y manipular materiales. Una combinación de control electrónico y diseño mecánico (mecatrónica) y análisis es necesario durante las fases de conceptualización y diseño preliminar. El prototipo construido es capaz de levantar una carga de dos libras, transportarla desde una estación de trabajo a otra y colocarla en un estante. El controlador electrónico desarrollado regula todos los aspectos del movimiento del manipulador. Este produce un arranque suave, un movimiento a velocidad constante y una parada suave. El proceso se repite de acuerdo a los requisitos y la programación del diseñador.

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### Introduction

This paper focuses on the mechanics and control of the most important form of the industrial robots, the mechanical manipulator. Generally, robotic manipulators are designed for stationary operation. However, for a variety of applications, such as inserting components of an assembling performed in different places, or mixing substances stored separately, movement capability is needed. In this project, mobility is added to the manipulator through an electronic controller (usually called locomotion system) designed specifically for moving the manipulator in specified ways.

The use of robots in the industry is directly related to the trends in the automation of the manufacturing process. The study of robotics is concerned with the desire to synthesize some aspects of human functions by the use of mechanisms, sensors, actuators and computers. It includes a wide range of different engineering fields, such as mechanical engineering, mathematics, control theory, electrical engineering and computer science.

Mechanical engineering provides methodologies for static and dynamic analysis. Mathematics supplies tools for describing spatial relationships. Control theory contributes tools for designing and evaluating algorithms to perform desired motions. Sensors and devices for interaction in the manipulator's environment are tools developed within the electrical engineering field. Computer science contributes the programming techniques and other important tools such as computer vision and artificial intelligence (AI)<sup>1, 2</sup>.

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<sup>1</sup> Craig, J., 1989, *Introduction to Robotics: Mechanics and Control*, Second edition, Addison-Wesley Publishing Company

<sup>2</sup> Gordon, M., 1987, *Robot Builder's Bonanza*, Tab Books/McGraw-Hill, Inc.



## Modular design of a mechanical manipulator

A generally used approach for robotics design is to construct individual robot components (modules) and combine these modules to make a functional machine. With modular design it is relatively simple to change and update the initial alternative design. [This design was conducted as part of a Mechanical Engineering Capstone Design course and modular design results in a suitable approach for integrating design groups (teamworks), each group focusing on a particular module].

The modular design of a mechanical manipulator, which performs certain mechanical tasks when actuated, involves the following systems or group of components: support structure (frame and chassis), mechanical manipulation, power transmission system, locomotion and navigation systems (driving control, electronic circuit boards, computer) and energy supply. Team members must understand each other's goals. This approach should generate a larger number of alternative designs, increasing the possibility of novel and creative ideas. Mechanical manipulation and locomotion systems will be discussed in detail in the next sections.

### Mechanical manipulation

The manipulator mechanism is a multi-degrees-of-freedom (mdof) mechanical device.<sup>3</sup> The mechanism designed in this project is a three-degrees-of-freedom manipulator, as shown in figure 1.

The primary task of a computer controlling a robot is determining the trajectory of the end effector or hand (gripper, screwdriver, painting nozzle) and decomposing it to separate joint motions. This process of analysis from the hand coordinate space to joint coordinate space is called inverse kinematics.

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<sup>3</sup> Wilson, C. and Sadler, P., 1993, *Kinematics and Dynamics of Machinery*, Second Edition, Harper Collins College Publishers

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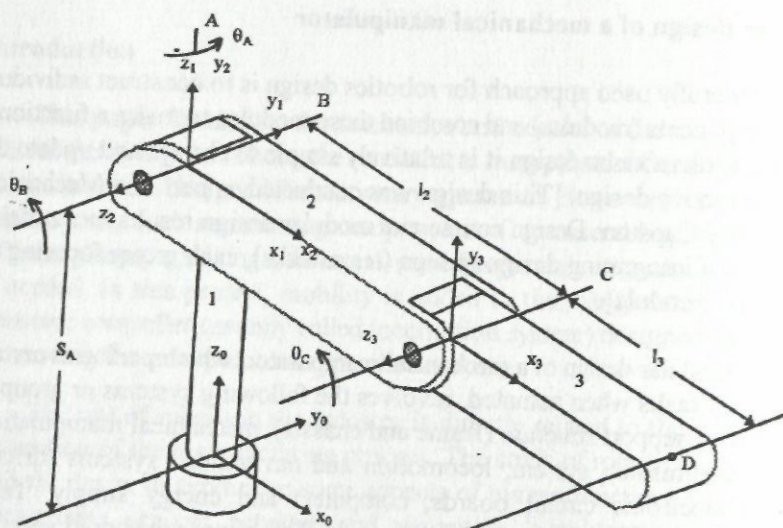


