A Quadcopter Test Bed for Academic and Research on Control Systems Design

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Abstract *— This article presents the development of a quadcopter platform to be used for future academic and research studies in control systems in the Mechanical Engineering Department at the Polytechnic University of Puerto Rico. This platform uses an ardupilot 2.6 technology based on the arduino 2560 mega developed by 3DR Robotics. All the software are open source available over the internet.*

Key Terms *— Ardupilot, Control Systems, Mission Planner, Quadcopter.*

INTRODUCTION

The quadcopter is a multirotor device that uses four equally spaced motors with propellers for lift. Recently this aircraft have become popular due to the advances in the control system technology and the electronics miniaturization allowing an onboard control system for stabilization purposes. This has created a new industry that goes from radio controlled enthusiasts, including aerial photography and surveillance, to military unmanned autonomous vehicle (UAV) applications. Recently commercial companies like amazon are researching the multirotor capabilities as a company delivery platform.

The quadcopters can be guided using a differential thrust mechanism by individually changing the motors revolutions to achieve roll, pitch and yaw, which are the three angles of rotation of the aircraft about the body frame center of mass. See Figure 1. This creates a more simple mechanism than changing the pitch in a helicopter propellers to change the angle of attack.

The quadcopter generate lift by increasing the four motors rpms to generate a perpendicular thrust vector strong enough to overcome the aircraft weight. In order to prevent the quadcopter from spinning in the yaw direction, two motors must spin in the clockwise directions and the other two in the counterclockwise direction. These principle can be used to rotate the quadcopter in the yaw direction.

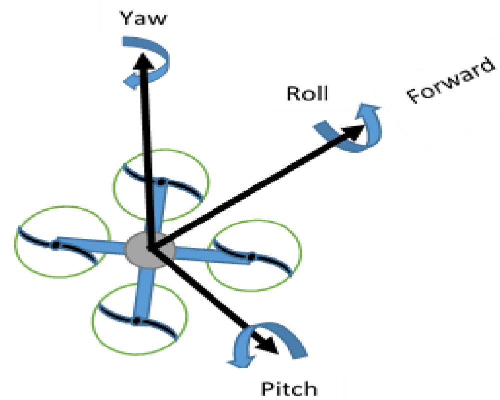

Figure 1
Graphical Representation of Yaw, Pitch and Roll

To rotate the pitch or roll we increase the rpm on two motors located on opposite directions, shown in Figure 2. These systems requires a control system to manipulate the motors rpm according the desired quadcopter movements.

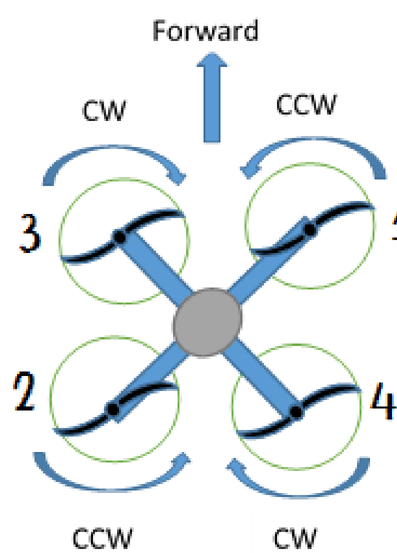

Figure 2
Quadcopter Rotors Configuration

FLIGHT DYNAMICS

Using Newton's law we can postulate, assuming no drag force due to air friction, that

$$F_{net} = F_g + RT_{prop} \quad (1)$$

where the F_{net} net force, F_g is the force exerted by the gravity and T_{prop} is the thrust generated by the propellers. Because there is an earth inertial frame (ground global coordinates) and the body frame (quadcopter local coordinates) rotation matrix R is needed to correlates both coordinates systems. The total rotation matrix becomes

$$R = \begin{bmatrix} C_\theta C_\psi & S_\psi S_\theta C_\psi - C_\phi S & C_\phi S_\theta C_\psi + S_\phi S_\psi \\ C_\theta S_\psi & C_\phi C_\psi + S_\phi S_\theta S_\psi & S_\theta C_\phi S_\psi - S_\phi C_\psi \\ -S_\theta & S_\phi C_\theta & C_\theta C_\phi \end{bmatrix} \quad (2)$$

where S means sine and C means cosine and the subscript symbol represents the angle.

