Dynamic Modeling and Control System Design for a Passenger Coaxial Dodecacopter

Mairim Nieves Nevárez
Master of Engineering in Mechanical Engineering
Prof. José R. Pertierra, MBA
Department of Mechanical Engineering
Polytechnic University of Puerto Rico

Abstract *— Personal Aerial Vehicles (PAVs) are drones that provide transportation to passengers. The airframe and propulsion systems of a manned dodecacopter, or Personal VTOL Vehicle, were previously developed by Capstone students from Polytechnic University of Puerto Rico. This paper explores the dynamic model and control system based on this design. The modeled airframe is a coaxial dodecacopter with six arms. It has 12 motors, one pair in each arm: the upper motor rotating in opposite direction from the lower motor. The plant model and PID controller were developed in Simulink. The results confirms that the Personal VTOL Vehicle has sufficient robustness to be further developed into a real-life model, and therefore, is a viable alternative for urban mobility, capable of transporting a person with minimal flight control experience.*

Key Terms *— Controls Engineering, Drones, Mathematical Modeling, Passenger Vehicle.*

PROBLEM STATEMENT

Aerial vehicles, such as drones, have proven to cover a wide range of capabilities, from hobby aircrafts to space and the military. The versatility and capabilities continue to increase with the introduction of new technology. There has been a rapid increase of much smaller Unmanned Aerial Vehicles (UAVs) in the market over the past decade. However, for manned aircraft, or Personal Aerial Vehicles (PAVs), the viability for urban mobility or manned explorations is yet to be explored. It is crucial to understand the current challenges to ensure their practicality and flexibility in day to day or specialized applications.

This design project focuses on building the dynamic model and control system of the Personal VTOL Vehicle Capstone developed by José Noel Caraballo, John M. Agosto Burgos, and Bilal M. Smaili Abounassif. The intent of this design is to explore the viability of this type of vehicle and areas of opportunity for optimizing the technology and aerodynamics.

LITERATURE REVIEW

History

Early developments of modern aerial vehicles are dated back to the 19th century, such as Sir George Cayley's research in aerodynamics which led to the design of a glider that had curved surface that generated lift [1]. In the upcoming decades, other designs included motors and propellers, as well as many changes in the aircraft and wing shapes, and with these improvements flight duration and stability increased as well.

Several advances in the aerodynamic design were achieved, as well as the technology to control equipment remotely with the emergence of radio communications. Great advances and applications of UAVs were mostly in the military and defense for many years until they evolved into the aircrafts known today.

As the capability and reliability increased, so did the availability of equipment, and the applications have been expanded beyond the government and the military. Remotely controlled aerial vehicles can be purchased at much lower costs and are now available for civilian use.

PUPR students developed for their Capstone Project a Personal VTOL Vehicle consisting of a coaxial dodecacopter with six arms that could transport a person. This paper focuses on the study of its Aerodynamics and develops the Control system design.

Aerodynamics Theory

The Personal VTOL Vehicle airframe geometry consists of a double-hex Dodecacopter with coaxial motors, and the configuration is similar to the Figure 1 below. The upper motors rotate clockwise (CW), and the lower coaxial counterparts rotate counterclockwise (CCW). Newton's third law states that for every force in nature there is an equal and opposite reaction. Therefore, this CW/CCW configuration is preferred to be used to minimize momentum. This reduces yaw rotation and provides stability during flight [2]. This configuration is going to be used to develop the equations of motion.

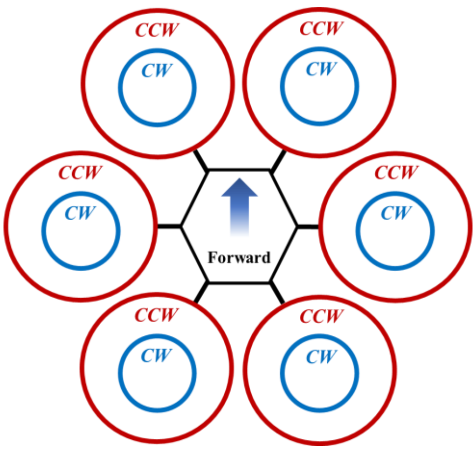

Figure 1

VTOL Motor Rotation Configuration

The mathematical model of a multirotor in space, such as the VTOL, are used to describe the Aerodynamic model, and contain 6 degrees of freedom, the first three degrees represent the position and the latter three represent its orientation. The position in space is determined by the Cartesian coordinate vector $\boldsymbol{\xi}=[x\ y\ z]^T$. Using the right-hand rule, the assumed direction of the Cartesian coordinate axes is North, East, Down (NED), respectively. Its orientation in space is described by the angles roll, pitch, and yaw $\boldsymbol{\eta} = [\varphi\ \theta\ \psi]^T$.