Figure 1. Three-degrees-of-freedom manipulator

As figure 1 shows, the position of a joint in space is referred to with different frames of reference. Each of these frames is a Cartesian reference frame having a particular set of  $x$ ,  $y$  and  $z$  coordinate axes. There are three revolute axes, identified as A, B, and C, with corresponding joint rotations  $\theta_A$ ,  $\theta_B$ ,  $\theta_C$ . In this design project, the values taken by these angles are produced by DC motors located at each revolute joint. For convenience, the reference frame is located at the base of the manipulator.

### *Coordinate transformation matrix: translation*

Matrix methods are useful for analyzing robotic manipulators<sup>4</sup> to represent spatial coordinates of joints. Figure 2 shows the coordinates of a

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<sup>4</sup> Wilson, C., and Sadler, P., 1993, *Kinematics and Dynamics of Machinery*, Second Edition, Harper Collins College Publishers

joint P with respect to reference frames, i and j. The origin of the frame j is located with respect to the origin of frame i through a three dimensional translation vector  ${}^i\{Q\}$ , where  ${}^i\{Q\}^T = \{Q_{xi}, Q_{yi}, Q_{zi}\}$ .

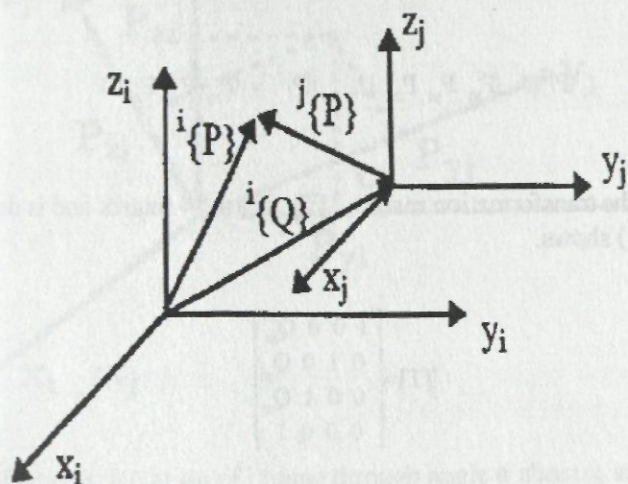


Figure 2. Frame j relative to frame i using the translation vector

The coordinates of any point P in space, expressed in the two reference frames are related by:

$$P_{xi} = P_{xj} + Q_{xi}, \quad P_{yi} = P_{yj} + Q_{yi}, \quad P_{zi} = P_{zj} + Q_{zi} \quad (1)$$

Expressing (1) using matrix notation, yields:

$${}^i\{P\} = {}^j\{P\} + {}^i\{Q\} \quad (2)$$



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or

$${}^i\{P\} = {}^j_j[T]{}^j\{P\} \quad (3)$$

where

$${}^i\{P\}^T = \{P_{xi} \ P_{yi} \ P_{zi} \ 1\}, \quad {}^j\{P\}^T = \{P_{xj} \ P_{yj} \ P_{zj} \ 1\} \quad (4)$$

Now, the transformation matrix,  ${}^i_j[T]$ , is a 4 \* 4 matrix and is defined as equation (5) shows.

$${}^i_j[T] = \begin{bmatrix} 1 & 0 & 0 & Q_{xi} \\ 0 & 1 & 0 & Q_{yi} \\ 0 & 0 & 1 & Q_{zi} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

### Coordinate transformation matrix: rotation

The rotational displacement considers the angular orientation of reference frame  $j$  relative to reference frame  $i$ . Figure 3 shows the  $j$  frame relative to the  $i$  frame by a rotation angle  $\alpha$  about the  $x_i$  axis.

