Thermoeconomics: Solving Stochastics Partial Differential Equations- From Mechanical Engineering to Financial Engineering

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Abstract — *The research explored and examined six diverse typical mechanical engineering methodologies such as CFD, finite element analysis, and design optimization, and their application to solve problems in econophysics using practical problems solved with enough detail to serve as an introduction to the theory of mechanical engineering techniques, and their correspondence analog concept in econophysics. The usefulness of the mechanical engineering methodologies utilized in another field of knowledge was demonstrated in sufficient substance to serve as an introductory basis for further studies for the interested researcher with a background in civil or mechanical engineering. This research could also serve as the foundation for an introductory course in econophysics at the fourth or fifth-year level in engineering in those engineering specialties. The objectives were to develop a deep comprehension of several of the econophysics models already being developed, prepare a basic introduction of numerical examples to complement that research and compare the results of such models against actual data results to evaluate their effectiveness. These objectives were met.*

Key Terms — *Econophysics Financial Engineering, Stochastics Partial Differential Equations, Thermoeconomics.*

INTRODUCTION

Arroyo Colón [1] developed a connection between thermodynamic physical concepts, such as heat reservoirs and thermal cycles, and economic ones such as supply and demand. His works belong to the emerging field of econophysics (also known as "thermo-economics") which applies theories and methods originally developed by physicists to solve complex, chaotic nonlinear stochastic problems in economics and the study of financial markets, posing that such markets are, after all, physical manifestations of the natural world, although created by human interactions.

The task here is to use some of the typical toolbox from mechanical engineering, such as finite element analysis, CFD (computational fluid dynamics), and thermodynamics, to solve typical problems in finance, such as optimal investment under stochastic uncertainty, capital budgeting under rationing, etc.

Two numerical cases using the most appropriate mechanical technique will be solved while explaining the analogy between the fields. It is expected that the cases will serve as a tutorial for the newly initiated mechanical engineer (ME) in the world of finance, and to illustrate in detail how a ME could translate the knowledge acquired in a new area of research.

BACKGROUND

Based on the field of econophysics, Arroyo Colón [1] proposed a model of economics using an analogy to thermodynamics while introducing the concept of the "Value Multiplier" as a fundamental addition to any such model in an attempt to make an analogy with the "Keynesian Multiplier". The idea of using thermodynamics and physics models to have a better understanding of the economy was intriguing. Jovanovic and Schinckus [2] defined the origin of the field of econophyics in "the mid-1990s, in the wake of some of the most recent advances in physics, a new approach to dealing with financial prices emerged [2]."

More recently, Ducournau, [3] considered the stock market as a physical system analogous to a fluid evolving in a macroscopic space subject to a force that influences its movement over time. Ducournau then showed theoretically that fluid mechanics equations can be used to describe stock market physical properties. Even more recently, Campuzan [4] provided an algorithm utilizing the Crank-Nicolson Method used in CFD to solve the PDE of a European call option. He also explained that this methodology is typically used in finance.

PROBLEM

It is proposed to investigate how diverse typical mechanical engineering methodologies such as CFD, and related optimization approaches could be applied to solve problems in finance and investment.

Research Description

Typical problems in thermo-economics (also known as econophysics) will be described in detail, and then they will be solved using one or more of the typical mechanical engineering tools, making emphasis on developing the models' analogies, their potentials, and limitations as well as possible future developments and applications. This is a new area of interdisciplinary study that promises great challenges and opportunities.

Research Objectives

This research is aimed at exploring and examining diverse typical mechanical engineering methodologies such as CFD, finite element analysis, and related optimization approaches and their application to solve problems in finance and investment.

Research Contributions

The usefulness of the mechanical engineering methodologies utilized in another field of knowledge will be demonstrated in sufficient substance to serve as an introductory basis for further studies for the interested researcher with a background in civil or mechanical engineering. It will also serve as the foundation for an introductory course at the fourth or fifth-year level in engineering in those engineering specialties.

METHODOLOGY

Several complete numerical cases examples of the methods and approaches used in mechanical engineering that have direct application in econophysics will be discussed with enough detail to serve as an introduction to the topics. Numerical examples will also be discussed as well as the possible parallels between the two fields. The didactical approach used will be first to solve a known or familiar problem in mechanical engineering using a typical mechanical engineering approach and then, an unfamiliar or new problem in econophysics will be solved using the same technique.