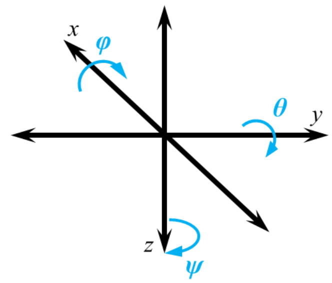

Figure 2

VTOL Orientation Configuration

To obtain a linear approximation of the dodecacopter model, the Euler angle 3 × 3 rotation matrix $\mathbf{R}_i^b$ is used to transform the coordinate position from the body frame *b* to the inertial frame *i*:

$$\mathbf{R}_i^b = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\varphi\sin\theta\cos\psi - \cos\varphi\sin\psi & \sin\varphi\sin\theta\sin\psi + \cos\varphi\cos\psi & \sin\varphi\cos\theta \\ \cos\varphi\sin\theta\cos\psi + \sin\varphi\sin\psi & \cos\varphi\sin\theta\sin\psi - \sin\varphi\cos\psi & \cos\varphi\cos\theta \end{bmatrix} \quad (1)$$

To transform from the inertial frame to the body fixed frame, the transpose of the rotation matrix $\mathbf{R}_b^i = [\mathbf{R}_i^b]^T$ is used [3].

As for most multirotor aerial vehicles, the main forces acting on the VTOL are the force of gravity and thrust from the motors. Other forces being considered are the rotor drag and air resistance. Assuming that the dodecacopter is a rigid body, the Newton-Euler equations are used to describe the vehicle dynamic model with the total forces **ΣF** and moments **ΣM**.

$$\begin{bmatrix} \boldsymbol{\Sigma}\mathbf{F} \\ \boldsymbol{\Sigma}\mathbf{M} \end{bmatrix} = \begin{bmatrix} mI_3 & 0_3 \\ 0_3 & \mathbf{I}_3 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{v}} \\ \dot{\boldsymbol{\omega}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\omega}^\wedge m\mathbf{v} \\ \boldsymbol{\omega}^\wedge \mathbf{I}_3\boldsymbol{\omega} \end{bmatrix} \quad (2)$$

- m is the mass of the vehicle
- I_3 is a 3 × 3 identity matrix
- **I** is the matrix form for the moments of inertia and products of inertia with respect to the vehicle's center of gravity:

$$\mathbf{I} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} \quad (3)$$

- **v** is the linear velocity
- $\boldsymbol{\omega}^\wedge$ is the angular velocity skew-symmetric cross-product matrix

$$\boldsymbol{\omega}\hat{} = \begin{bmatrix} 0 & -\dot{\psi} & \dot{\theta} \\ \dot{\psi} & 0 & -\dot{\varphi} \\ -\dot{\theta} & \dot{\varphi} & 0 \end{bmatrix} \quad (4)$$

- $\boldsymbol{\omega}$ is the angular velocity
- $\dot{\mathbf{v}}$ is the inertial acceleration
- $\dot{\boldsymbol{\omega}}$ is the angular acceleration

Control Theory

The control system manipulates an input signal to obtain a desired output or result. The closed loop control system, which is the model of for the dodecacopter, maintains the system stability by monitoring the observed state against the desired state, and adjusting the variables to reach a set point. The main components for a feedback control loop are the controller, plant, and feedback elements. The controller or actuator computes and executes the correction of the signal error. The plant is the system or process being controlled, it could be the physical system or a mathematical representation of it. The feedback loop provides information to the controller about the actual state of the system and is mainly composed of sensing elements.

For the dodecacopter, the mathematical representation is provided by the equations developed in the following section and MATLAB/Simulink is the software tool used in the development of the plant model and the controller. The plant model for the double-hex dodecacopter corresponds to a nonlinear system, therefore the Euler angles are being used to obtain a linear approximation. The inputs for the controller are the hover thrust, roll, pitch, and yaw moments. The controller will provide the desired output throttle of the twelve rotors.