From figure 3 we obtain the following equations which relate the coordinates of an arbitrary point,

$$P_{xi} = P_{xj}, \quad P_{yi} = P_{xy} \cos \alpha - P_{yz} \sin \alpha, \quad P_{zi} = P_{yz} \sin \alpha + P_{xy} \cos \alpha \quad (6)$$

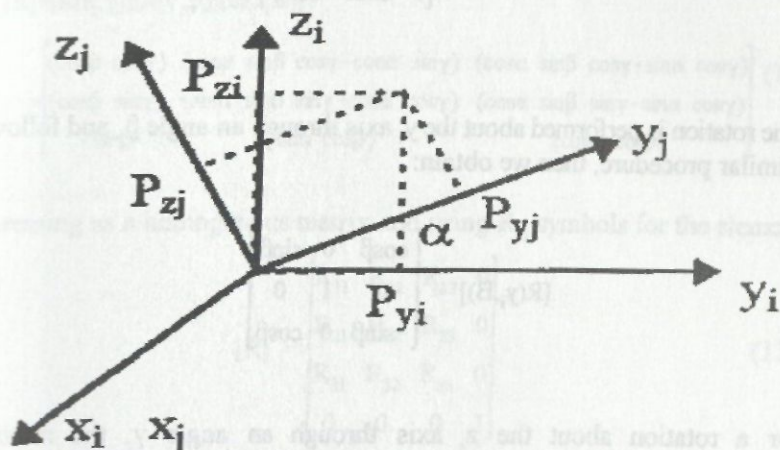


Figure 3. Rotation of j frame through angle  $\alpha$  about  $x_i$  axis

Using matrix notation,

$$\begin{bmatrix} P_{xi} \\ P_{yi} \\ P_{zi} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\beta \end{bmatrix} \quad (7)$$

The rotational transformation matrix  $[R(x_i, \alpha)]$  defines a rotation about the  $x_i$  axis through an angle  $\alpha$ , and is given by:

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$$[R(x_i, \alpha)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix} \quad (8)$$

If the rotation is performed about the  $y_i$  axis through an angle  $\beta$ , and following a similar procedure, then we obtain:

$$[R(y_i, \beta)] = \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix} \quad (9)$$

For a rotation about the  $z_i$  axis through an angle  $\gamma$ , the rotational transformation matrix is as follows:

$$[R(z_i, \gamma)] = \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (10)$$

The orientation of the reference frame  $i$  can be expressed as a combination of a rotation through angle  $\alpha$  about the  $x_i$  axis, followed by a rotation  $\beta$  about the  $y_i$  axis, followed by a rotation  $\gamma$  about the  $z_i$  axis, where the origins of frames  $i$  and  $j$  are coincident. Symbolically, these transformations are expressed as:

$${}^i\{P\} = {}^i_j[R]{}^j\{P\} \quad (11)$$



where

$${}^j[R] = [R(z, \gamma)][R(y, \beta)][R(x, \alpha)]$$

$$= \begin{bmatrix} (\cos\beta \cos\gamma) (\sin\alpha \sin\beta \cos\gamma - \cos\alpha \sin\gamma) (\cos\alpha \sin\beta \cos\gamma + \sin\alpha \cos\gamma) \\ (\cos\beta \sin\gamma) (\sin\alpha \sin\beta \sin\gamma + \cos\alpha \cos\gamma) (\cos\alpha \sin\beta \sin\gamma - \sin\alpha \cos\gamma) \\ -\sin\beta & (\sin\alpha \cos\beta) & (\cos\alpha \cos\beta) \end{bmatrix} \quad (12)$$

Expressing as a homogenous matrix and using  $R_{i,j}$  symbols for the elements,

$$[R] = \begin{bmatrix} R_{11} & R_{12} & R_{13} & 0 \\ R_{21} & R_{22} & R_{23} & 0 \\ R_{31} & R_{32} & R_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (13)$$

The analysis of a manipulator involves the determination of the hand trajectory and the transformation of its coordinates into joint coordinates. In a general case, a particular point may require a combination of a rotational and a translational orientation. Assuming that the translational transformation is executed first and the rotational transformation second, then equations (5) and (12) are used to represent these transformations, following this sequence, as shown in equation (13).