The thrust vector can be defined as

$$T_{prop} = k\omega^2 \quad (3)$$

where k is the propellers lift coefficient and ω is the angular velocity of the motor, then the net force due to the four propellers are

$$T_{prop} = k \begin{bmatrix} 0 \\ 0 \\ \sum_{i=1}^{4} \omega^2 \end{bmatrix} \quad (4)$$

Then substituting in equation 2 and 3 in equation 1 and F_{net}=ma and solving for acceleration vector we have

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} + \frac{\sum_{i=1}^{4} \omega^2}{m} \begin{bmatrix} C_\phi S_\theta C_\psi + S_\phi S_\psi \\ S_\theta C_\phi S_\psi - S_\phi C_\psi \\ C_\theta C_\phi \end{bmatrix} \quad (5)$$

If we formulate an equation in terms of torques then using the Euler equation for rigid bodies we have

$$\boldsymbol{\tau} = \mathbf{I}\dot{\omega} + \omega \times (\mathbf{I}\omega) \quad (6)$$

where **I** is the moment of inertia matrix. Assuming symmetry in the frame body, the inertial matrix becomes

$$\mathbf{I} = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} \quad (7)$$

Substituting equation 7 in equation 6 we have

$$\begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} \begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{bmatrix} + \begin{bmatrix} (I_y - I_z)\omega_y\omega_z \\ (I_z - I_x)\omega_x\omega_z \\ (I_x - I_y)\omega_x\omega_y \end{bmatrix} \quad (8)$$

For roll (ϕ) we can relate the torques as

$$\tau_\phi = L(T_{prop(1)} + T_{prop(4)} - T_{prop(3)} - T_{prop(2)}) \\ or \\ \tau_\phi = Lk(\omega_1^2 + \omega_4^2 - \omega_3^2 - \omega_2^2) \quad (9)$$

where L is the quadcopter radius. The motor angular velocity ω can be related to the voltage. For the pitch (θ)

$$\tau_\theta = L(T_{prop(2)} + T_{prop(4)} - T_{prop(1)} - T_{prop(3)}) \\ or \\ \tau_\theta = Lk(\omega_2^2 + \omega_4^2 - \omega_1^2 - \omega_3^2) \quad (10)$$

and for the yaw (ψ)

$$\tau_\psi = L(T_{prop(1)} - T_{prop(4)} + T_{prop(3)} - T_{prop(2)}) \\ or \\ \tau_\psi = Lk(\omega_1^2 + \omega_2^2 - \omega_3^2 - \omega_4^2) \quad (11)$$

Substituting equations 9, 10 and 11 in equation 8 and solving for $\dot{\omega}$ we have

$$\begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{bmatrix} = Lk \begin{bmatrix} (\omega_1^2 + \omega_4^2 - \omega_3^2 - \omega_2^2)/I_x \\ (\omega_2^2 + \omega_4^2 - \omega_1^2 - \omega_3^2)/I_y \\ (\omega_1^2 - \omega_4^2 + \omega_3^2 - \omega_2^2)/I_z \end{bmatrix} + \begin{bmatrix} (I_y - I_z)\omega_y \omega_z / I_x \\ (I_z - I_x)\omega_x \omega_z / I_y \\ (I_x - I_y)\omega_x \omega_y / I_z \end{bmatrix} \quad (12)$$

Also the rate of change roll, pitch and yaw can be related to the angular velocity as

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \mathbf{Q} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \quad (13)$$

where **Q** is a projection matrix defined as

$$Q = \begin{bmatrix} 1 & \sin\phi \tan\theta & \cos\phi \tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi / \cos\theta & \cos\phi / \cos\theta \end{bmatrix} \quad (14)$$

Finally the state space form are equations 5, 12, 13 and

$$\begin{aligned} \dot{x} &= v_x \\ \dot{y} &= v_y \\ \dot{z} &= v_z \end{aligned} \quad (15)$$

$$\begin{aligned} a_x &= \frac{\sum_{i=1}^{4} \omega^2}{m} (\cos\phi \sin\theta \cos\psi + \sin\phi \sin\psi) \\ a_y &= \frac{\sum_{i=1}^{4} \omega^2}{m} (\sin\theta \cos\phi \sin\psi - \sin\phi \cos\psi) \\ a_z &= -mg + \frac{\sum_{i=1}^{4} \omega^2}{m} \cos\theta \cos\phi \end{aligned} \quad (16)$$