RESULTS & DISCUSSION

Two cases will be discussed in detail. The first case is an optimization example, and the second case is a Computational Fluids Dynamics (CFD) problem.

First Case: An Optimization in Mechanical Engineering Example. Optimum Design of a Rectangular Beam

A beam of rectangular cross section as shown in Figure 1 is subjected to a maximum bending moment of M and a maximum shear of V. The bending stress in the beam is calculated as:

$$\sigma=6M/(bd^2) \quad (1)$$

and the average shear stress in the beam is calculated as:

$$\tau=3V/2bd \quad (2)$$

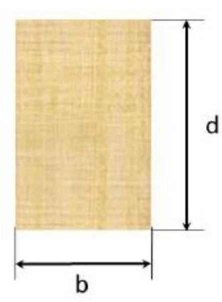
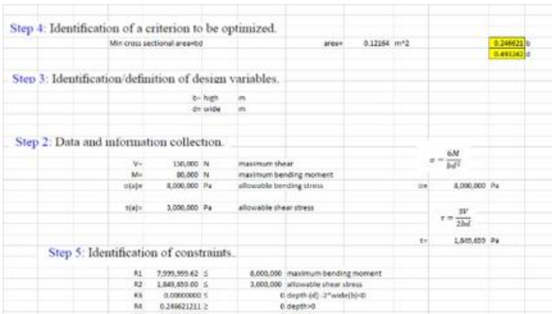


Figure 1
Rectangular Beam for Linear Programming Example. Source: Course Notes From ME6360 Optimization in Engineering [5].

Where d is height, and b is the width of the beam. It is also desirable to have the depth of the beam not exceed twice its width. The formulation of the standard design optimization model for minimum cross-sectional will be done and solved it with the Excel solver using this data: M = 140 kN m, V = 24 kN, allowable bending stress σa=165 MPa; allowable shear stress τa=50 MPa. This is Step 1 in Arora's [6] recommended design optimization model, see Table 1.

Table 1

Optimization Model Steps. Source Own Elaboration


Step 4: Identification of a criterion to be optimized.
Min cross sectional area=bd
area= 0.12164 m^2
b
d
Step 3: Identification/definition of design variables.
b= high m
d= wide m
Step 2: Data and information collection.
V= 130,000 N maximum shear
M= 80,000 N maximum bending moment
σ(a)= 8,000,000 Pa allowable bending stress
τ(a)= 3,000,000 Pa allowable shear stress
σ= 8,000,000 Pa
Step 5: Identification of constraints.
R1 7,999,999.62 ≤ 8,000,000 maximum bending moment
R2 1,849,659.00 ≤ 3,000,000 allowable shear stress
R3 0.00000000 ≤ 0 depth (d) -2*wide(b)<0
R4 0.246621211 ≥ 0 depth>0
R5 0.493242423 ≥ 0 wide>0

Solution to the Problem

The standards steps for the design optimization model are (Arora [6]):

Step 1: Project/Problem Description. Simply put, to state the problem to be solved in as clearly and organized a manner as possible. See Table 1. For this example, that would be the problem statement given before.

Step 2: Data and Information Collection: Gather the available data in an organized way. In this case, Table 2 shows the data from the statement:

Table 2

Initialization Data. Source Own Elaboration

V=	24,000	N	maximum shear
M=	140,000	N	maximum bending moment
σ(a)=	165,000,000	Pa	allowable bending stress
τ(a)=	50,000,000	Pa	allowable shear stress

Step 3: Definition of Design Variables. The important variables must be identified correctly. From Figure1 is obtained, b= high and d=wide.

Step 4: Optimization Criterion. Arora states this step as no. 4, the researcher does not agree with that assessment, the recommendation would be to always set the optimization criteria as the second step, that will almost always help in the identification of the design variables (Step 3) and will easily flow into the data collection process (Step 2). The goal in this example is to minimize the cross-sectional area=bd

Step 5: Formulation of Constraints. This is the most difficult task as the wording of the problem statement must be carefully translated to a set of inequations so the problem could be solved using linear programming standard format.