#### *Link-to-link transformation matrix*

The design of a mechanical manipulator implies the selection of parameters to satisfy the constraints imposed on the design. In previous sections, we defined the transformations needed to represent a particular point relative to a fixed reference frame. Considering that the manipulator designed consists on a set of interconnected links, it is necessary to determine the

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transformation which defines frame  $i$  relative to frame  $j$  attached to neighboring links as function of the link parameters.

$$\begin{aligned}
 {}^i\{P\} &= \begin{bmatrix} 1 & 0 & 0 & Q_{xi} \\ 0 & 1 & 0 & Q_{yi} \\ 0 & 0 & 1 & Q_{zi} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} R_{11} & R_{12} & R_{13} & 0 \\ R_{21} & R_{22} & R_{23} & 0 \\ R_{31} & R_{32} & R_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} {}^j\{P\} \\
 &= \begin{bmatrix} R_{11} & R_{12} & R_{13} & Q_{xi} \\ R_{21} & R_{22} & R_{23} & Q_{yi} \\ R_{31} & R_{32} & R_{33} & Q_{zi} \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \end{aligned} \tag{13}$$

The transformation from reference frame  $j$  to frame  $k$  is given by equation (13), substituting the angular displacement as shown in figure 4, we obtain equation (14).

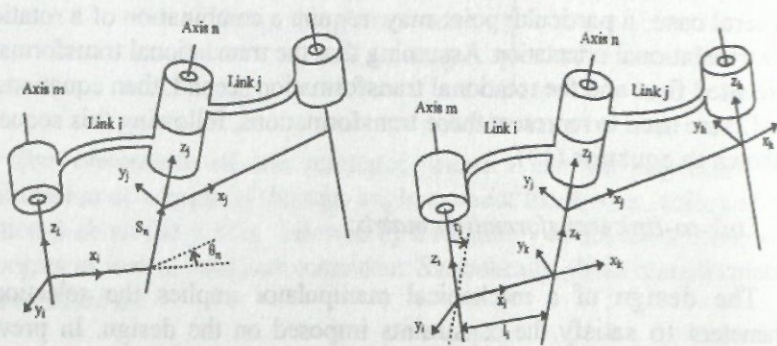


Figure 4. Joint variables ( $S_n$ ,  $\theta_n$ ) associated with joint  $n$  of the manipulator

$${}^k_j[T] = \begin{bmatrix} \cos\theta_n & -\sin\theta_n & 0 & 0 \\ \sin\theta_n & \cos\theta_n & 0 & 0 \\ 0 & 0 & 1 & S_n \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (14)$$

The transformation from frame  $k$  to frame  $i$  is obtained by a rotation  $\tau_i$  about the axis  $x_i$  and a translation  $l_i$  along  $x_i$ :

$${}^i_k[T] = \begin{bmatrix} 1 & 0 & 0 & l_i \\ 0 & \cos\tau_i & -\sin\tau_i & 0 \\ 0 & -\sin\tau_i & \cos\tau_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (15)$$

The final transformation is obtained multiplying the matrices given by equations (14) and (15) as follows:

$${}^i_j[T] = {}^i_k[T] {}^k_j[T] = \begin{bmatrix} \cos\theta_n & -\sin\theta_n & 0 & l_i \\ \cos\tau_i \sin\theta_n & \cos\tau_i \cos\theta_n & -\sin\tau_i & -S_n \sin\tau_i \\ \sin\tau_i \sin\theta_n & \sin\tau_i \cos\theta_n & \cos\tau_i & S_n \cos\tau_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (16)$$

As figure 1 shows, the reference frame 0 is located at the base of the mechanical manipulator. The objective is to obtain the transformation matrices that relate the spatial orientation of the links connecting all joints of the manipulator. Following the previous procedure, we can obtain the transformation matrices for the three-degrees-of-freedom manipulator designed in this project. Note that all joints considered are revolutes. From figure 1, the following parameters that define the links are identified:  $l_0 = 0$ ,



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$l_1 = 0, \tau_0 = 0, \tau_1 = 90^\circ, \tau_2 = 0, S_B = 0$  and  $S_C = 0$ . Substituting these values into equation (15), we get:

$$\begin{aligned}
 {}^1_0[T] &= \begin{bmatrix} \cos\theta_A & -\sin\theta_A & 0 & 0 \\ \sin\theta_A & \cos\theta_A & 0 & 0 \\ 0 & 0 & 1 & S_A \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 {}^2_1[T] &= \begin{bmatrix} \cos\theta_B & -\sin\theta_B & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin\theta_B & \cos\theta_B & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 {}^3_2[T] &= \begin{bmatrix} \cos\theta_C & -\sin\theta_C & 0 & l_2 \\ \sin\theta_C & \cos\theta_C & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \end{aligned} \tag{17}$$

