Thermoeconomics: Solving Stochastics Partial Differential Equations- From Mechanical Engineering to Financial Engineering

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Abstract

The research explored and examined six diverse typical mechanical engineering methodologies such as CFD, finite element analysis, and design optimization, and their application to solve problems in econophysics using practical problems solved with enough detail to serve as an introduction to the theory of mechanical engineering techniques, and their correspondence analog concept in econophysics. The usefulness of the mechanical engineering methodologies utilized in another field of knowledge was demonstrated in sufficient substance to serve as an introductory basis for further studies for the interested researcher with a background in civil or mechanical engineering. This research could also serve as the foundation for an introductory course in econophysics at the fourth or fifth-year level in engineering in those engineering specialties. The objectives were to develop a deep comprehension of several of the econophysics models already being developed, prepare a basic introduction of numerical examples to complement that research and compare the results of such models against actual data results to evaluate their effectiveness. These objectives were met..

Key Terms. Econophysics Financial Engineering, Stochastics Differential Equations, Thermoeconomics.

Introduction

Arroyo Colón [1] developed a connection between thermodynamic physical concepts such as heat reservoirs and thermal cycles and economic ones such as supply and demand. His works belong to the emerging field of econophysics (also known as "thermo-economics") that applies theories and methods originally developed by physicists to solve complex, chaotic nonlinear stochastic problems in economics and the study of financial markets, posing that such markets are, after all, physical manifestations of the natural world, although created by human interactions. The task here is to use some of the typical toolbox from mechanical engineering, such as finite element analysis, CFD (computational fluid dynamics), and thermodynamics, to solve typical problems in finance, such as optimal investment under stochastic uncertainty, capital budgeting under rationing, etc. Two numerical cases using the most appropriate mechanical technique will be solved while explaining the analogy between the fields. It is expected that the cases will serve as a tutorial for the newly initiated mechanical engineer (ME) in the word of finance and to illustrate in detail how an ME could translate the knowledge acquired in a new area of research.

Background

Based in the field of econophysics, Arroyo Colón [1] proposed a model of economics using an analogy to thermodynamics while introducing the concept of the "Value Multiplier" as a fundamental addition to any such model to make an analogy with the "Keynesian Multiplier". Intriguing use of thermodynamics and physics models to have a better understanding of the economy. Jovanovic and Schinckus, [3] defined the origin of the field of econophysics in "the mid- 1990s, in the wake of some of the most recent advances in physics, a new approach to dealing with financial prices emerged.". Ducournau, [3] considered the stock market as a physical system analogous to a fluid evolving in a macroscopic space subject to a force that influences its movement over time and then showed theoretically that fluid mechanics equations can be used to describe stock market physical properties. Campuzan [4] used the Crank-Nicolson Method from CFD to solve the PDE of a European call option while stating that this is typically used in finance.

Problem

Research Description

It is proposed to investigate how diverse typical mechanical engineering methodologies such as CFD, and related optimization approaches could be applied to solve problems in finance and investment. Typical problems in thermo-economics (also known as econophysics) will be described in detail, and then they will be solved using one or more of the typical mechanical engineering tools, making emphasis on developing the models' analogies, their potentials, and limitations as well as possible future developments and applications. This is a new area of interdisciplinary study that promises great challenges and opportunities.

Research Objectives

It is aimed at the exploration and examination of diverse typical mechanical engineering methodologies such as CFD, finite element analysis, and related optimization approaches and their application to solve problems in finance and investment.

Research Contributions

The usefulness of the mechanical engineering methodologies utilized in another field of knowledge will be demonstrated in sufficient substance to serve as an introductory basis for further studies for the interested researcher with a background in civil or mechanical engineering. It will also serve as the foundation for an introductory course at the fourth or fifth-year level in engineering in those engineering specialties.

Methodology

Several complete cases numerical examples of the methods and approaches used in mechanical engineering that have direct application in econophysics and will be discussed with enough detail to serve as an introduction to the topics. Numerical examples will also be discussed as well as the possible parallels between the two fields. The didactical approach used will be first to solve a known or familiar problem in mechanical engineering using a typical mechanical engineering approach and then, an unfamiliar or new problem in econophysics will be solved using the same technique.

Results and Discussion

Illustrative Case: Velocity distribution in two parallel plates: A Mechanical Engineering Example of the CFD Implicit Method

Let's consider a fluid bounded by two parallel plates extended to infinity as seen in Figure 1. The planar walls and the fluids are initially at rest. Now, the lower wall is suddenly accelerated in the x direction, as illustrated in the figure. The Navier-Stoke equations for this problem may be expressed as:

∂u/∂t=ν(∂^2 u)/(∂y^2) (1)

The fluid is oil with a kinematic viscosity of 0.000217 m2/s, and the spacing between plates is 40 mm. The velocity of the lower wall is specified as Uo=40 m/s. A solution for the velocity is to be obtained up to 1.08 seconds using implicit discretization and using 30 times intervals.