Standard Design Optimization Model:

Find an n-vector x= (x1, x2…. xn) of n design variables to:

Minimize a cost function:

$$f(x)= (x_1, x_2 \ldots . x_n) \quad (3)$$

Subject to the p equality constraints

$$h_j(x)= (x_1, x_2 \ldots . x_n) =0; j=1 \text{ to } p \quad (4)$$

and the m inequality constraints

$$g_i(x)= (x_1, x_2 \ldots . x_n) \leq 0; i=1 \text{ to } m \quad (5)$$

In particular as Table 3 shows:

Table 3

Problem Restrictions. Source Own Elaboration

R1	7,999,999.62	≤	8,000,000	maximum bending moment
R2	1,849,659.00	≤	3,000,000	allowable shear stress
R3	0.00000000	≤	0	depth (d) -2*wide(b)<0
R4	0.246621211	≥	0	depth>0
R5	0.493242423	≥	0	wide>0

This format is perfect for the Excel Solver feature, see Figure 2:

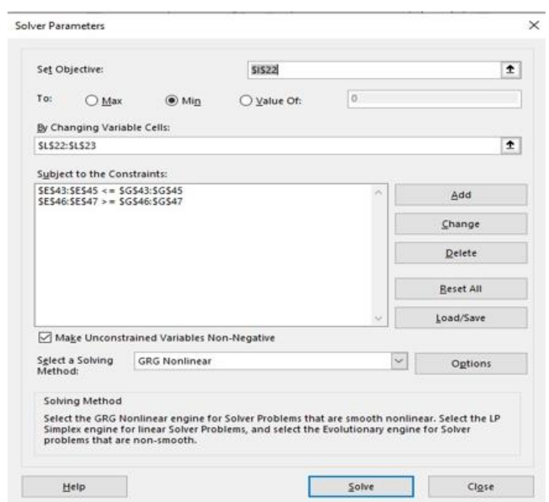


Figure 2

Excel Solver Set-up. Source Own Elaboration

Where the set objective is cell I22 where the area calculation takes place, and it is set to be minimized by changing the set of variables in L22 and L23 (high and wide respectively). Subject to the set of constraints equations listed from R1 to R5 (some to be less than or equal to and others to be set more than or equal to). Also, note the algorithm used is a nonlinear one, this is not an issue because most nonlinear algorithm work with linear situations without any concern. It is also of great importance to ensure the variables are non-negative (that is, wide and high cannot be negative numbers). Once set up as stated, the solver algorithm takes care of the solution by finding the minimum area equal to 0.1262 m^2 and its variables: wide =0.4932 m and high =0.2466 m.

Optimization in Econophysics Example: Optimum Funding of a Pension Liability

Baker [7] provides a typical retirement problem faced by financial analysts when recommended options to cover the expected financial needs of retirees whose social security, 401K, and other investments are found inadequate to satisfy future expenses.

Step 1: Project/Problem Description In this example, the fund is committed to a set of annual payments stretching over 14 years, starting at the end of year as shown in Table 4:

Table 4
Dollar Amount in'000.00 Needed Per Year. Baker [7]

Year	00	01	02	03	04	05	06	07
$000	0	12	14	15	16	18	20	21
Year	08	09	10	11	12	13	14	
$000	22	24	25	30	31	31	31	

The fund consists of the combination of three bonds (See Table 5) that will be bought initially with certain amount of cash (whose minimum is the main question to be ascertained) and then, each years the proceeds coming from the bonds' annual coupons payments plus the face value redeemed at the end of each set of bonds must match the amount of cash needed. The funds will be kept at a bank account that pays 4% annual interest.

Table 5
Bonds Data. Baker [7]

	Current Price	Annual Coupon	Years to Maturity	Face Value
Bond1	$ 980	$ 60	5	$ 1,000
Bond2	$ 970	$ 65	11	$ 1,000
Bond3	$ 1,050	$ 75	14	$ 1,000

Step 2: Data and Information Collection. For simplicity, the analyst selected just three bonds which have different maturities (due date when the face value of $1,000.00 will be cashed or redeemed) and different current prices and annual coupon payments. That is, you would buy certain amounts of bonds no. 1 (x1, amount that the optimization problem will state as part of the solution) at $980.00 each, totalizing x1*980 dollars needed today for the buying transaction, and those will pay annually pay you a total of x1*60 from the C1 coupon payments for a duration of M1=5 years when, in addition of the coupon payment due you will receive a total of x1*1,000 from redeeming the bond's face value.

The selection of the bond is not discussed here nor in the original paper but that is another crucial process where mathematical algorithms and experience are required. Here, as the main point is the illustration of the technique, stating the features of the three selected bonds will suffice as shown in Figure 3.