Point D, as shown in figure 1, defines the coordinates of the hand or gripper. Combining the transformations (17) as in equation (16), we can find the transformation matrix of point D expressed in terms of the reference frame 0, that is,

$${}^1_0\{P\}_D = {}^1_3\{T\} {}^3\{P\}_D \tag{18}$$

where,

$${}^0_1[T] = {}^0_1[T] \quad {}^1_2[T] \quad {}^2_3[T] = \begin{bmatrix} \cos\theta_A & -\sin\theta_A & 0 & 0 \\ \sin\theta_A & \cos\theta_A & 0 & 0 \\ 0 & 0 & 1 & S_A \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_B & -\sin\theta_B & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \sin\theta_B & \cos\theta_B & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_C & -\sin\theta_C & 0 & l_2 \\ \sin\theta_C & \cos\theta_C & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (19)$$

The coordinates of point D in reference frame 3 are:

$${}^3\{P\}_D = \begin{Bmatrix} l_3 \\ 0 \\ 0 \\ 1 \end{Bmatrix} \quad (20)$$

Multiplying equations (19) and (20), the final coordinates of the point D expressed in terms of the reference frame 0 are:

$${}^0\{P\}_D = {}^0_3[T] \quad {}^3\{P\}_D = \begin{Bmatrix} l_2 \cos\theta_A \cos\theta_B + l_3 \cos\theta_A \cos(\theta_B + \theta_C) \\ l_2 \sin\theta_A \cos\theta_B + l_3 \sin\theta_A \cos(\theta_B + \theta_C) \\ S_A + l_2 \sin\theta_B + l_3 \sin(\theta_B + \theta_C) \\ 1 \end{Bmatrix} = \begin{Bmatrix} x \\ y \\ z \\ 1 \end{Bmatrix} \quad (21)$$

The solution for angles  $\theta_A$ ,  $\theta_B$ , and  $\theta_C$  can be obtained squaring each of the equations (22), (23) and (24) as follows (Note that the coordinates of the point D, x, y and z are considered as known variables):

$$x = l_2 \cos\theta_A \cos\theta_B + l_3 \cos\theta_A \cos(\theta_A + \theta_C) \quad (22)$$

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$$y = l_2 \sin\theta_A \cos\theta_B + l_3 \sin\theta_A \cos(\theta_A + \theta_C) \quad (23)$$

$$z - S_A = l_2 \sin\theta_B + l_3 \sin(\theta_B + \theta_C) \quad (24)$$

$$x^2 + y^2 + (z - S_A)^2 = l_2^2 + l_3^2 + 2 l_2 l_3 \cos\theta_C \quad (25)$$

from which

$$\theta_C = \cos^{-1} \left( \frac{x^2 + y^2 + (z - S_A)^2 - l_2^2 - l_3^2}{2 l_2 l_3} \right) \quad (26)$$

Dividing (23) by (22) yields

$$\theta_C = \tan^{-1} \left( \frac{y}{x} \right) \quad (27)$$

To obtain  $\theta_B$ , equations (22) and (24) must be rewritten and solved simultaneously, as follows,

$$\frac{x}{\cos\theta_A} = (l_2 + l_3 \cos\theta_C) - l_3 \sin\theta_C \sin\theta_B \quad (28)$$



$$z - S_A = l_3 \sin\theta_C \cos\theta_B + (l_2 + l_3 \cos\theta_C) \sin\theta_B \quad (29)$$

Solving for  $\sin\theta_B$  and  $\cos\theta_B$ , and combining, we obtain:

$$\theta_B = \tan^{-1} \left( \frac{(z - S_A)(l_2 + l_3 \cos\theta_C) \cos\theta_A - x l_3 \sin\theta_C}{x(l_2 + l_3 \cos\theta_C) + (z - S_A) l_3 \sin\theta_C \cos\theta_A} \right) \quad (30)$$

In the previous kinematic analysis, the quantities  $l_2$ ,  $l_3$  and  $S_A$  are constrained by the design dimensions of the manipulator. The joint angles  $\theta_A$ ,  $\theta_B$  and  $\theta_C$  are time dependent variables. In this project these angular positions are controlled using electronic sensors, disc decoders and microswitches.

### Locomotion and navigation systems

Locomotion and navigation systems provide the movement capability and direction control of the robot. These systems work together to guide the manipulator through the different obstacles that it encounters when moving and keeps the manipulator on track.

