Dynamic analysis of rigid pavements and bridges under moving vibrating trucks

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Abstract

The computer program UPR-PAVI2 was developed to perform the dynamic analysis of a pavement slab or a bridge superstructure loaded with moving vibrating trucks. UPR-PAVI2 can model the soil, slab and truck, considering their stiffness and geometry. This paper presents the theory used by UPR-PAVI2 with emphasis on the modification of the soil-slab-truck stiffness matrix required to consider the truck moving along the slab and the effect of the surface roughness in the force vector.

Sinopsis

El programa UPR-PAVI2 se desarrolló para hacer análisis dinámico de un losa de pavimento una superestructura de puente cargada de camiones en movimiento. UPR-PAVI2 puede modelar el suelo, la losa y el camión, considerando su rigidez y geometría. Este documento presenta la teoría que usa UPR-PAVI2 para modificar la matriz de rigidez del suelo-losa-camión que se requiere para considerar el camión moviendose a lo largo de la losa y el efecto en la rugeza de la superficie en el vector de fuerza.

Introduction

The internal forces in pavements or superstructure of bridges produced by moving trucks are obtained through a dynamic analysis. The computer program UPR-PAVI2 was developed to perform the dynamic analysis of this type of structures (Tito, 1996). This paper presents the theoretical background of this program. The study emphasizes the modification required on the soil-slab-truck stiffness matrix to consider the truck moving along the slab and the effect of the surface roughness on the force vector. The other components of the general dynamic equation, such as mass, damping, soil-slab stiffness and force are also presented (Tito, 1996).

Structural model

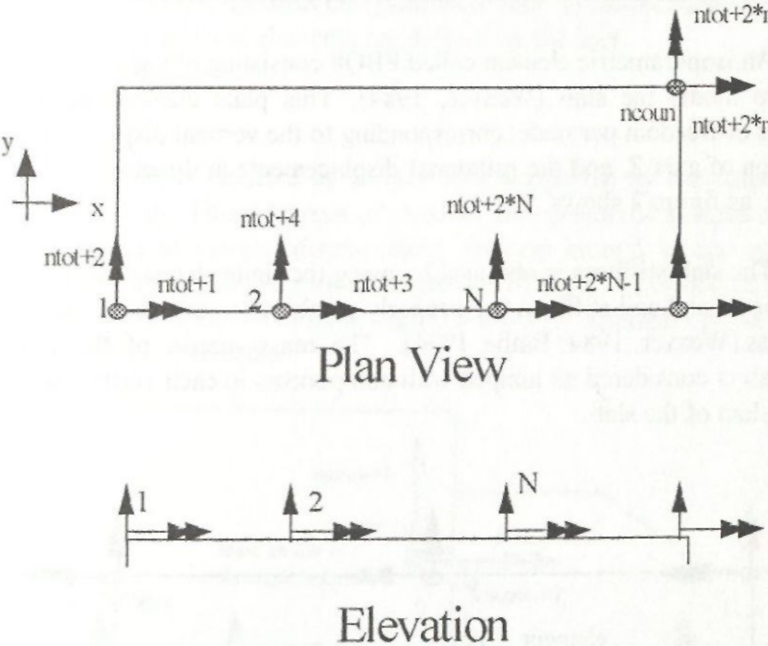
Global degrees of freedom

The slab is divided using quadratic isoparametric finite elements of 8 nodes. Degrees of freedom of vertical displacement coordinates are numbered first, followed by the truck coordinates and finally the rotational coordinates. Figure 1 shows a schematic of the numeration of slab coordinates, and the global degrees of freedom considered in each node. The axis X corresponds to the direction in which the truck moves.

Soil or support model

The soil is modeled as springs concentrated in the nodes. For pavement model the modulus of this spring was the result of the modulus of subgrade reaction (k) applied in the influence area of the node (Winkler foundation). The soil damping is considered for dynamic analysis.

When the superstructure of a bridge is studied the supports can be modeled by using springs. If the support is rigid, it can be modeled by using large spring stiffness. Zero-stiffness springs are assigned to free nodes.



ncoun: number of slab vertical coordinates
ntot : ncoun + number of truck coordinates.