Figure 3
Set-up of the LP System. Source Own Elaboration

Step 3: Definition of Design Variables. The decision variables are the initial amount to invest in each of the three bonds and the amounts that will be saved in each of the 14 years the investment scheme will be working. In this way, the algorithm establishes the minimum amount of money initially needed to fund the plan and set up the initial amount that will be in the bank as well as the amounts that will be kept each year in the bank.

Step 4: Optimization Criterion. The goal here is to have the minimum initial amount of money to buy the, also minimum, needed amount of cash (that is, the minimum capitalization) that will make the yearly retirements feasible.

Step 5: Formulation of Constraints. In this example, the constraints must be solved as equations, not as inequations. The reason is that for each and every year an exact amount of money is needed (any excess of money in a particular year will be saved in the bank account) and, it is intended to work that, in the last years all the money from the bank is retired (to complete that year requirements) so at the end the bank account must be closed.

The format is perfect for the Excel Solver feature as shown in Figure 4:

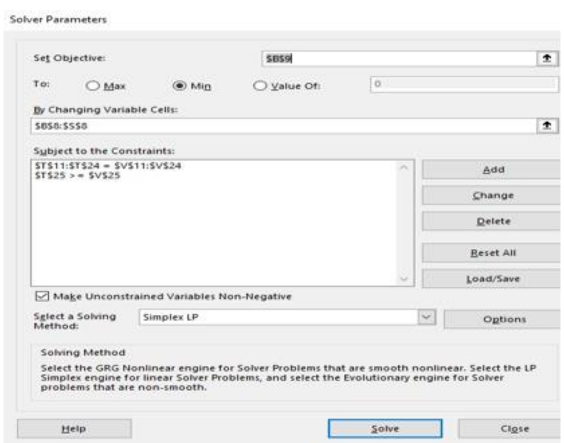


Figure 4

Excel Solver Set-up. Source Own Elaboration

Where the set objective is cell B9 where the area calculation takes place, and it is set to be minimized by changing the set of variables from B8 to S8 (the initial amount of money to invest in each of the 3 bonds and to save in the bank account every year for the entire investment scheme). Subject to the set of constraints equations listed from T11 to T24 (all equal to the amount of money needed in a particular year- cells V11 to V24 but the last one, to provide for the case that any extra money will be saved in the bank account; situation that would physically then be required to retire). Also note the algorithm used is a the linear one. It is also of great importance to ensure the variables are non-negative (that is, wide and high cannot be negative numbers). Once set up as stated, the solver algorithm takes care of the solution by finding the minimum amount of initial proceeds needed $186,768.40 and the amount of bonds needed from each class as: .73.695, 77.208 and 28.837 with an initial bank account setup with $9,376.30.

It is to be noted that, in practice, it is impossible to buy 73.695 bonds, you either buy 73 or 74; that is, this problem is really one situation of "integer optimization". Now, that is easily resolved by just adding the additional integer constraints to the three initial bonds; thus, not allowing the bonds to have a non-integer value.

Second Case: Shear Stress of Viscous Flow Over a Flat Plate: A Mechanical Engineering Example

Computational Fluids Dynamics (CFD) is a numerical method approximation approach. The typical case is a continuous differential equation (it could be linear or nonlinear) that is approximated (discretized) by a set of linear algebra equations and then, these equations are iteratively solved at different discrete points (a grid) until some minimum error is achieved.

A numerical example from the CFD notes course will illustrate.

Consider the viscous flow of air over a flat plate. At a given station on the flow direction, the variation of the flow velocity u, in the direction perpendicular to the plane (the y direction as per Figure 5 is given by the expression:

$$u=1582(1-e^{(-y/L)}) \quad (6)$$

Where L=characteristics length = 1 in. The units of u are feet per second (ft/s). The viscosity coefficient $\mu=3.7373x10^{(-7)}$ slug/(ft*s). Use this equation to provide the value of u at discrete grid points equally spaced in the y direction with $\Delta y=0.1$ in. The values of u are discrete values at the discrete grid points locate at y= 0.0,0.1.0.2 and 0.3 (obtained from a numerical finite-difference solution) were found as (see Table 5):

Table 5
Values of U at the Discrete Grid Points. Source: Notes from ME 6160: Computational Fluid Dynamics [8]

	y	u
1	0	0
2	0.1	150.5472
3	0.2	286.7679
4	0.3	410.0256

Using the discrete values, calculate the shear stress at the wall □w in three different ways, namely (see Figure 5 and Table 6):

a) Using a first-order one-side difference
b) Using the second-order one-side difference
c) Exact value with the formula