The navigation of the robot can be classified in two categories: the obstacle avoidance system and the tracking system. The avoidance function is generally accomplished by using infrared photo transistor sensors or ultrasonic sensing devices. The tracking function can be achieved with a variety of sensors or mechanical track (similar to a train track). For robotics applications, the mechanical tracks are usually not desired because re-routing the robot is a complex and difficult task.

The approach followed with this particular application is the line tracing system. This system consists of a reflective tape on the floor which the robot detects and whose direction the robot corrects to follow the line. The line track can be changed easily when desired.

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### *Direction control of the manipulator*

In this design project we used two driving DC motors, two wheels and a caster to provide stability to the support structure. The wheels provide forward and backward movements and steer the manipulator left and right. Figure 5 shows the DC motors, the wheels and the caster as mounted in this design.

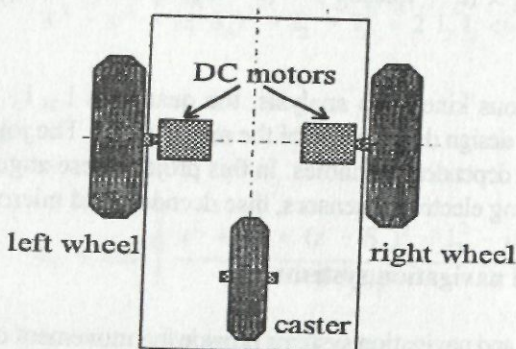


Figure 5. Locomotion system

The direction control of the turning wheels can be accomplished by changing the rotational speed direction of the DC motor. For example, we can reverse the moving direction by switching the power lead connections to the DC power supply. Stopping one motor causes the robot to turn in that direction, and reversing the rotation of the motors relative to one another causes the robot to turn by spinning on its wheel axis.

One method to control the direction of rotation of the motors is using double-pole double-throw (DPDT) relays (fig. 6). Figure 7 shows the complete driver circuit for a relay-controller motor. An advantage of using relays is that they can easily be driven by digital signals. Logical 1 (HIGH) turns the relay on and logical 0 (LOW) turns the relay off. The activation of the relays is completed by the navigation system as explained in section 4.

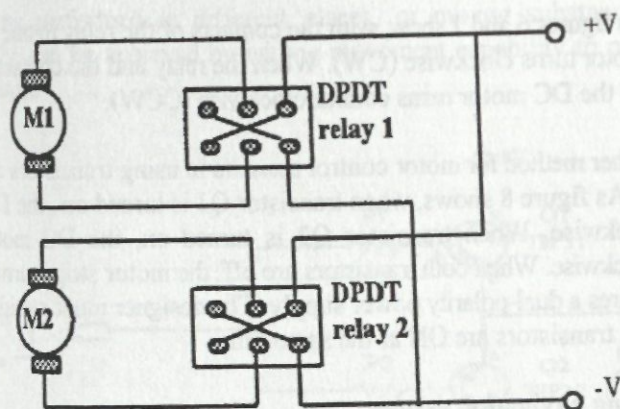


Figure 6. DPDT relay control of the DC driven motors

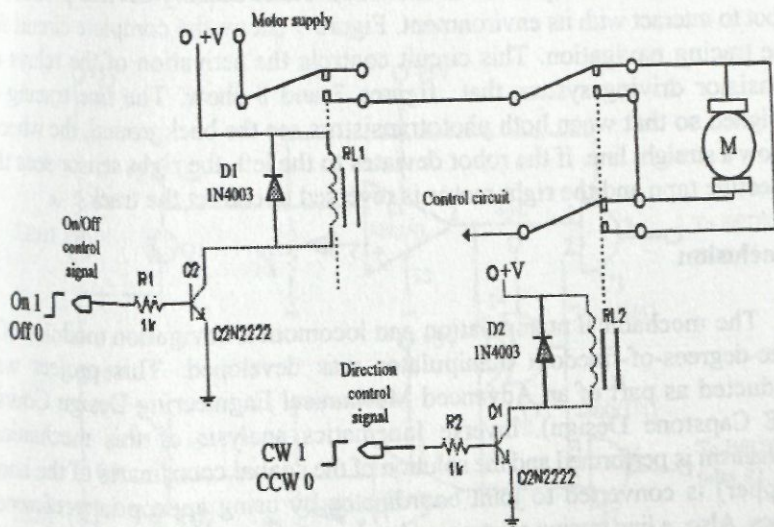


Figure 7. ON/OFF and direction relay controller



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As in figures 6 and 7 show, with the contacts of the relay in one position, the DC motor turns clockwise (CW). When the relay and the contacts change positions, the DC motor turns counterclockwise (CCW).