Figure 1. Numeration of slab coordinates 1

This model produces a lumped soil stiffness matrix. The components of this matrix are added to the vertical global degree of freedom of each node.

Beams and slab model

In the case of bridges, the beams are modeled by using the stiffness matrix of a one-dimensional beam element. Each beam is defined between two nodes of the plates using the same global degree of freedom.

An isoparametric element called PBQ8 consisting of eight nodes was used to model the slab (Weaver, 1984). This plate element has three degrees of freedom per node, corresponding to the vertical displacement in direction of axis *Z*, and the rotational displacements in direction of *X* and *Y* axes, as figure 2 shows.

The slab stiffness is obtained by using the finite element theory. The slab, beam and soil stiffness are properly combined to obtain the soil-slab stiffness (Weaver, 1984: Bathe, 1996). The mass matrix of the beams and slab is considered as lumped with components in each vertical degree of freedom of the slab.

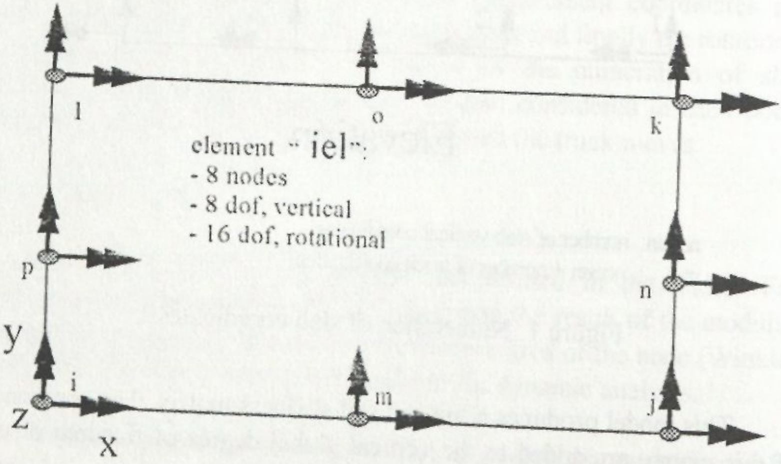


Figure 2. Eight node finite element PBQ8

Dowel and tie model

Dowels and ties act as reinforcement joining two slabs separated by a construction joint. A fixed-ended beam spanning between the nodes at opposite ends of the construction joint were used to model these elements. The properties of these elements are defined by the user.

Truck model

The truck is modeled as a rigid body supported by the suspension system and tires. Three degrees of freedom are considered at mass center, corresponding to vertical displacement, rotation around *y*, and rotation around *x*, respectively. A vertical degree of freedom is considered at the tire and truck suspension system. Figure 3 shows a schematic of the truck and its degrees of freedom.

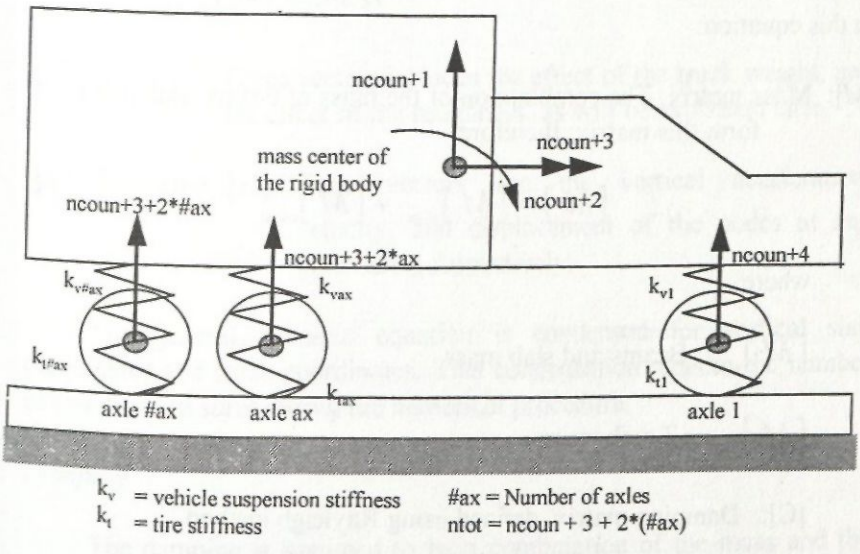


Figure 3. Truck model

The interaction between the truck stiffness and soil-slab stiffne explained later. The truck mass matrix is considered lumped, components in each degree of freedom. The truck damping also ca considered. This model provides a realistic modeling of the truck.