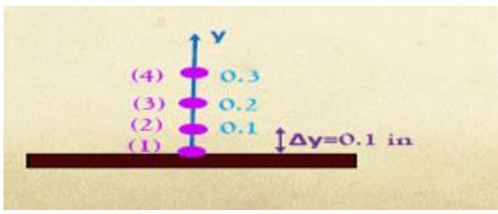


Figure 5
Problem to be Solved. Source Class Notes From ME 6160: Computational Fluid Dynamics [8]

Table 6
Data (EXCEL). Source: Own Elaboration

				L=	1	in	μ=	3.7373E-07	slug/(ft*s)
	u=	1582	1-	exp(-y/L)	ft/s				
	y	u				delta_y=	0.1		
1	0	0							
2	0.1	150.5472							
3	0.2	286.7679							
4	0.3	410.0256							

a) First Order Difference Approximation (Figure 6):

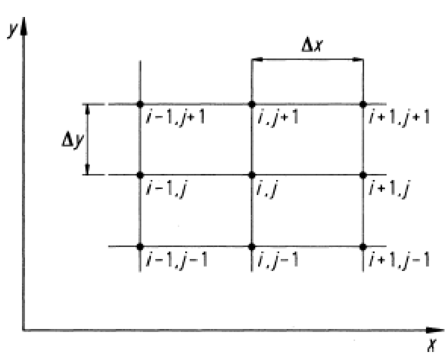


Figure 6
Typical Grid. Anderson [9]

The grid is solved for a point (i, j) which distances itself from other points by a difference Δx and Δy. For simplicity, just using a 1D grid, a Taylor expansion around that point results in:

$$u_{i+1,j}=u_{i,j}+\left(\frac{\partial u}{\partial x}\right)_{i,j}\Delta x+\left(\frac{\partial^2 u}{\partial x^2}\right)_{i,j}\frac{(\Delta x)^2}{2!}+\left(\frac{\partial^3 u}{\partial x^3}\right)_{i,j}\frac{(\Delta x)^3}{3!}+\dots \qquad (7)$$

Assuming higher orders terms than Δx are negligible, it is obtained:

$$u_{i+1,j}\approx u_{i,j}+\left(\frac{\partial u}{\partial x}\right)_{i,j}\Delta x \qquad (8)$$

Solving for the derivative,

$$\left(\frac{\partial u}{\partial x}\right)_{i,j}\approx\frac{u_{i,j}-u_{i-1,j}}{\Delta x} \qquad (9)$$

That could be as precise as wanted as $\Delta x \to 0$.

Therefore, for the example, then:

$$\left(\frac{\partial u}{\partial x}\right)_j\approx\frac{u_2-u_1}{\Delta y}\approx\frac{150.5472\,(ft/s)-0}{0.1\,in}\approx 1{,}505.472\,(ft/(s*in)) \qquad (10)$$

$$\tau_w\approx 3.7373*10^{-7}\,(slug/(ft*s))*(1{,}505.472\,(ft/(s*in)))*12\,(in/(1\,ft))\approx 6.7517*10^{-3}\,(lb/(ft^2)) \qquad (11)$$

b) Second Order Difference Approximation:

Another approach that can be used for viscosity (speed, etc.) is to assume that, at the boundaries the equation can be approximated by a parabolic linear equation. If this could be the case, then:

$$u=a+by+cy^2 \qquad (12)$$

$$u_2=a+b(\Delta y)+c(\Delta y)^2 \qquad (13)$$

$$u_3=a+b(2\Delta y)+c(2\Delta y)^2 \qquad (14)$$

And,

$$\partial u/\partial y=b+2cy \qquad (15)$$

At the boundary,

$$u_1=a+b(0)+c(0)^2=a \qquad (16)$$

and

$$=b+2c(0)=b \qquad (17)$$

Solving for b

$$b=\frac{-3u_1+4u_2-u_3}{2\Delta y}=\left.\frac{\partial u}{\partial y}\right|_1 \qquad (18)$$

In this case, then

$$\left.\frac{\partial u}{\partial y}\right|=\frac{-3u_1+4u_2-u_3}{2\Delta y}=\frac{-3(0)+4(150.5472)(ft/s)-286.7679(ft/s)}{2(.1)(in)}=1{,}577.104\,ft/(s*in) \qquad (19)$$

$$\tau_w\approx 3.7373*10^{-7}\,(slug/(ft*s))*(1{,}577.104\,(ft/(s*in)))*12\,(in/(1\,ft))\approx 6.7517*10^{-3}\,(lb/(ft^2))\approx 7.0729*10^{-3}\,(lb/(ft^2)) \qquad (20)$$

c) Exact Value:

Using the formula given as the “exact value” of the flow velocity, u, then:

$$u=1582*(1-e^{-y/L}) \qquad (21)$$

$\partial u/\partial y=1582/L*(e^{(-y/L)})=1582/1*(e^{(-y/1)})$ (22)

At the boundary,

$\partial u/\partial y=1582*(e^{(-(0))})=1582$ ft/(s*in) (23)

$\tau_w \approx 3.7373*10^{(-7)}$ (slug/(ft*s)))*(1,582(ft/(s*in))*12(in/(1 ft))≈6.7517*10^(-3) (lb/(ft^2))≈7.0949*10^(-3) (lb/(ft^2)) (24)

The percentage errors from the exact value are estimated as:

$\varepsilon_1=1-(6.7517*10^{(-3)})/(7.0949*10^{(-3)})\approx 4.84\%$ (25)

$\varepsilon_2=1-(7.0729*10^{(-3)})/(7.0949*10^{(-3)})\approx 0.31\%$ (26)

In summary (see Table 7):

Table 7
Summary of the Case. Source: Own Elaboration

Methodology	$\frac{\partial u}{\partial y}$	τ_w	Error (Difference from Exact Value)
First Order Difference Approximation	$1,505.472(\frac{ft}{s*in})$	$6.7517 * 10^{-3} (\frac{lb}{ft^2})$	4.84%
Second Order Difference Approximation	1,577.104 ft/(s * in)	$7.0729 * 10^{-3} (\frac{lb}{ft^2})$	0.31%
Exact Value	1582 ft/(s * in)	$7.0949 * 10^{-3} (\frac{lb}{ft^2})$	0

Velocity distribution in two parallel plates: A Mechanical Engineering Example of the CFD Implicit Method

Let's consider a fluid bounded by two parallel plates extended to infinity as seen in Figure 7. The planar walls and the fluids are initially at rest. Now, the lower wall is suddenly accelerated in the x direction, as illustrated in Figure 7. The Navier-Stoke equations for this problem may be expressed as:

$\partial u/\partial t=\nu(\partial^2 u)/(\partial y^2)$ (27)

The fluid is oil with a kinematic viscosity of 0.000217 m2/s, and the spacing between plates is 40 mm. The velocity of the lower wall is specified as Uo=40 m/s. A solution for the velocity is to be obtained up to 1.08 seconds using implicit discretization and using 30 times intervals as shown in Figure 7.

a) The initial conditions, u= Uo for y=0 and u= 0 for 0<y< h)┤

b) The boundary conditions, u= Uo for y=0 and u= 0 for y= h)┤

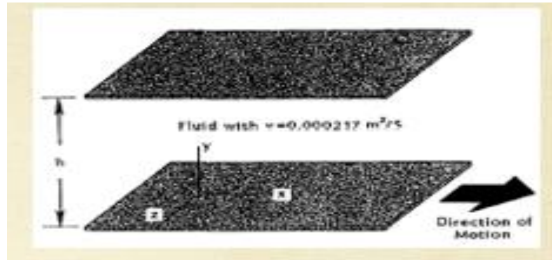


Figure 7
2 Parallel Plates. Class notes from ME 6160: Computational Fluid Dynamics [8]

Theory

There are two methods widely used to solve PDEs: the explicit and the implicit approaches. Both are based on the discretization of the PDE and then solving the resulting difference equations in a recurrent manner.

The extra complexity of the implicit methodology is compensated for by its superior stability.

The explicit method of solution describes an unknown value at a certain grid point depending on the known values at neighboring grid points; in particular after discretization, for the diffusion equation (which is of parabolic type), the recurrent difference equation is:

$T_i^{(n+1)}=T_i^n+s(T_{(i+1)}^n-2T_i^n+T_{(i-1)}^n)$ (28)

Thus, it can be calculated $T_i^{(n+1)}$-values at the first-time row depending on the boundary and initial values at $T_i^n=0$ line. Similarly, the values at the second time row depend on the values that were calculated for the first row, and so on. This methodology is illustrated in Figure 8.