Another method for motor control consists in using transistors as simple switches. As figure 8 shows, when transistor Q1 is turned on, the DC motor turns clockwise. When transistor Q2 is turned on, the DC motor turns counterclockwise. When both transistors are off, the motor stops turning. This setup requires a dual-polarity power supply. The designer must avoid the case when both transistors are ON at the same time.

### Line tracing navigation system

A line tracing navigation system consists in a reflective tape placed on the floor, a set of optical sensors or light emission diodes (LEDs) and infrared photo transistors incorporated in the robot. These sensory devices permit the robot to interact with its environment. Figure 9 shows the complete circuit for line tracing navigation. This circuit controls the activation of the relays or transistor driving system that figures 7 and 8 show. The line tracing is designed so that when both phototransistors see the background, the wheels follow a straight line. If the robot deviates to the left, the right sensor sees the reflective tape and the right motor is reversed to correct the track.

### Conclusion

The mechanical manipulation and locomotion-navigation modules of a three-degrees-of-freedom manipulator was developed. This project was conducted as part of an Advanced Mechanical Engineering Design Course (ME Capstone Design). Inverse kinematics analysis of this mechanical mechanism is performed and the solution of the spatial coordinates of the hand (gripper) is converted to joint coordinates by using appropriate reference frames. Also, a line tracing electronic circuit was developed for controlling the tracking of the manipulator. This circuit provides the manipulator with a means to interact with its environment and follows a reflective tape on the floor. A variety of applications, such as inserting components of an

assembling performed in different places, or mixing substances stored separately, can be achieved by adding movement capability to mechanical manipulators.

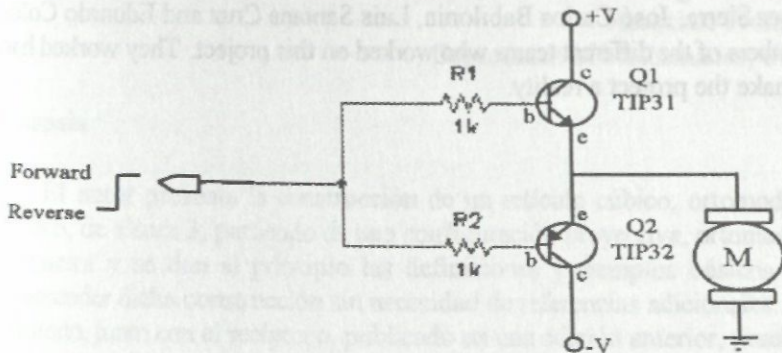


Figure 8. Use of transistors to control the direction of a DC motor

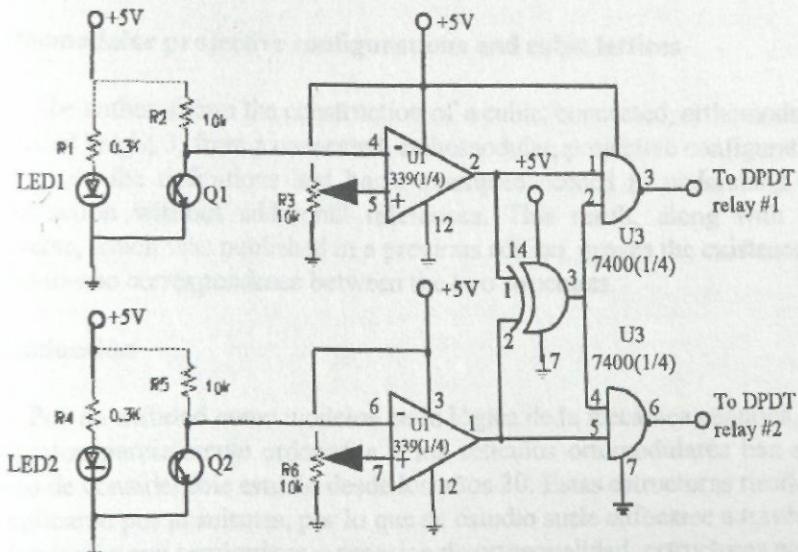


Figure 9. Line tracing circuit

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