Aircraft Wing Spanwise Air-Load Distribution using MATLAB

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Abstract — *Aircraft design is a complex task that involves the interaction between aerodynamics, propulsion, flight mechanics and structures. In particular, the structural design tasks involve the consideration of a significant number of load conditions that affect the performance of the aircraft from the moment it takes off to the execution of its mission and eventual landing. A critical aspect in the design of aircraft structures is the determination of the aerodynamic loads. These loads are especially important for the design of air lifting components such as wings and tails. The required level of detail and accuracy for these loads depend on the stage in the design process. For example, during the concept and preliminary design stages, the aerodynamic loads could be estimated using simplified analytical approaches such as the Schrenck method, Diederich method, and Fourier series, among others. The purpose of this paper is to discuss the development of a Matlab© tool to determine the air loads for the preliminary design of wing structures using the Shrenck method. The tool requires the user to provide basic geometric information about the wing and airfoil aerodynamic data. The output provides the wing lift, drag, shear, moment, and torsional loads distributions. This information can be used then by the structural designer to perform the structural evaluation of the wing structure. As shown in the article, the results compare very well to published results.*

Key Terms — *Loads to Airplane, Lift to Span-wise, Matlab Airplane, Air-load Distribution.*

WING LOAD DISTRIBUTION

Loads on the wing are made up of aerodynamics lift and drag forces, as well as concentrated or distributed weight of wing-mounted engines, stored fuel, weapons, structural elements, etc [1]. Figure 1 show the span-wise wing-loading distribution.

The FAA (in Advisory Circular) shows different methodologies for calculating the airloads distribution such as: ANC – 1, Schrenk's NACA TM-948, Sherman's NACA TN-732, etc. [2].

SCHRENK'S APPROXIMATION METHOD

This method is used to approximate to span-wise lift distribution on the wing see figure 1. The fundamental idea of this method is made that the real lift distribution lies between an ideal distribution independent of the wing shape and a distribution determined in a simple manner by the wing shape. The ideal distribution is that with minimum induced drag and constant induced downwash velocity that is, for the usual monoplane, the elliptic distribution; while the distribution dependent on the shape is proportional to angle of attack at each position of the wing. [1]

Schrenk assume that wings with no aerodynamics twist, constant airfoil section on the wing, and the distribution is the average of:

- A load distribution representing the actual planform shape and
- An elliptical distribution of the same span area.

This approximate solution is accepted by the Civil Aeronautics Administration (CAA) and 14 CFR part 23, sub part C. [2].

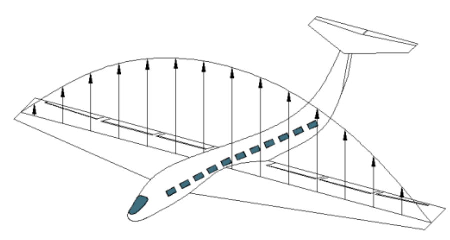


Figure 1
Spanwise Lift Distribution

As a consequence of the finite aspect ratio of any wing, the lift distribution will vary along the wingspan, from a maximum near its root to a minimum near its tip. [3] See Figure 2, the method shown that the span-wise lift distribution should be proportional to the shape of the wing planform. In the case of an elliptical planform, the local chord distribution, c(y) given as:

$$ccl(y) = \frac{4S}{\pi b}\sqrt{1-\left(\frac{2y}{b}\right)^2} \quad (1)$$

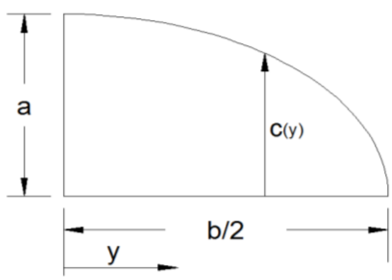


Figure 2
Elliptical Distribution

Where: S is the total area of wing, b is the span wing.

In the Peery's book explain that the untwisted wing occur when zero-lift chords of all airfoil sections lie in the same plane, the wing has no aerodynamics twist [5]. Normally the wings have thinner airfoil sections near the tip than near the root, also the airfoil section to along the wing has no the same angle of attack, this characteristic is for advertise to stall, but these different angles produce aerodynamic twist, see Figure 3.