a) The initial conditions, $t = 0 \begin{cases} u = Uo\ for\ y = 0 \\ u = 0\ for\ 0 < y \leq h \end{cases}$ (2)

b) The boundary conditions, $t \geq 0 \begin{cases} u = Uo\ for\ y = 0 \\ u = 0\ for\ y = h \end{cases}$ (3)

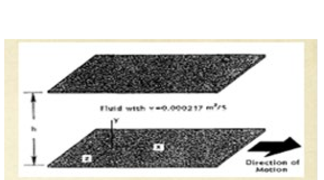

Figure 1

2 Parallel Plates. Class notes from ME 6160: Computational Fluid Dynamics [6]

Theory

There are two methods widely used to solve PDE's: the explicit and the implicit approach. Both are based on the discretization of the PDE and then, solving the resulting difference equations in a recurrent manner.

The extra complexity of the implicit methodology is compensated for by its superior stability.

The explicit method of solution describes an unknown value at a certain grid point depending on the known values at neighboring grid points; after discretization, for the diffusion equation (which is of parabolic type), we get the recurrent difference equation:

$$T_i^{n+1} = T_i^n + s(T_{i+1}^n - 2T_i^n + T_{i-1}^n) \quad (4)$$

Thus, we can calculate T_i^{n+1}values at the first-time row depending on the boundary and initial values at T_i^n=0 line. Then, similarly we can calculate the values at the second time row depending on the values that were calculated for the first row, and so on. See Figure 2

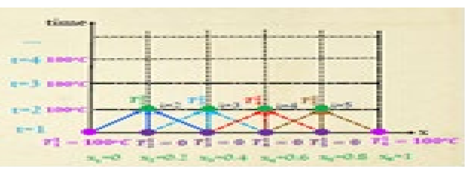

Figure 2

Use of Explicit Method: Illustration of the Calculation Procedure. Class notes from ME 6160: Computational Fluid Dynamics [6]

This methodology suffers from innate instability; to ensure a stable (convergent) solution, we must select:

$$s = \alpha \frac{\nu \Delta t}{(\Delta x)^2} \quad (5)$$

On the other hand, the implicit method is unconditionally stable but suffers from the need of an extra subroutine to solve the linear equations that are created. This causes it to take a lot of computing power. For this we can use the Thomas algorithm, among others. The Implicit methodology results in the recurrent formula:

$$u_j^{n+1} = u_j^n + \left(\frac{\nu \Delta t}{(\Delta x)^2}\right)\left(u_{j+1}^{n+1} - 2u_j^{n+1} + u_{j-1}^{n+1}\right) \quad (6)$$

But the coefficients are calculated from:

$$u_j^n = u_j^{n+1} - \left(\frac{\nu \Delta t}{(\Delta x)^2}\right)(u_{j+1}^{n+1}) + 2\frac{\nu \Delta t}{(\Delta x)^2}(u_j^{n+1}) - \frac{\nu \Delta t}{(\Delta x)^2}(u_{j-1}^{n+1}) \quad (7)$$

For simplicity, we can call,

$$a_j^n = -\left(\frac{\nu \Delta t}{(\Delta x)^2}\right) \quad (8)$$

$$b_j^n = \left(1 + 2\frac{\nu \Delta t}{(\Delta x)^2}\right) \quad (9)$$

$$c_j^n = -\frac{\nu \Delta t}{(\Delta x)^2} \quad (10)$$

Thus, the discretized equation is:

$$u_j^n = a_j^n(u_{j+1}^{n+1}) + b_j^n(u_j^{n+1}) + c_j^n(u_{j-1}^{n+1}) \quad (11)$$

Coding these equations in Matlab, we obtained the veloocity profile in Figure 3:

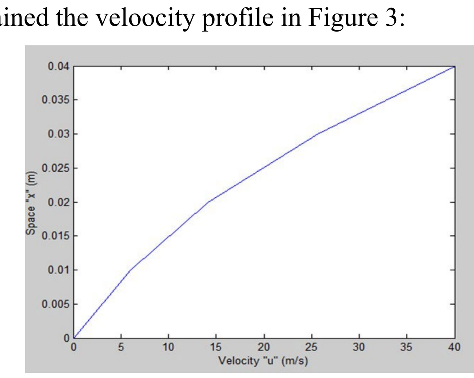

Figure 3

Implicit Methodology Velocity Profile. Source: Own Elaboration

The CFD Approach Applied to Solve the Black-Scholes Ordinary Differential Equation (ODE): An Econophysics Example

We can recall the Black-Scholes PDE as:

$$\frac{\partial V(S,t)}{\partial t} + \left(\frac{1}{2}\sigma^2 S(t)^2\right)\frac{\partial^2 V(S,t)}{\partial S^2} + (r * S(t))\frac{\partial V(S,t)}{\partial S} - r * V(S,t) = 0 \quad (12)$$