Methodology

The equations for the main forces and moments acting on the Dodecacopter are the following:

Gravity: $$F_g = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} \quad (5)$$

Thrust (per each rotor j): $$F_{T,j} = \begin{bmatrix} 0 \\ 0 \\ \Sigma k_T \omega_j^2 \end{bmatrix} \quad (6)$$

Torque (per each rotor *j*): $$M_{D,j} = \begin{bmatrix} 0 \\ 0 \\ \Sigma k_D \omega_j^2 \end{bmatrix} \quad (7)$$

To calculate the moments acting on the Dodecacopter, it is considered the distance l and location of each rotor from the center of gravity. The clockwise rotation (CW) is assumed to be positive, while counterclockwise (CCW) is assumed negative.

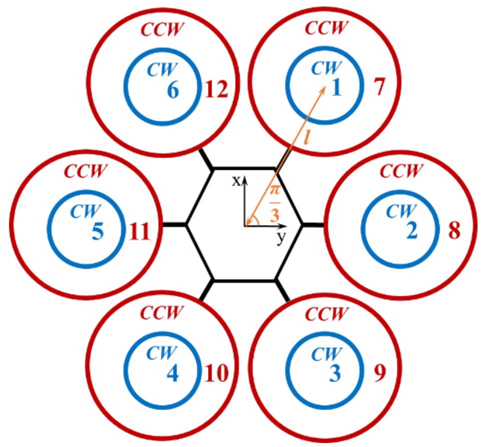

Figure 3

Configuration for the Moments on the Dodecacopter

The model was designed to accept the input throttle, roll, pitch, and yaw from a Logitech Extreme 3D Pro Joystick. The following equations define the input vector $U = [U_1\ U_2\ U_3\ U_4]^T$ [4]:

U_1	Hover Thrust	$U_1 = \Sigma k_T \omega_j^2 = \Sigma T_j = (T_1+T_2+T_3+T_4 + T_5+T_6+T_7+T_8 + T_9+T_{10}+T_{11}+T_{12}$	(8)
U_2	Roll Moment	$U_2 = l\,(-T_2 - T_8+T_5+T_{11}+\cos\frac{\pi}{3}(-T_1-T_7-T_3-T_9+T_4+T_{10}+T_6+T_{12}))$	(9)
U_3	Pitch Moment	$U_3 = l\,(\sin\frac{\pi}{3}(T_1+T_7-T_3-T_9-T_4-T_{10}+T_6+T_{12}))$	(10)

U_4	Yaw Moment	$U_4 = \frac{k_D}{k_T}(\Sigma T_{CW} - \Sigma T_{CCW}) = \frac{k_D}{k_T}(T_1+T_2+T_3+T_4+T_5+T_6-T_7-T_8-T_9-T_{10}-T_{11}-T_{12})$	(11)

The force of gravity in terms of the body frame the rotational matrix $\mathbf{R}_i^b$ is used:

$$F_g^b = \mathbf{R}_i^b F_g = \mathbf{R}_i^b \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} = \begin{bmatrix} -mg\ \sin\theta \\ mg\ \sin\varphi\cos\theta \\ mg\cos\varphi\cos\theta \end{bmatrix} \tag{12}$$

With the Newton-Euler equations, the total forces F_x, F_y, F_z and linear accelerations are obtained:

Force F_x	$F_x = m\ddot{x} + m\dot{z}\dot{\theta} - m\dot{y}\dot{\psi}$	(13)
Force F_y	$F_y = m\ddot{y} + m\dot{x}\dot{\psi} - m\dot{z}\dot{\varphi}$	(14)
Force F_z	$F_z = m\ddot{z} + m\dot{y}\dot{\varphi} - m\dot{x}\dot{\theta}$	(15)
Linear Acceleration $\ddot{x}$	$\ddot{x} = \dot{y}\dot{\psi} - \dot{z}\dot{\theta} - g\sin\theta$	(16)
Linear Acceleration $\ddot{y}$	$\ddot{y} = \dot{z}\dot{\varphi} - \dot{x}\dot{\psi} + g\sin\varphi\cos\theta$	(17)
Linear Acceleration $\ddot{z}$	$\ddot{z} = \dot{x}\dot{\theta} - \dot{y}\dot{\varphi} + g\cos\varphi\cos\theta$	(18)

The system is assumed to be a rigid, symmetric body, therefore, the moments and angular accelerations for the model are:

Moment M_x	$M_x = \ddot{\varphi}(I_{xx}) + (-I_{yy}\dot{\psi})\dot{\theta} + (I_{zz}\dot{\theta})\dot{\psi}$	(19)
Moment M_y	$M_y = \ddot{\theta}\,(I_{yy}) + (I_{xx}\dot{\psi})\dot{\varphi} + (-I_{zz}\dot{\varphi})\dot{\psi}$	(20)
Moment M_z	$M_z = \ddot{\psi}\,(I_{zz}) + (-I_{xx}\dot{\theta})\dot{\varphi} + (I_{yy}\dot{\varphi})\dot{\theta}$	(21)
Angular Acceleration $\ddot{\varphi}$	$\ddot{\varphi} = \frac{M_x - (-I_{yy}\dot{\psi})\dot{\theta} - (I_{zz}\dot{\theta})\dot{\psi}}{(I_{xx})}$ $\ddot{\varphi} = \frac{lU_2 - (-I_{yy}\dot{\psi})\dot{\theta} - (I_{zz}\dot{\theta})\dot{\psi}}{(I_{xx})}$	(22)
Angular Acceleration $\ddot{\theta}$	$\ddot{\theta} = \frac{M_y - (I_{xx}\dot{\psi})\dot{\varphi} - (-I_{zz}\dot{\varphi})\dot{\psi}}{(I_{yy})}$ $\ddot{\theta} = \frac{lU_3 - (I_{xx}\dot{\psi})\dot{\varphi} - (-I_{zz}\dot{\varphi})\dot{\psi}}{(I_{yy})}$	(23)
Angular Acceleration $\ddot{\psi}$	$\ddot{\psi} = \frac{M_z - (-I_{xx}\dot{\theta})\dot{\varphi} - (I_{yy}\dot{\varphi})\dot{\theta}}{(I_{zz})}$ $\ddot{\psi} = \frac{lU_4 - (-I_{xx}\dot{\theta})\dot{\varphi} - (I_{yy}\dot{\varphi})\dot{\theta}}{(I_{zz})}$	(24)

The equations developed are used to build the plant model in MATLAB/Simulink. This is used to design, tune, and test the control system.

To design the controller, first the open-loop model of the motors was implemented in Simulink to calculate the transfer function. In the real world, the motor's input is the voltage, and its output is the rotation speed. The mathematical model represented the input voltage as a step response. The motor model was represented by the dynamic equations below in Laplace domain [5]:

$$P(s) = \frac{\dot{\Theta}(s)}{V(s)} = \frac{K}{(Js+b)(Ls+R)+K^2} \tag{25}$$

- J is the individual rotor moment of inertia
- b is the motor damping coefficient
- K is the motor torque and electromotive force constant
- R and L are the motor's electrical resistance and inductance, respectively

The controller selected for this project is a Proportional-Integral-Derivative controller. The transfer function is

$$C(s) = K_p + \frac{K_i}{s} + K_d s \tag{26}$$

- K_p is the proportional gain which increases the system response
- K_i is the integrative gain which corrects the steady-state error over time
- K_p is the derivative gain which reduces the systems oscillation and overshoot

Aerodynamic models contain a closed-loop feedback subsystem, which may include sensors such as an inertial measurement unit, camera, ultrasonic sensors, or pressure sensors [6]. These components provide the estimated altitude and attitude of the vehicle, but also add noise signals to the system. In this project, the calculated states of the vehicle will represent the closed loop feedback instead.

RESULTS AND DISCUSSION

The mathematical representation of this project was baselined from the Simulink model for the Parrot Minidrone example [6]. This project focused on building the dynamic model with the twelve coaxial motors (VTOL block) and the Flight Controller block.