UPR-PAVI2 can also perform a static analysis of the truc different locations. This is useful to find the amplification factor, whic defined as the ratio between the dynamic and static response.

General dynamic equation

The following general dynamic equation is used to model problem of a truck moving on a slab (Clough and Penzien, 1993):

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F\}$$

In this equation:

[*M*]: Mass matrix. The combination of the mass of beams, slab and tru form this matrix; therefore:

$$[M] = [M]_{slab} + [M]_{truck}$$ (

where:

$[M]_{slab}$: Beams and slab mass

$[M]_{truck}$: Truck mass

[C]: Damping matrix, defined using Rayleigh method.

[K]: Stiffness matrix of the soil-slab-truck system. It is a combination of the stiffness of soil, slab, dowel, tie, beam and truck:

$$[K]=[K]_{soil}+[K]_{slab}+[K]_{dowel}+[K]_{truck} \quad (3)$$

where:

$[K]_{soil}$: Soil stiffness

$[K]_{slab}$: Beams and slab stiffness

$[K]_{dowel}$: Dowel, tie or beam stiffness

$[K]_{truck}$: Truck stiffness

{F}: Force vector, includes the effect of the truck weight, and the effect of the roughness, as will be explained later.

$\{\ddot{u}\}, \{\dot{u}\}$ and $\{u\}$:These vectors are the vertical acceleration, velocity, and displacement of the nodes at any given time, respectively.

The general dynamic equation is condensed for vertical slab coordinates and truck coordinates. This condensation reduces the number of equations to solve during the numerical procedure.

Damping

The damping is assumed to be a combination of the mass and the stiffness matrices as defined by the Rayleigh method. The Rayleigh damping (Clough and Penzien, 1993) is given by the following expression:

$$[C] = \alpha_0 [M] + \alpha_1 [K]$$

where:

$$\begin{Bmatrix} \alpha_0 \\ \alpha_1 \end{Bmatrix} = \frac{2\xi}{\omega_m + \omega_n} \begin{Bmatrix} \omega_m \omega_n \\ 1 \end{Bmatrix}$$

ξ is the damping ratio. It was assumed that the same dampi ratio applies to both control frequencies (ω_m, ω_n).

ω_m is a fundamental frequency of the system.

ω_n is the highest modal frequency that contributes to the dynami response.

Using these expressions, the system damping was considered as th sum of the soil, slab and truck damping. The following equations were used; the soil damping equation,

$$[C]_{soil} = \frac{2\xi_{soil}}{\omega_1 + \omega_n} [K]_{soil} \quad (6)$$

beam and slab damping:

$$[C]_{slab} = \frac{2\xi_{slab}}{\omega_1 + \omega_n} * \omega_1 \omega_n [M]_{slab} + \frac{2\xi_{slab}}{\omega_1 + \omega_n} * [K]_{slab} \quad (7)$$

truck damping:

$$[C]_{truck} = \frac{2\xi_{truck}}{\omega_1 + \omega_n} * \omega_1 \omega_n [M]_{truck} + \frac{2\xi_{truck}}{\omega_1 + \omega_n} * [K]_{truck} \quad (8)$$

Thus, the damping matrix is:

$$\left[C\right] = \left[C\right]_{soil} + \left[C\right]_{slab} + \left[C\right]_{truck} \tag{9}$$

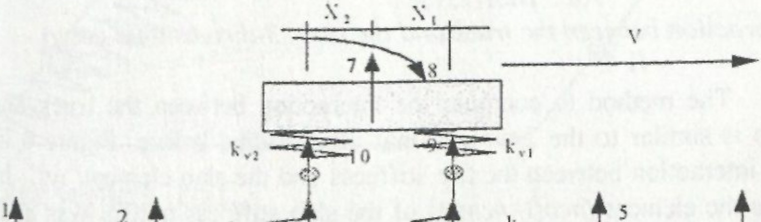
In these equations the most significant frequencies were used. The damping matrix was dependent on the soil-slab-truck stiffness matrix.