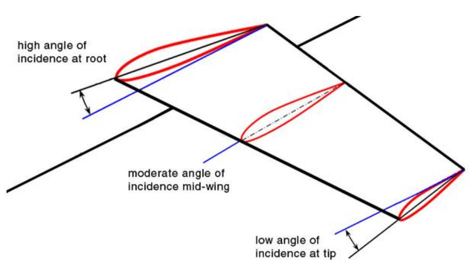


Figure 3
Aerodynamic Twist

The air-load distribution for wings with aerodynamic twist should be obtained in two parts. The first part, called the basic lift distribution, is obtained for the angle of attack at which the entire wing has no lift, and the second part, called the additional lift distribution, can be obtained by Schrenk's method [5].

MATLAB TOOL

Using the equations from Schrenk's Method for determinate the Spanwise air-load distribution and considerations for ANC-1(1) [4].

Was developed a tool in the GUI Matlab see figure 5, this tool has the following parts:

- Wing Geometry.
- Additive Lift Distribution.
- Basic Lift distribution.
- Stall Cl calculation.
- Correction Factor (τ)
- Spanwise Coefficient Distribution
- Spanwise Air-Loads Distribution for specified CL and true airspeed V

And the tool GUI has a flowchart as shown in Figure 4.

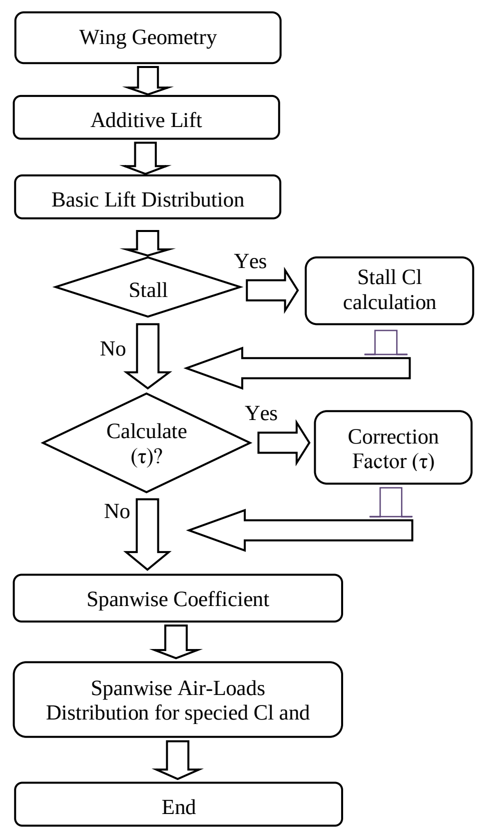


Figure 4
Air-Load Distribution Flowchart

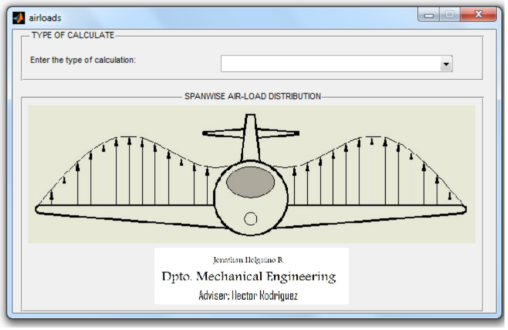


Figure 5
Air-Load Distribution Main Window

Wing Geometry

In this part, the user enters a number of wing stations, a number of points that define the Leading edge and Trailing edge as shown in Figure 6.

The module calculates; total wing span area, the mean aerodynamic chord (*mac*), coordinates in leading edge from *mac*, and aspect ratio.

The equations used to calculate are the following:

$$mac = \frac{2}{S}\int_0^{b/2} c^2 dy \quad (2)$$

$$y_{mac} = \int_0^{N0dy} \frac{y \cdot c}{S/2} dy \quad (3)$$

$$x_{mac} = \int_0^{Nody} \frac{x \cdot c}{S/2} dy \quad (4)$$

$$AR = \frac{b^2}{S} \quad (5)$$

Where: c is the chord and It's function of the y, x are the coordinates in leading edge, y is the coordinate discretized of the large to span, s is the total wing span area., *Nody* is the number of part the wing span, and AR is the aspect ratio.