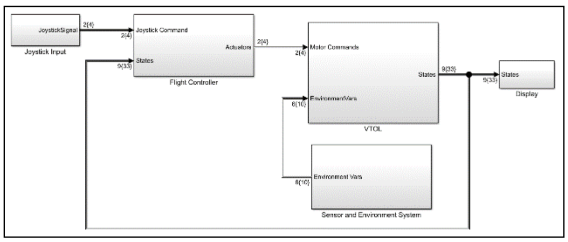

Figure 4

Simulation Model for the VTOL

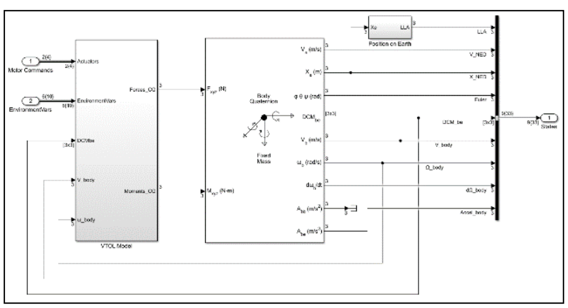

Figure 5

VTOL Airframe Model

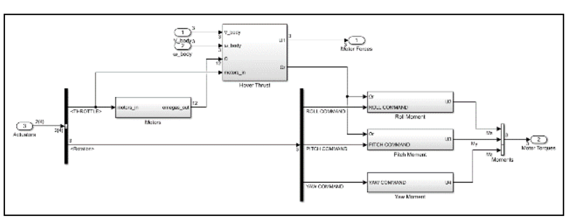

Figure 6

VTOL Model for the Forces and Moments

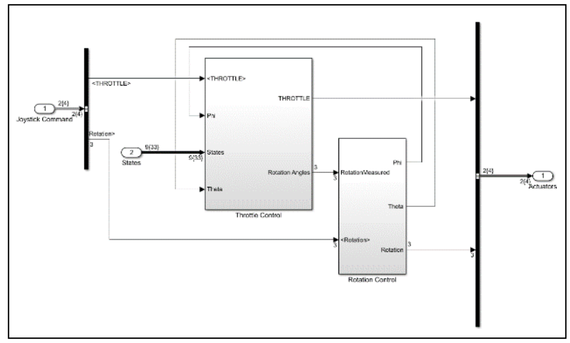

Figure 7

VTOL Flight Controller Model

The PID controller for the motors and rotation angles was designed and tuned in Simulink. The motor transfer function and the feedback controller were mathematically represented using the Simulink model below:

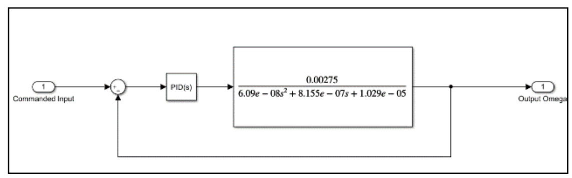

Figure 8

VTOL Motor PID Controller Model

The PID block facilitates the tuning of the proportional, integral, and derivative constants of the model by automatic tuning, and provides the option to manually tune the response as well. This approach was applied for the motor controller.

It is required that for this model, a settling time of less than 2 seconds be accomplished to stabilize the motors response. For the motor model, as well as the rotational angles, the overshoot is required to be less than or equal to 1% and reach a steady state of less than 1%. The following figure presents the results of the tuned motor PID control. By using K_p = 0.003856, K_i = 0.02615, and K_d = 0.0001407, these requirements were met.

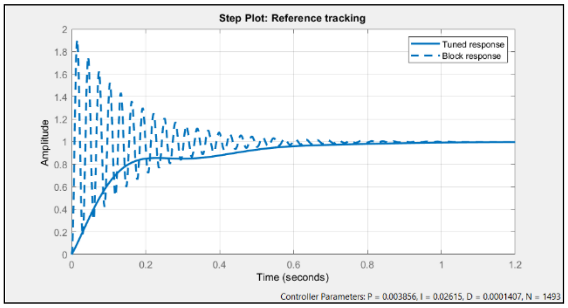

Figure 9

VTOL Motor PID Step Response

The PID control for the rotation angles was tuned manually. The table below presents the controller gains and the step response for each angle, demonstrating that the controller requirements were satisfied.

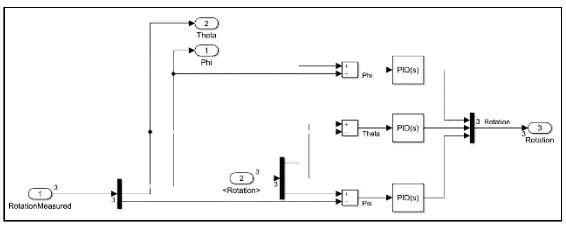

Figure 10

VTOL Rotational Angle PID Controller Model

Table 1

Rotational Angle PID Controller Gains

Rotation Angle	K_p	K_i	K_d
φ	1.0	0.0001	0.001
θ	1.0	0.0001	0.001
ψ	0.99	0.001	0.01

CONCLUSIONS

Aerial Vehicles, such as the mini drones, have been explored in several applications. Despite their known stability with primarily four motors, the fast maneuverability of these devices is something that cannot be implemented in a drone that carries human passengers. This project focused on keeping the human safety factor in mind, and it demonstrated that by applying Newton-Euler equations of motion and implementing a PID controller in the motors' thrust, the roll, pitch, and yaw angles, a stable and robust design can be obtained with twelve motors.

As part of future work, the model can be further developed with the design of various subsystems such as power and sensor subsystem, which provide better estimation of the vehicle's behavior. Although this may add complexity, it will significantly increase the model accuracy to the real-world environment.

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