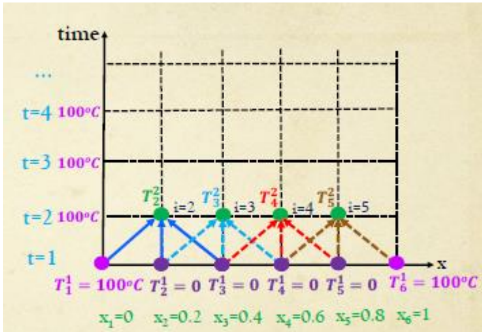


Figure 8

Use of Explicit Method: Illustration of the Calculation Procedure. Class Notes From ME 6160: Computational Fluid Dynamics [8]

This methodology suffers from innate instability; in order to ensure a stable (convergent) solution, it must be selected:

s=α vΔt/ 〖(Δx)〗^2 (29)

On the other hand, the implicit method is unconditionally stable but suffers from the need of an extra subroutine to solve the linear equations that are created. This causes it to take a lot of computing power. For this, the Thomas algorithm can be used, among others.

The Implicit methodology results in the recurrent formula

u_j^(n+1)=u_j^n+(vΔt/ 〖(Δx)〗^2)(u_(j+1)^(n+1)-2u_j^(n+1)+u_(j-1)^(n+1)) (30)

But the coefficients are calculated from:

u_j^n=u_j^(n+1)-(vΔt/(Δx)^2)(u_(j+1)^(n+1))+2 vΔt/(Δx)^2 〖(u〗_j^(n+1))-vΔt/(Δx)^2 〖(u〗_(j-1)^(n+1)) (31)

For simplicity,

a_j^n=-(vΔt/(Δx)^2) (32)

b_j^n=(1+2 vΔt/(Δx)^2) (33)

c_j^n=-vΔt/(Δx)^2 (34)

Thus, the discretized equation is:

u_j^n=a_j^n (u_(j+1)^(n+1))+b_j^n 〖(u〗_j^(n+1))+c_j^n 〖(u〗_(j-1)^(n+1)) (35)

The CFD Approach Applied to Solve the Black-Scholes Ordinary Differential Equation (ODE): An Econophysics Example

Recall the Black-Scholes PDE as:

(∂V(S,t))/∂t+(1/2 σ^2 〖S(t)〗^2) (∂^2 V(S,t))/(∂S^2)+(r*S(t)) (∂V(S,t))/∂S-r*V(S,t)=(36)

The approximation using the recurrent difference equation:

(V_(n.j+1-) V_(n.j))/Δt+a_(n,j) ((V_(n+1This meth.j) 〖-2V〗_(n.j)+V_(n-1.j))/(ΔS^2))+b_(n,j) ((V_(n+1.j)-V_(n-1.j))/2ΔS)+c_(n,j)(V_(n.j))=o(Δt,ΔS^2) (37)

After substituting the specifics, then:

V_(n.j+1)=1/2 (σ^2 n^2-rn)ΔtV_(n-1.j)+[1-(σ^2 n^2+r)Δt 〖]V〗_(n.j)+1/2(σ^2n^2+rn)ΔtV_(n+1.j) +o(Δt^2,ΔtΔS^2) (38)

The equation holds for n=1,2, 3...N-1 since neither V-1,j nor V-n+1,j are defined. There will be N-1 equations and N+1 unknown. Thus, two more equations to solve the system are needed. These are the boundary conditions imposed on the system for when n=0 and 0 and n=N. They depend on the particular case, for instance, to price a European call option.

At S=0 (asset price with no value) then V0,j =0. By necessity if the asset is worth nothing, the value of an option to buy it must also be nothing.

When S->∞ (a very big asset price. Say Smax), then the value at that moment of the call option ->Smax- E*e^(-r*(T-t))). That is, the value of the call option is the difference between the value of that asset less its expected cost. Therefore, the upper boundary condition would be: VN,j =N*ΔS-Ee^(-r*j*(Δt)).

An analogous approach is used for the put option. Only the work on the call option in this case will be done. Figure 9 and Figure 10 show the resulting graphs.