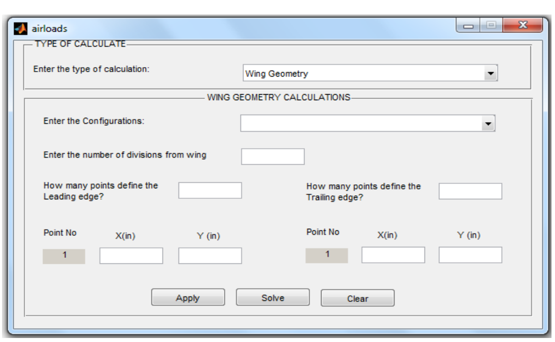


Figure 6
Wing Geometry

Additive Lift Distribution

In this section, the user enter, a number of wing stations, distance of the wing station regarding wing root in inches, and slope of the section lift with respect to the angle of attack in degree angles m_0, as shown in Figure 7.

In this case the module calculates; the cc_{la1} is the chord per additional lift coefficient corresponding to $C_L = 1$ for the entire wing, and its corresponding additional lift coefficient c_{la1}.

The equations used to calculate are the following:

$$cc_{la1} = \frac{1}{2}\left[\frac{m_0 c}{\bar{m}_o} + \frac{4S}{\pi b}\sqrt{1-\left(\frac{2y}{b}\right)^2}\right] \quad (6)$$

$$\bar{m}_o = \frac{\int_0^{\frac{b}{2}} m_0 c \cdot dy}{S/2} \quad (7)$$

$$c_{la1} = \frac{cc_{la1}}{c} \quad (8)$$

Where $\bar{m}_0$, is the average slope of section lift coefficients, m_0 is the slope of the section lift coefficient in respect to angle of attack in degree.

These equations from Peery's book [5].

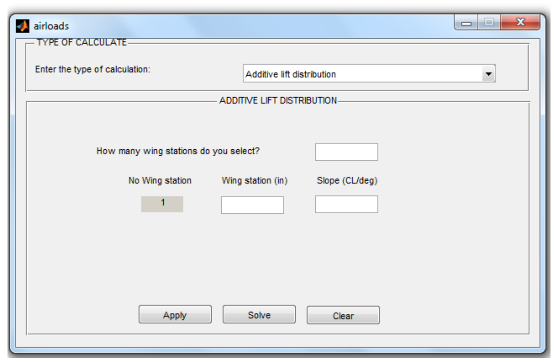


Figure 7
Additive Lift Distribution

Basic Lift Distribution

As shown Figure 9, the user enter a number of wing stations, distance of the wing station regarding wing root in inches, the wing station reference angle, this reference is formed to chord-waterline regarding to angle zero of each section, and continuity or discontinuity value.

The output data shows, the wing angle of attack for zero lift, the basic lift coefficient, and the equations used to calculate this module are following:

$$\alpha_{w0} = \frac{\int_0^{b/2} m_0 \cdot \alpha_{ar} \cdot c \cdot dy}{\int_0^{b/2} m_0 c \cdot dy} \quad (9)$$

$$\alpha_a = \alpha_{ar} - \alpha_{w0} \quad (10)$$

$$cc_{lb}(rounded\ off) = \frac{1}{2} c m_0 \alpha_a \quad (11)$$

Where; α_{w0} is the wing angle of attack for zero lift, this angle is solved for the absolute angle of attack α_a, the α_{ar} is the reference angle regarding angle zero lift, and cc_{lb}(rounded off) is the chord per basic lift coefficient but rounded off.

When shown in the Figure 8, the wing has length l of discontinuity between flap and aileron.

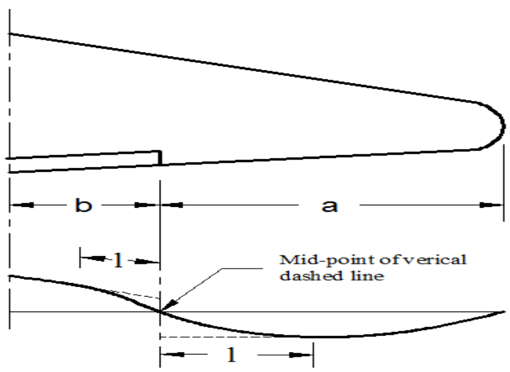


Figure 8
Wing with Discontinuity Flap-Aileron

$$l = \frac{a}{2} \quad if\ a > b \quad (12)$$

$$l = \frac{b}{2} \quad if\ a < b \quad (13)$$

When, the abrupt change in the curve of cc_{lb} at the outboard end of the flap as shown in the figure (8), the curve is faired so that the positive area removed is equal to the negative area removed, and the total wing lift remains zero [1].