Effect of the moving vibrating truck on the stiffness matrix

The truck changes its position at each time interval depending on the velocity and path; for that reason the stiffness matrix of the system soil-slab-truck changes during the dynamic analysis. The path is defined by the user, and it can vary in the transversal direction if the approach angle is different from zero.

Interaction between a truck and a beam: 2-dimensional case.

Figure 4 shows a simplified example of a two dimensional truck on a beam modeled with 6 nodes and 5 beam elements. The truck changed from node to node according to the velocity. Table 1 shows the stiffness matrix of the beam-truck system for the position indicated in figure 4. According to the stiffness matrix theory, the tire stiffness interacted with the truck at the vertical coordinates of the beam elements that support the truck directly. At the other coordinates there was no interaction.

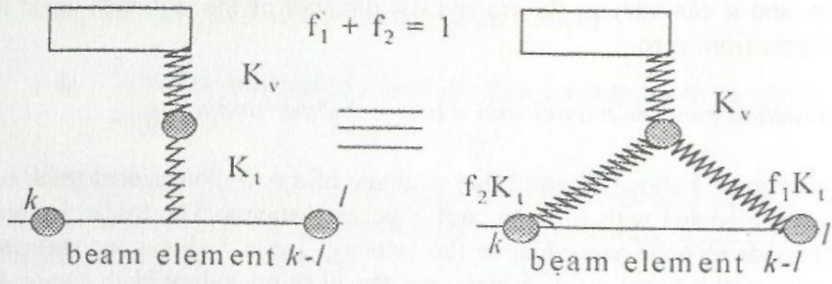


k_{ti} = tire stiffness, i:1,2; k_{vi} = truck suspension stiffness, i:1,2
V = truck velocity
X_i = absolute distance from truck gravity center to its axles

Figure 4. Simplified model of a truck on a beam

Each element of column '*i*' of the stiffness matrix is obtained l determining the force needed at each coordinate due to a unit displaceme at coordinate '*i*' whereas the displacement at the other coordinate '*j*' zero. The truck and the slab are assumed to be in contact all the time f this derivation.

When the truck axle is on the beam element *k-l* between the nod '*k*' and '*l*', the tire stiffness is distributed to both nodes according to tl shape function of the beam element. This is schematized in figure 5.



Where f_i is the shape function. i=1.2

Figure 5. Tire between the nodes

Interaction between the truck and the slab: 3-dimensional case

The method to compute the interaction between the truck and tl slab is similar to the 2-dimensional case studied before. Figure 6 show the interaction between the tire stiffness and the slab element '*iel*'. In thi case the element *(nco(t).nco(i))* of the slab stiffness matrix was added t the contribution of the tire in that node:

$$k_{(nco\ (t),nco\ (i))} = k_{(nco\ (t),nco\ (i))} + f_{(i)} \frac{k_t}{n}; \qquad (10)$$

where:

nco(t)	is the coordinate of the tire *t*.
nco(i)	is the vertical coordinate of the node *'i'*.
$f_{(i)}$	is the shape function of the element PBQ8 for the node *i* computed for the coordinates *(x,y)* of the point where the load is acting.
k_t	is the tire stiffness.
n_t	is the number of interaction points between the tire and the slab. These points are located in the tire imprinting area, which is defined by the tire pressure and the load.