$$\begin{aligned} \dot{\omega}_x &= \frac{Lk}{I_x}(\omega_1^2 + \omega_4^2 - \omega_3^2 - \omega_2^2) + \frac{(I_y - I_z)\omega_y \omega_z}{I_x} \\ \dot{\omega}_y &= \frac{Lk}{I_y}(\omega_2^2 + \omega_4^2 - \omega_1^2 - \omega_3^2) + \frac{(I_z - I_x)\omega_x \omega_z}{I_y} \\ \dot{\omega}_z &= \frac{Lk}{I_z}(\omega_1^2 - \omega_4^2 + \omega_3^2 - \omega_2^2) + \frac{(I_x - I_y)\omega_x \omega_y}{I_z} \end{aligned} \quad (17)$$

$$\begin{aligned} \dot{\phi} &= \omega_x + \omega_y \sin\phi \tan\theta + \omega_z \cos\phi \tan\theta \\ \dot{\theta} &= \omega_y \cos\phi - \omega_z \sin\phi \\ \dot{\psi} &= \omega_y \sin\phi / \cos\theta + \omega_z \cos\phi / \cos\theta \end{aligned} \quad (18)$$

If we assume the same initial inertial frame in the quadcopter and the ground then

$$\begin{aligned} \dot{\phi} &= \omega_x \\ \dot{\theta} &= \omega_y \\ \dot{\psi} &= \omega_y \end{aligned} \quad (19)$$

For more complex system refer to [1], [2], and [3].

MODEL SIMULATION

The simple mathematical model developed in the previous section was implemented using Simulink. The nonlinear differential equations presented in equations 16 and 17 can be seen in Figure 3 in a Simulink graphical representation. The model was tested using some arbitrary set of parameters, like the mass, moments of inertia and propellers lift coefficient, and computed such that the quadcopter begins to lift at 2000 rpms.

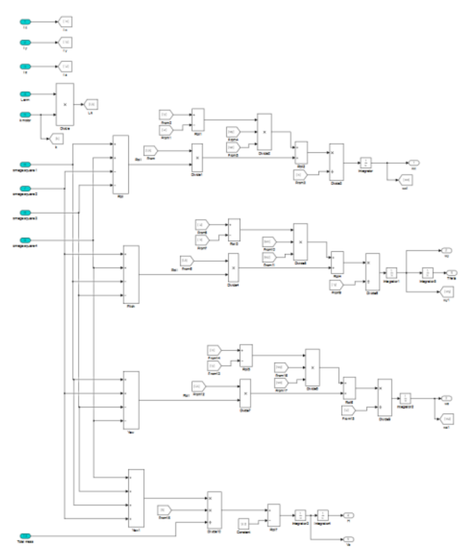

Figure 3
Simulink Block Diagram for the Differential Equations 16 and 17

The system was simulated by setting the motors 1 and 2 rpms to 2010 while keeping motors 3 and 4 to 2000 rpms, refer to Figure 2 for motors configuration as shown in Figure 4. The yaw (blue line) and the altitude (black line) increase in time as expected while keeping the roll (red line) and pitch (green line) equal to zero. Both values continue to increase in time because there is no control system to stop the quadcopter at a specific altitude or yaw.

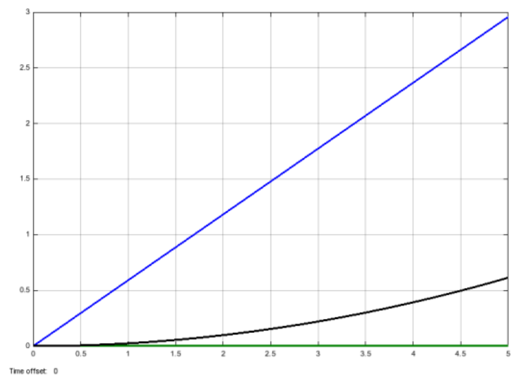

Figure 4
Model Response to an Increase in Rmps on Motors 1 and 2 While Keeping Motors 3 and 4 Steady

By increasing motors 2 and 4 to 2010 rpm while keeping the 1 and 3 to 2000 rpms the system increase the pitch (green line) and its altitude (black line) as shown in Figure 5 as expected.

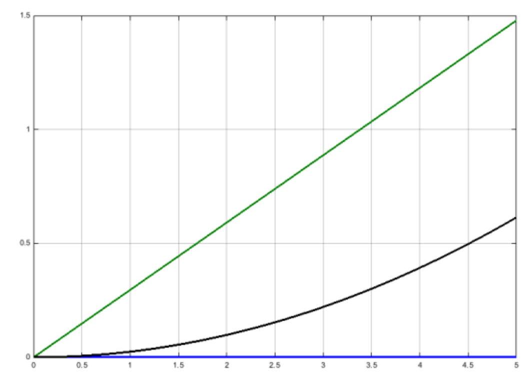

Figure 5
Model Response to an Increase in Rmps on Motors 2 and 4 While Keeping Motors 1 and 3 Steady

By increasing the rpms on motors 1 and 4 to 2010 while keeping motors 2 and 3 to 2000 rpms the system increase the roll (red line) and its altitude (black line) as shown in Figure 6.