Which we can approximate using the recurrent difference equation:

$$\frac{V_{n.j+1} - V_{n.j}}{\Delta t} + a_{n,j}\left(\frac{V_{n+1.j} - 2V_{n.j} + V_{n-1.j}}{\Delta S^2}\right) + b_{n,j}\left(\frac{V_{n+1.j} - V_{n-1.j}}{2\Delta S}\right) + c_{n,j}(V_{n.j}) = o(\Delta t, \Delta S^2) \quad (13)$$

After substituting the specifics, we obtain:

$$V_{n.j+1} = \frac{1}{2}(\sigma^2 n^2 - rn)\Delta t V_{n-1.j} + [1 - (\sigma^2 n^2 + r)\Delta t]V_{n.j} + \frac{1}{2}(\sigma^2 n^2 + rn)\Delta t V_{n+1.j} + o(\Delta t^2, \Delta t \Delta S^2)) \quad (14)$$

The equation holds for n=1,2, 3...N-1 since $V_{-1,j}$ nor $V_{-n+1,j}$ are defined. There will be N-1 equations and N+1 unknown. Thus, we need two more equations to solve the system. These are the boundary conditions imposed on the system for when n=0 and 0 and n=N. They depend on the case, for instance if we were to price a European call option.

At S=0 (asset price with no value) we have V0,j =0. By necessity if the asset is worth nothing, the value of an option to buy it must also be nothing.

When S->∞ (a very big asset price. Say Smax), then the value at that moment of the call option -->S_{max}- $E * e^{-r*(T-t)}$.That is, the value of the call option is the difference between the value of that asset less its expected cost. Therefore, the upper boundary condition would be: $V_{N,j} = N * \Delta S - Ee^{-r*j*(\Delta t)}$. Coding these equations in Matlab, we obtained Figure 4 and Figure 5:

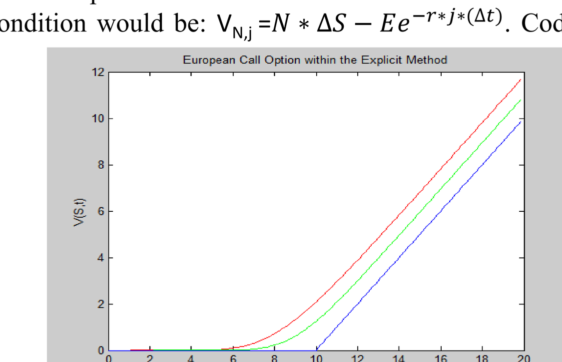

Figure 4

Solution for the European Call Option by Using an Explicit Method. Source: Own Elaboration

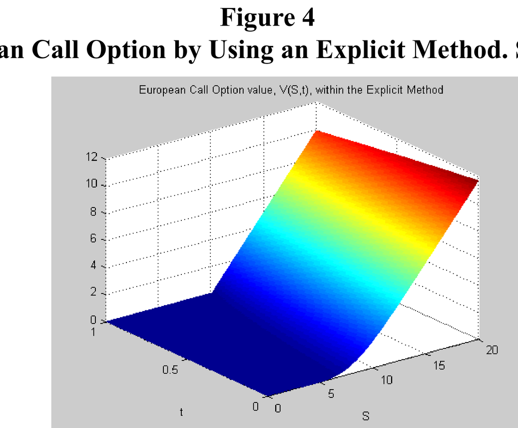

Figure 5

Solution for the European Call option by Using an Explicit Method. Source: Own Elaboration.

Blue represents the value of the option at expiration, green half a year before expiration and red one year before expiration (price at the time the contract is signed).

Conclusions

We explored and examined diverse typical mechanical engineering methodologies such as CFD, finite element analysis, and related optimization approaches and their application to solve problems in econophysics using practical problems solved with enough detail to serve as an introduction to the theory of mechanical engineering techniques, and their correspondence analog concept in econophysics.

Several complete cases numerical examples of the methods and approaches used in mechanical engineering that have direct application in econophysics were discussed with enough detail to serve as an introduction to the topics. Those numerical examples were discussed as well as the parallels between the two fields. The didactical approach used was first to solve a known or familiar problem in mechanical engineering using a typical mechanical engineering approach and then, an unfamiliar or new problem in econophysics was solved using the same technique.

The different models tested for each technique were discussed in detail in each case-example chapter. Visual graphs showing the results and illustrating that the mechanical engineering approach in each case is directly applicable to the econophysics situation were shown. For every case, we discussed the mathematical models from the mechanical engineer/standard physics and econophysics methodologies and then compared them.

Our objectives were, as previously stated, to develop a deep comprehension of several of the econophysics models already being developed, prepare a basic introduction of numerical examples to complement that research, and compare the results of such models against actual data results to evaluate their effectiveness. These objectives were met.

Future Work

The usefulness of the mechanical engineering methodologies when utilized in another field of knowledge was demonstrated in sufficient substance to serve as an introductory basis for further studies for the interested researcher with a background in civil or mechanical engineering. This research could also serve as the foundation for an introductory course in econophysics at the fourth or fifth-year level in engineering in those engineering specialties.

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