According by H.C. McMaster [6], the equations for cc_{lb}(faired) are following:

$$\theta = \frac{\pi}{2 \cdot l}\left(\int_0^{b/2} dy - y_{lin}\right) \quad (14)$$

$$y_{lin} = y_{disc} - l \quad (15)$$

$$cc_{lb}(Faired) = \left(cc_{lb} - cc_{lb}(ave)\right) \cdot ||\cos(\theta)|| + cc_{lb}(ave) \quad (16)$$

Where θ is the angle used in cosine function in the basic lift coefficient at faired, y_{disc} is the butt line at discontinuity of flap and aileron, and $cc_{lb}(ave)$ is the average element lift at discontinuity.

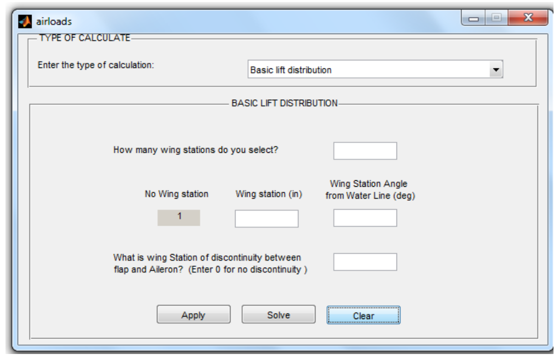


Figure 9
Additive Lift Distribution

Stall Cl Calculation

In this part, require enter the wing stations, the first maximum lift coefficient C_{l1max}, first Reynolds number and second maximum lift coefficient C_{l2max} and its Reynolds number, shown as Figure 10.

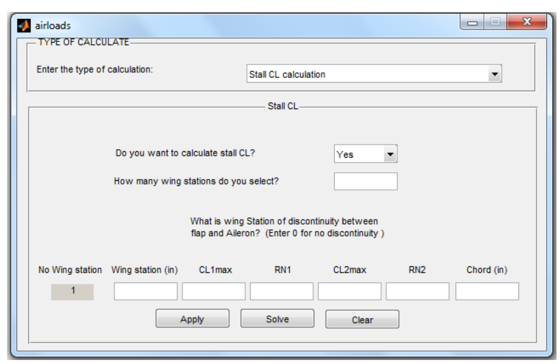


Figure 10
Stall Lift Coefficient Cl

Correction Factor (τ)

According of Peery's book τ is a correction factor which accounts for the deviation of the planform from an ellipse [1].

In this module shown as in the Figure 11, used the equation according to ANC-1(1) [4].

$$\tau_0 = 0.026209 - 1.26146\lambda + 3.05385\lambda^2 - 2.8027\lambda^3 + 0.976801\lambda^4 \quad (17)$$

$$\tau_1 = 0.112203 - 0.575843\lambda + 1.08306\lambda^2 - 0.696856\lambda^3 + 0.194241\lambda^4 \quad (18)$$

$$\tau_2 = 0.0302789 + 0.0294027\lambda - 0.470926\lambda^2 + 0.880983\lambda^3 - 0.394766\lambda^4 \quad (19)$$

If $\gamma = 0$, then $\tau = \tau_0$

If $\gamma = 0.1$, then $\tau = \tau_1$ (20)

If $\gamma = 0.2$, then $\tau = \tau_2$

If $\gamma = 1$, then $\tau = 0$

For γ that are between the values of the conditionals equations (20),

For $0 < \gamma < 0.1$ then, $\tau = \tau_0 + \gamma(\tau_1 - \tau_0)/0.1$

For $0.1 < \gamma < 0.2$ then, $\tau = \tau_1 + (\gamma - 0.1)(\tau_2 - \tau_1)/0.1$

For $0.2 < \gamma < 1$ then, $\tau = \tau_2 - \tau_2(\gamma - 0.2)/0.8$

For $0.2 < \gamma < 1$ then, $\tau = \tau_2 - \tau_2(\gamma - 0.2)/0.8$

(21)

$$\lambda = \frac{c_t}{c_s} \quad (22)$$

Where, $\boldsymbol{\lambda}$, is the ratio between the tip chord and the centerline root chord, $\boldsymbol{\gamma}$, is the ratio of rounded tip length lo length of semi-span as shown in the Figure 12.