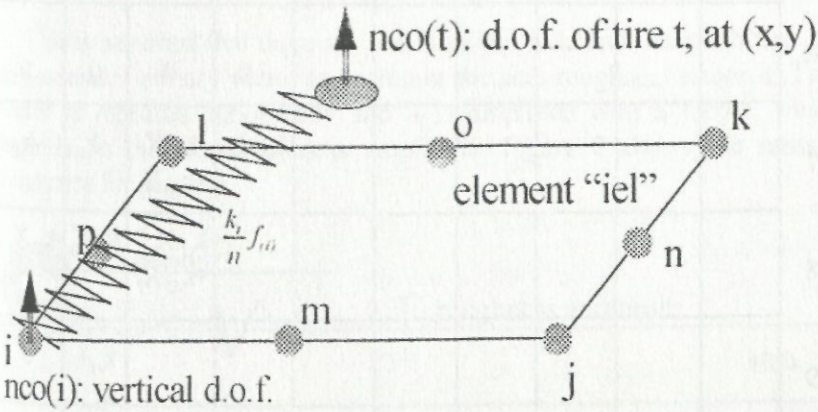


Figure 6. Distribution of tire stiffness on nodes

The stiffness matrix of the system is modified at each step according to the movement of the truck along the structure. The corresponding damping matrix is also modified.

Table 1. Stiffness matrix of the beam-truck system

	1	2	3	4	5	6	7	8	9	10
1	k_{11}	k_{12}	0	0	0	0	0	0	0	0
2		k_{22}	k_{23}	0	0	0	0	0	0	0
3			$k_{33}+k_{t2}$	k_{34}	0	0	0	0	0	$-k_{t2}$
4				$k_{44}+k_{t1}$	k_{45}	0	0	0	$-k_{t1}$	0
5					k_{55}	k_{56}	0	0	0	0
6						k_{66}	0	0	0	0
7							$k_{v1}+k_{v2}$	$-k_{v1}.X_1$ $+k_{v2}.X_2$	$-k_{v1}$	$-k_{v2}$
8								$k_{v1}.X_1^2$ $+k_{v2}.X_2^2$	$k_{v1}X_1$	$-k_{v2}X_2$
9									$k_{v1}+$ k_{t1}	0
10										$k_{v2}+$ k_{t2}

In this table: k_{ij} = elements of beam stiffness matrix, i,j:1...6

Force vector

The truck weight and the slab roughness contribute to the force vector {F} in the dynamic analysis.

Truck weight

The truck weight is added directly to the force vector {F}. This weight is applied at coordinates of the tire; therefore it is not distributed into the tire imprinting area. The imprinting area of the tire serves to distribute the tire stiffness on the slab, as explained before. It is assumed that each axle has two tires and each tire takes half of the axle's weight. More than one axle in the same location can be defined to represent axles with more than two tires (tandem tires).

Effect of roughness on force vector

It is assumed that there are two tires per axle and that each tire has an associated unitary vector to represent the slab roughness under it. This vector is repeated periodically and it is amplified with a factor, which depends on the real roughness amplitude. Figure 7 shows the unitary roughness for one tire.

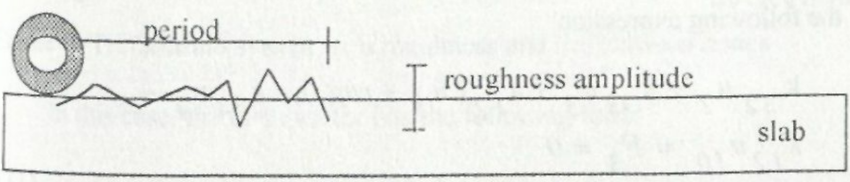


Figure 7. Slab roughness model

Figure 8 shows a simplified model of a bidimensional truck on a beam with roughness. The tire is over node 3 (global coordinate three). By definition of rigidity the coefficients of the stiffness matrix are found.