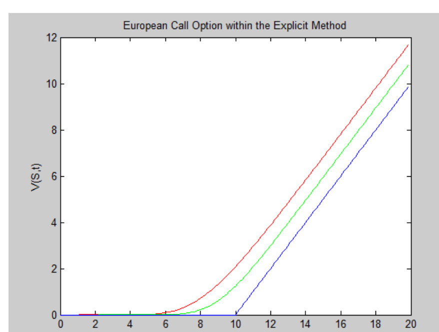


Figure 9

Solution for the European Call Option by Using an Explicit Method. Source: Own Elaboration

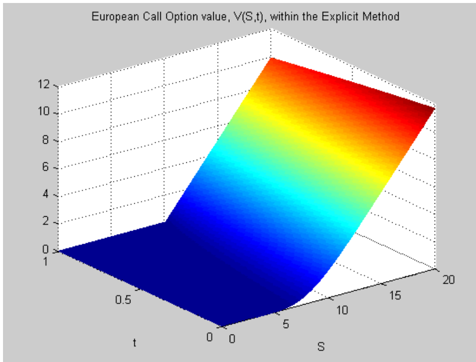


Figure 10
Solution for the European Call option by Using an Explicit Method. Source: Own Elaboration

Blue represents the value of the option at expiration, green half a year before expiration, and red one year before expiration (price at the time the contract is signed).

CONCLUSIONS

The exploration and examination of diverse typical mechanical engineering methodologies such as CFD, finite element analysis, and related optimization approaches and their application to solve problems in econophysics using practical problems solved with enough detail to serve as an introduction to the theory of mechanical engineering techniques, and their correspondence analog concept in econophysics was performed.

Several complete numerical cases examples of the methods and approaches used in mechanical engineering that have direct application in econophysics were discussed with enough detail to serve as an introduction to the topics. Those numerical examples were discussed as well as the parallels between the two fields. The didactical approach used was first to solve a known or familiar problem in mechanical engineering using a typical mechanical engineering approach and then, an unfamiliar or new problem in econophysics was solved using the same technique.

The different models tested for each technique were discussed in detail in each case-example chapter. Visual graphs showing the results and illustrating that the mechanical engineering approach in each case is directly applicable to the econophysics situation, were shown. For every case, the mathematical models from the mechanical engineer/standard physics and econophysics methodologies were discussed and then compared.

The objectives were, as previously stated, to develop a deep comprehension of several of the econophysics models already being developed, prepare a basic introduction of numerical examples to complement that research, and compare the results of such models against actual data results to evaluate their effectiveness. These objectives were met.

FUTURE WORKS

The usefulness of the mechanical engineering methodologies utilized in another field of knowledge was demonstrated in sufficient substance to serve as an introductory basis for further studies for the interested researcher with a background in civil or mechanical engineering. This research could also serve as the foundation for an introductory course in econophysics at the fourth or fifth-year level in engineering in those engineering specialties.

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REFERENCES

[1] L. B. Arroyo-Colón, “A thermal model of the economy,” M.S. Thesis, Physics, Universidad de Puerto Rico, Mayagüez, Puerto Rico, 2010. Available: https://scholar.uprm.edu/server/api/core/bitstreams/0cbb336f-776d-467d-be3e-c258bbd08af5/content

[2] F. Jovanovic, and C. Schinckus, *Econophysics and financial economics: An emerging dialogue*, New York: Oxford University Press, 2017.

[3] G. Ducournau, *Stock market's physical properties description based on Stokes law*. arXiv:2103.00721, 2021.

[4] F. Capuzan, (2023) *The Crank-Nicolson Method in Layman's terms* [online]. Available: https://www.linkedin.com/posts/florian-campuzan_cranknicolsonmethod-quantitativefinance-optionpricing-activity-7122413852792602624-doLp/?utm_source=share&utm_medium=member_android.

[5] I. Torres-Pizarro, *Course notes from ME 6340: Optimization in Engineering WI-21 Dr. Bernardo Restrepo*, Universidad Politécnica de Puerto Rico, San Juan, PR, 2021

[6] J. S. Arora, *Introduction to Optimum Design*,4th Ed. Elsevier, 2017.

[7] K. Baker, "Gaining Insight in Linear Programming from Patterns in Optimal Solutions." *INFORMS Transactions in Education*, vol.1, num.1, 4-17, 2000, doi.org/10.1287/ited.1.1.4.

[8] I. Torres-Pizarro, *Course notes from ME 6160: Computational Fluid Dynamics. FA-23 Dr. Moisés Ángeles*, Universidad Politécnica de Puerto Rico, San Juan, PR, 2023

[9] J. D. Anderson Jr., J. Degroote, G. Degrez, E. Dick, R. Grundmann and J. Vierendeels, *Computational Fluid Dynamics: An Introduction*, 3rd. Ed. Springer, 2009.