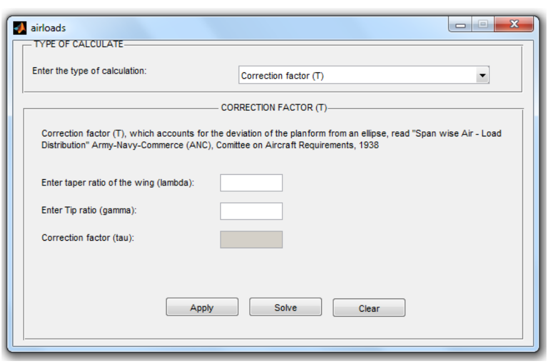


Figure 11
Correction Factor t

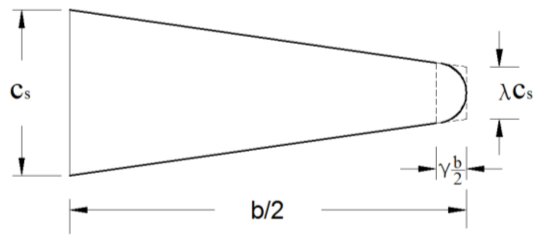


Figure 12
Ratio γ

Spanwise Coefficient Distribution

As shown in the Figure 13 of this module, the user can be to use for calculates the aerodynamics coefficient distribution to along the wing and the input data requirements are: τ is the correction factor, the total lift coefficient of wing C_L, the wing station with their drag coefficients and their moment coefficients.

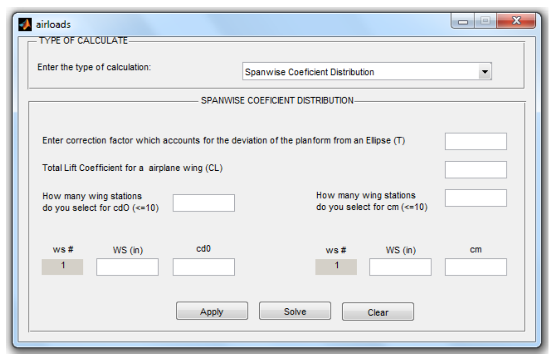


Figure 13
Spanwise Coefficient Distribution

In case to use the value of $\boldsymbol{\tau}$ evaluated in correction factor module, the input of $\boldsymbol{\tau}$ in Span-wise.

Coefficient Distribution is no necessary, because the space of $\boldsymbol{\tau}$ will be disable, but charged the previous analysis. The other form for determined the $\boldsymbol{\tau}$ value is using the Figure 14.

For the drag and moment coefficient values is found in diagrams the airfoils making up the wing.

This module use data output of previous modules, therefore calculate previous modules is necessary according flowchart from the Figure 4.

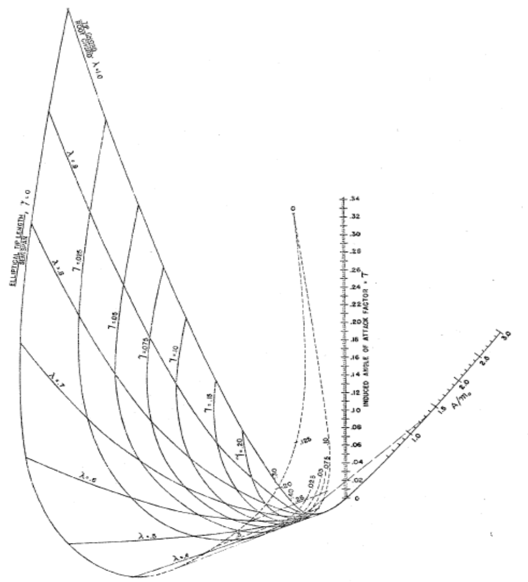


Figure 14 [4]

Correction Factor for Induced Angle of Attack, τ

The equations to use for this module are: The value of lift coefficient per unit of span is:

$$c_l = C_L c_{la1} + c_{lb} \tag{23}$$

With the value of factor correction τ, the values of lift curve slopes can be solved as.

$$m = \frac{m_0 \cdot 180/\pi}{1+\left(\frac{m_0 \cdot 180}{\pi^2 AR}\right)(1+\tau)} \tag{24}$$

Then, the new angle of attack solve to such as.

$$\alpha_a = \frac{C_L}{m \cdot \pi/180} \tag{25}$$

The induce drag coefficient for an elliptical wing is

$$C_{Di} = C_L \, \alpha_i \tag{26}$$

The induce angle is defined as:

$$\alpha_i = \alpha_a - \frac{c_l}{m_0} \tag{27}$$

Where C_L, is the lift coefficient on the wing, c_l is the section lift coefficient, and m_0 is the slope of section lift coefficient.