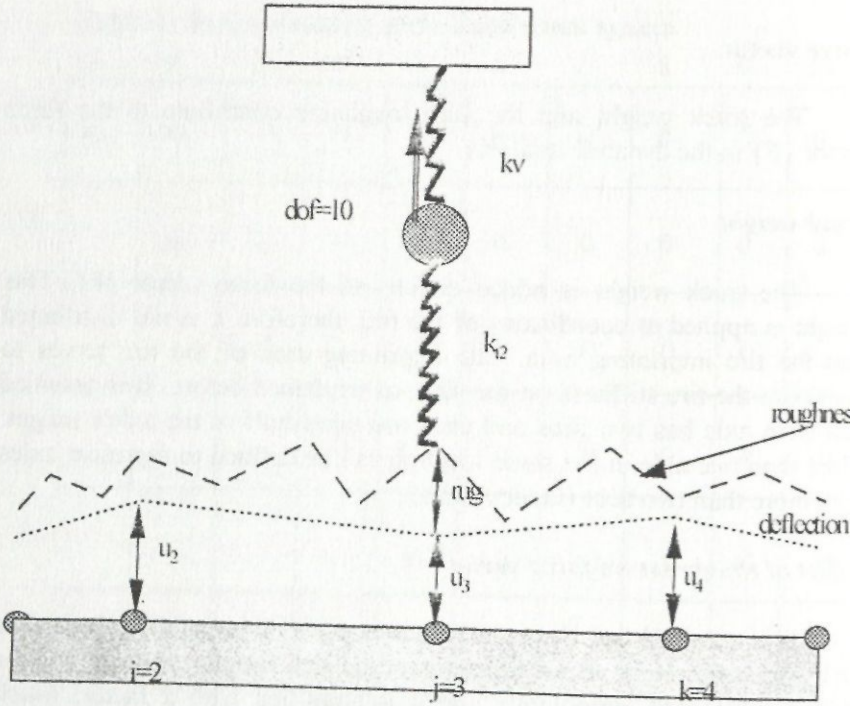


Figure 8. Truck-beam system with roughness

For example, the static equilibrium equation in node 3 is given by the following expression:

$$k_{32}u_2 + k_{33}u_3 + k_{12}(u_3 + rug) + k_{34}u_4 - k_{12}u_{10} = F_3 = 0$$

$$k_{32}u_2 + k_{33}u_3 + k_{12}u_3 + k_{34}u_4 - k_{12}u_{10} = 0 - k_{12}rug \qquad (11)$$

The roughness contribution to the force vector is computed by adding the component $-k_{t2}\,rug$ to the force vector; only the coordinates of the nodes of the element that support a tire are affected.

When the tire is between two nodes, the roughness effect is distributed using the shape function of the element. Figure 9 shows a schematic of this situation.

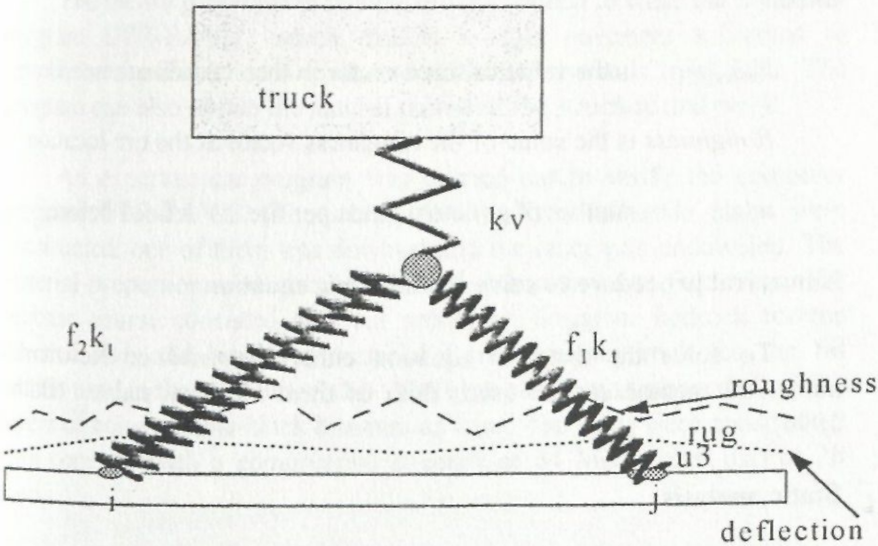


Figure 9. Truck-beam system with roughness and tire between nodes

In this case, the force vector has the following form:

$$\{F\} = \{F\} + \begin{Bmatrix} -k_t.rug.f_i \\ -k_t.rug.f_j \end{Bmatrix}, \quad f_i + f_j = 1 \qquad (12)$$

In the case of the slab with roughness, the effect of the roughne distributed to the eight nodes of the element according to its s function. To compute the force vector the following equation is used:

$$F_{(nco\ (i))} = F_{(nco\ (i))} - f_{(i)} \frac{k_t * roughness}{n};$$

where:

$F_{(nco(i))}$ is the value of force vector in the coordinate *nco(i).*

Roughness is the value of the roughness vector at the tire locatior

n, number of punctual loads per tire, as defined before.