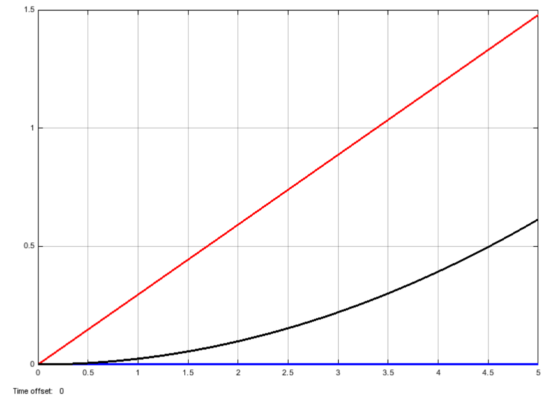

Figure 6
Model Response to an Increase in Rmps on Motors 1 and 4 While Keeping Motors 2 and 3 Steady

QUADCOPTER HARDWARE

On table 1 we can see a list of parts used for the test bed and the system weights around 1400g. The XXD A2212 brushless motors with the 1045 propellers, shown in Figure 7. To control the brushless motors we need Electronic Speed controllers or ESC. The Hobbypower 3OA, shown in Figure 7 on the left side, was chosen to control the motors rpm's by the main control board.

Table 1
Parts Description and Weight

# of parts	Description	Weight (grams)
1	Frame body (X525)	803
4	XXD A2212 brushless motors	48g/per motor (192 g total)
4	30A ESC speed controller	25g/per ESC (100 g total)
1	Power distribution board	33 g
4	1045R/L propellers	11g each (44 total)
1	3DR Ardupilot Mega 2.6 with external compass	30g
1	Futaba R2006GS Receiver	8.3g
	Connection cables	
1	Turnigy 2200mAh 3S 20 Lipo Battery pack	188 g
		Total weight 1398.3 g

Figure 7
A2212 Brushless Motor with Aluminum Mounting Frame, ESC and Propellers

Also there is a power distribution circuit that connects the main power source to each of the motors, shown in Figure 8 on the upper right corner. The selected body was a light carbon glass with aluminum arms rods and shock absorbing landing legs. The power source is a Turnigy 2200 mAh with 3 cells Lithium Polymer battery, shown in Figure 8.

Figure 8
Quadcopter Frame (Center), Power Module (Upper Left), Power Distribution Board (Upper Right) and Battery Pack

The main control system is an ardupilot 2.6 board based on Arduino mega running an open source firmware. The ardupilot includes internally a gyro, an accelerometer and a barometer as sensors needed to implement a control systems. An external compass and GPS are added to allow the system to perform unmanned mission. The receiver is a 2.4 GHz with 6 channel Futaba R2006GS, all shown in Figure 9. Figure 10 shows the final assembly.

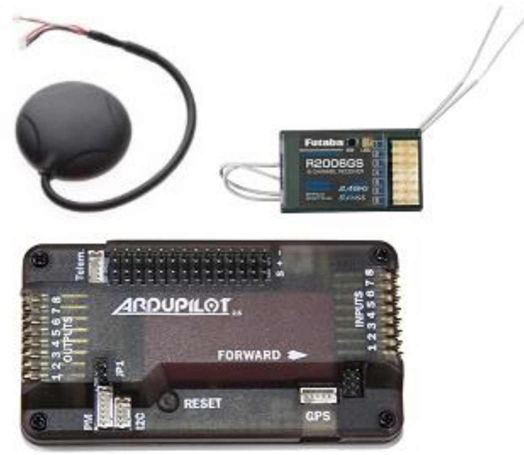

Figure 9
Ardupilot, External GPS (Upper Left) and 2.5GHz 6 Channel Receiver

Figure 10
Final System Configuration

The firmware was installed on the ardupilot using an open source application called Mission Planner developed by Michael Oborne, which is also used to calibrate the sensors and to set the control system parameters. This applications also allows a wireless connection to the quadcopter by using a telemetry system to data log and trace its behavior from a ground station, in this case a personal computer.

CONCLUSION

A mathematical model of a simple quadcopter was developed and verified for specific cases. It will be used, once is fully validated, to calculate the control system parameters needed for flight stabilization purposes. A test-bed platform was built and tested for motor's response. More testing is needed to fine tune the system. Up to this point the system is behaving as expected

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