See Figure 15 for the typical angles that form the section wing.

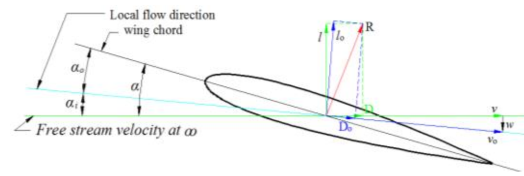


Figure 15

Typical Wing Angles

Spanwise Airloads Distribution for Specified Cl and True Velocity V

In this part, the users enter a value of true velocity from aircraft, the waterline distance of 25% from root chord, and the angle dihedral see Figure 16.

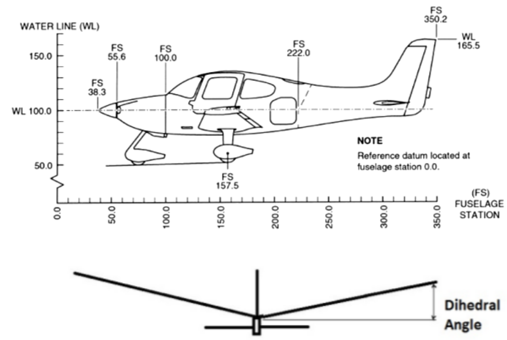


Figure 16

Water Line and Dihedral Angle

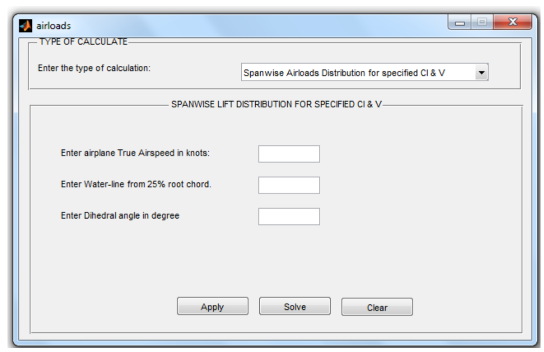


Figure 17

Spanwise Coefficient Distribution

As shown in the Figure 17, the distribution the lift W, is the distance function of the wing, and the shear force V, is related to the resultant load as.

$$W = \frac{dV}{dy} \tag{28}$$

The bending moment M, acting on the element is related to the shear force by

$$V = \frac{dM}{dy} \tag{29}$$

In integral form,

$$V = \int W dy \quad (30)$$

And

$$M = \int V dy \quad (31)$$

These integrals can be approximated by sums, namely,

$$V = \sum_i^N W_i \Delta y \quad (32)$$

And

$$M = \sum_i^N V_i \Delta y \quad (33)$$

The similar form is calculated for the torsion force.

The distribution of W, M and dy is shown as in the Figure 18

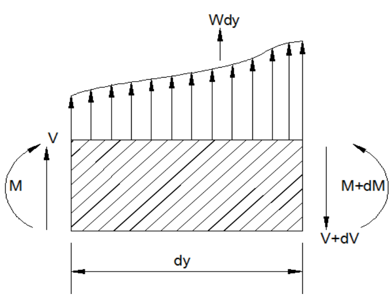


Figure 18
Schematic Representation of the Shear Force and Bending Moment on a Span-wise Element of the Wing

SAMPLE TOOL AIRLOADS

The tool Matlab was validate using data input-output from H.C. McMaster, “FAR 23 LOADS” [6], for the Wing Aerodynamic Coeficient module, the data input has 3 coordinates in Leading edge and 2 coordinates in the Trailing edge shown as Figure 19.

For the Additive Lift Coefficient Distribution Module is defined for 2 wing station, the first station in the root wing and the second station in the tip wing, and with its slope of lift curve shown as Figure 20.

The Basic Lift distribution Coefficient module is defined to 4 wing stations and their slope of the lift section regarding the angle of attack in degree angles, shown as Figure 21.

The Stall Cl coefficient module the data input has 4 wing station, 2 Reynolds Number, C_{lmax}, and length chord for each airfoil section discretized along to wing, shown as Figure 22.

For the Wing Air Coefficient Distribution module the data input is 2 wing stations, their profile drag coefficient, and their moment coefficients as shown Figure 23a. The data output shown 3 graphics the coefficients aerodynamics such as: c_l, c_d, and c_{di} along the wing, see Figure 23b.