Numerical procedure to solve the dynamic equation

To solve the dynamic equation either Wilson-θ or Newmarl numerical method can be used. Both of them are very stable (Batl 1996).

Static analysis

The static analysis is performed by locating the truck at differe points along the structure. The truck stiffness is not considered for th analysis. The tire load is distributed in the imprinting area considering ' number of punctual loads applied in the same number of contact points. I order to optimize the computation process, a force matrix for all the truc locations along the structure is obtained, permitting the solution of onl one equation to find the corresponding displacements.

Internal forces

The displacements $\{u\}_i$ corresponding to each time interval are computed. The internal forces are computed by using these displacements and the finite element theory.

Comparison between experimental results and UPR-PAVI2

The theory presented in this paper was applied to write the computer program UPR-PAVI2, which models a rigid pavement subjected to temperature gradient, dead load, and static and dynamic truck load. The program can also obtain the natural period of the structure and truck.

An experimental program was carried out to verify the computer program UPR-PAVI2 (Tito et al. 1997). Two full-scale slabs were constructed; one of them was doweled and the other was undoweled. The material properties were obtained through laboratory and field tests. The subbase course consisted of a cut section in limestone bedrock for the doweled slab and borrowed material from limestone rock for the fill section used in the undoweled slab. The base course was composed of two layers of non-erodable black bituminous base. The slabs were constructed with concrete with a compressive strength of 34 Mpa (5000 psi) at 28 days.

A loaded truck generated the established repetitions by moving over the instrumented slabs at different velocities. The truck produced static and dynamic deflections that were measured by using a data acquisition system. The dynamic characteristics of this truck were measured before the tests.

The replication and behavior of the response were analyzed, and validated data were compared with the response obtained using UPR-PAVI2. Figure 10 shows a typical comparison between the experimental and theoretical deflections. In general, the displacements computed using

UPR-PAVI2 was in agreement with the experimental data. These results validate the theory detailed in this paper (Tito et al, 1997).

Conclusions and recommendations

According to the theory presented the, displacements and internal forces in pavements and bridge superstructures are a function of the vehicle and structure characteristics. The computer program UPR-PAVI2 obtained with this theory and validated with experimental data is versatile and reliable, permitting the study of the dynamic behavior of pavements and bridge superstructures.

In order to apply the computer program UPR-PAVI2, it is necessary to know the dynamic characteristics of the soil, slab and truck. This data can be obtained from experimental procedures.

Finally, the computer program UPR-PAVI2 can be used to find the slab deflections and stresses produced by real trucks. This structural knowledge will permit a realistic design of rigid pavements.

Acknowledgment

Financial support is gratefully acknowledged from the Puerto Rico Economic Development Administration - Science and Technology Board, from the Puerto Rico Highway and Transportation Authority (PRHTA), and from Civil Infrastructure Research Center of the University of Puerto Rico.