Finally the ultimate module Figure 24a, the Air-loads Distribution enter CL from wing, the true velocity and others data shown in the Figure 24a, and the output data with their graphics of load along the wing shown in Figure 24b-e.

```
 WING AERODYNAMIC COEFICIENT, CRUISE
=====================================
Input data
-----------
Leading Edge Coordinates
  Point        XLE          YLE
    1       45.00000      0.00000
    2       64.31300     46.50000
    3       72.00000    201.00000

Trailing Edge Coordinates
  Point        XTE          YTE
    1      146.00000      0.00000
    2      116.00000    201.00000

Output data
------------
Area Total                = 26513.446
Mean Aerodynamic Chord = 69.246
YBAR                      = 87.854
XLE                       = 63.641
Aspect ratio              = 6.095
Span Total                = 402.000

  Element        Y            C
--------------------------------
    1         5.0250     98.16295
    2        15.0750     92.48885
    3        25.1250     86.81475
    4        35.1750     81.14065
    5        45.2250     75.46655
    6        55.2750     73.00041
    7        65.3250     71.00038

    9        85.4250     67.00032
   10        95.4750     65.00030
   11       105.5250     63.00027
   12       115.5750     61.00024
   13       125.6250     59.00021
   14       135.6750     57.00018
   15       145.7250     55.00015
   16       155.7750     53.00013
   17       165.8250     51.00010
   18       175.8750     49.00007
   19       185.9250     47.00004
   20       195.9750     45.00001
```

Figure 19
Sample – Wing Aerodynamic Geometry

```
WING AERODYNAMIC COEFICIENT, CRUISE
===============================================================
ADDITIVE LIFT DISTRIBUTION
==============================================================

Input Data
-----------
Wing       Wing      Slope of
Station #  Station   Lift Curve
   1          0        0.1075
   2        201        0.1075

Output Data
-----------

Element       YE          CC(LA1)      C(LA1) for CL = 1.00061
----------------------------------------------------
   1        5.02500     91.05578     0.927598
   2       15.07500     88.11359     0.952694
   3       25.12500     85.06548     0.979851
   4       35.17500     81.90982     1.009479
   5       45.22500     78.64409     1.042105
   6       55.27500     76.86877     1.052991
   7       65.32500     75.20829     1.059266
   8       75.37500     73.42356     1.064104
   9       85.42500     71.50690     1.067262
  10       95.47500     69.44848     1.068433
  11      105.52500     67.23572     1.067229
  12      115.57500     64.85227     1.063148
  13      125.62500     62.27653     1.055531
  14      135.67500     59.47916     1.043491
  15      145.72500     56.41887     1.025795
  16      155.77500     53.03449     1.000648
  17      165.82500     49.22852     0.965263
  18      175.87500     44.82711     0.914838
  19      185.92500     39.45386     0.839443
  20      195.97500     31.82982     0.707329
```

Figure 20

Sample – Additive Lift Distribution

```
   BASIC LIFT DISTRIBUTION
=====================================================
Input Data
-----------
 Item  Wing Station   Angle
  1       0.000       5.000
  2      46.500       4.577
  3     109.279       4.028
  4     201.000       1.900
Output Data
-----------
Wing angle of attack for zero lift = 3.988147
Element  Ref Angle  alphawo      cclb       clb
---------------------------------------------
  1      4.95429    0.96614    5.09762    0.05193
  2      4.86287    0.87472    4.34847    0.04702
  3      4.77144    0.78330    3.65509    0.04210
  4      4.68002    0.69187    3.01748    0.03719
  5      4.58860    0.60045    2.43563    0.03227
  6      4.50026    0.51212    2.00943    0.02753
  7      4.41238    0.42423    1.61897    0.02280
  8      4.32449    0.33634    1.24742    0.01808
  9      4.23660    0.24846    0.89476    0.01335
 10      4.14872    0.16057    0.56099    0.00863
 11      4.06083    0.07268    0.24612    0.00391
 12      3.88193   -0.10622   -0.34827   -0.00571
 13      3.64876   -0.33939   -1.07628   -0.01824
 14      3.41559   -0.57255   -1.75417   -0.03077
 15      3.18242   -0.80572   -2.38192   -0.04331
 16      2.94926   -1.03889   -2.95955   -0.05584
 17      2.71609   -1.27206   -3.48704   -0.06837
 18      2.48292   -1.50523   -3.96440   -0.08091
 19      2.24975   -1.73839   -4.39162   -0.09344
 20      2.01658   -1.97156   -4.76872   -0.10597
There is no Discontinuity between Flap and Aileron

Element   cc_lb      c_lb  Faired
  1      5.09762    0.05193
  2      4.34847    0.04702
  3      3.65509    0.04210
  4      3.01748    0.03719
  5      2.43563    0.03227
  6      2.00943    0.02753
  7      1.61897    0.02280
  8      1.24742    0.01808
  9      0.89476    0.01335
 10      0.56099    0.00863
 11      0.24612    0.00391
 12     -0.34827   -0.00571
 13     -1.07628   -0.01824
 14     -1.75417   -0.03077
 15     -2.38192   -0.04331
 16     -2.95955   -0.05584
 17     -3.48704   -0.06837
 18     -3.96440   -0.08091
 19     -4.39162   -0.09344
 20     -4.76872   -0.10597
```