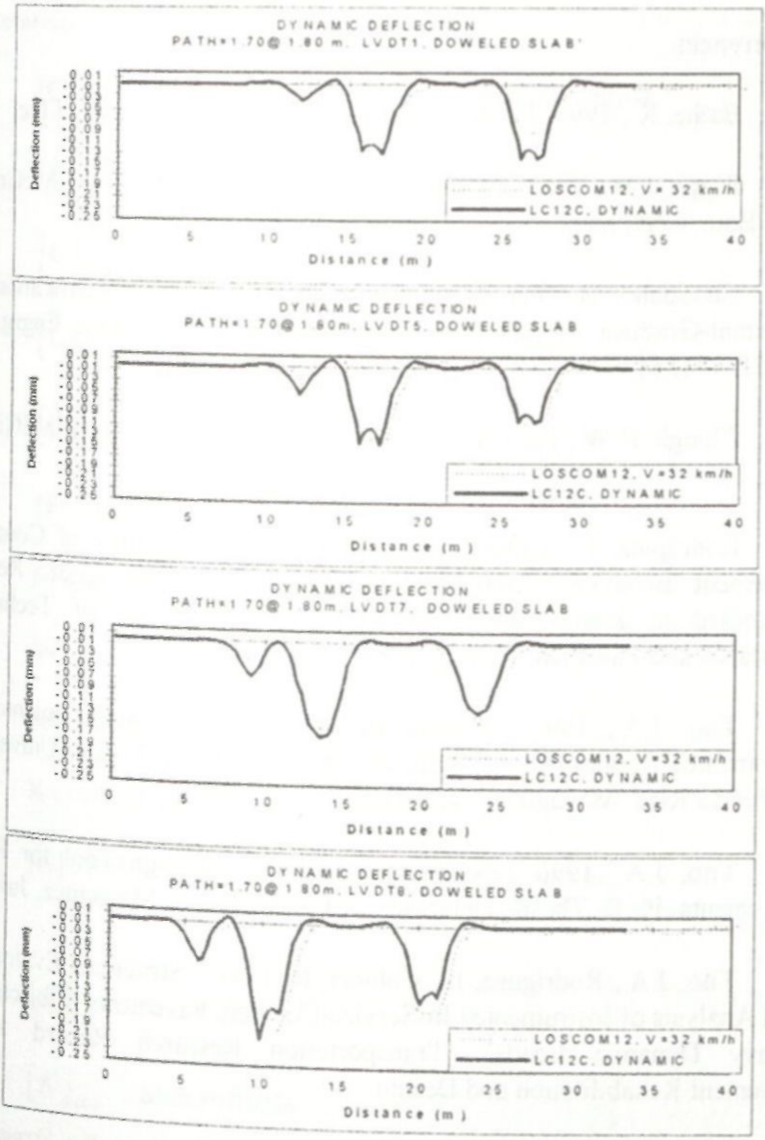


Figure 10. Comparison between a typical experimental test (LOSCOM12) and UPR-PAV12 result (LC12C) (Tito, 1997).

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Notation

[C]: Damping matrix, defined using Rayleigh method.

$[C]_{soil}$: Soil damping.

$[C]_{slab}$: Slab damping.

$[C]_{dowel}$: Dowel, tie or beam damping.

$[C]_{truck}$: Truck damping.

{F} : Force vector.

f_i : shape function.

iel : slab element '*ijklmnop*'.

k_v : vehicle suspension stiffness.

k_t : tire stiffness.

[K] : Stiffness matrix of the soil-slab-truck system.

$[K]_{soil}$: Soil stiffness.

$[K]_{slab}$: Slab stiffness.

$[K]_{dowel}$: Dowel, tie or beam stiffness.

$\left[K\right]_{truck}$: Truck stiffness.

k_{ij} : elements of beam stiffness matrix.

$[M]$: Mass matrix.

$\left[M\right]_{slab}$: Slab mass.

$\left[M\right]_{truck}$: Truck mass.

n : number of punctual loads used for distributing the tire load o the slab.

ncoun : number of slab vertical coordinates.

nco(t) : coordinate of the tire t.

ntot : *ncoun* plus truck coordinates.

rug : roughness amplitude.

roughness : roughness vector at tire location.

u_i : displacements in node I due dynamic loads.

$\{\ddot{u}\}$, $\{\dot{u}\}$ and $\{u\}$:vectors of vertical acceleration, velocity, and displacement of the nodes at any given time, respectively.

V : truck velocity.

X_i : absolute distance from truck gravity center to its axles.

#ax : number of axles.

ξ : damping ratio.

ω_m : a fundamental frequency of the system.

ω_n : highest modal frequency contributing to dynamic response.

$f_{(i)}$: shape function of the element PBQ8 for the node i computed for the coordinates (x,y) of the point where the load is acting.