Figure 21

Sample – Basic Lift Distribution

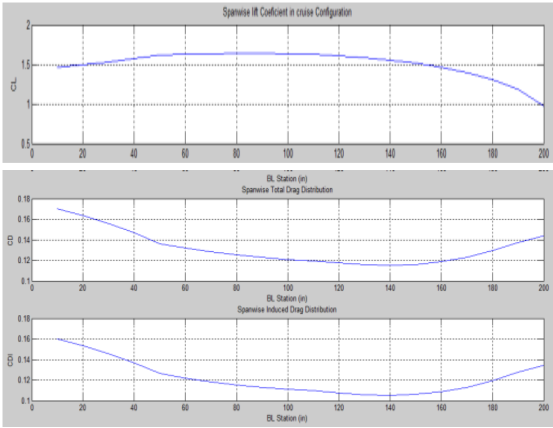
```
    Stall CL
=============================================================

Input Data
-----------
Select Wing Stas, 2CLMAXS & Reinolds NO. and chord

Item    Wing Sta    CLMAX1    RN1      CLMAX2    RN2      CHORD
  1       0.000     1.450   3000000   1.660   9000000   101.000
  2      46.500     1.460   3000000   1.680   9000000    74.746
  3     109.279     1.480   3000000   1.700   9000000    62.253
  4     201.000     1.500   3000000   1.740   9000000    44.000

Output Data
------------

For 70 mph
item     RN3      CLMAX3
  1    5514282   1.562843
  2    4080896   1.518539
  3    3398818   1.503487
  4    2402261   1.455641

CL MAX (STALL) FOR EACH ELEMENT
Item      YE       ELEMENT CL MAX
  1     5.02500     1.558055
  2    15.07500     1.548480
  3    25.12500     1.538904
  4    35.17500     1.529329
  5    45.22500     1.519753
  6    55.27500     1.516435
  7    65.32500     1.514025
  8    75.37500     1.511616
  9    85.42500     1.509206
 10    95.47500     1.506797
 11   105.52500     1.504387
 12   115.57500     1.500203
 13   125.62500     1.494960
 14   135.67500     1.489718
 15   145.72500     1.484475
 16   155.77500     1.479232
 17   165.82500     1.473990
 18   175.87500     1.468747
 19   185.92500     1.463505
 20   195.97500     1.458262

Wing Stall CL Distribution
Wing Stall CL = 1.420
Item       YE         CL
  1      5.02500    1.359844
  2     15.07500    1.390315
  3     25.12500    1.423691
  4     35.17500    1.460554
  5     45.22500    1.501643
  6     55.27500    1.512244
  7     65.32500    1.516367
  8     75.37500    1.518465
  9     85.42500    1.518194
 10     95.47500    1.515122
 11    105.52500    1.508700
 12    115.57500    1.493329
 13    125.62500    1.470056
 14    135.67500    1.440547
 15    145.72500    1.403063
 16    155.77500    1.355074
 17    165.82500    1.292648
 18    175.87500    1.209015
 19    185.92500    1.090176
 20    195.97500    0.891362
```

Figure 22

Sample – Stall Cl

```
 WIng Aero Coeficient Distribution CL
==========================================================

Input Data
-----------
TAU = 0.050
Select Wing Stations and their profile Drag Coeficients

Item     WS       DC
  1     0.000    0.010
  2   201.000    0.010

Select Wing Stations and their Moment Coeficients
Item     WS       MC
  1     0.000   -0.030
  2   201.000   -0.030

Output Data
------------

SPANWISE AERO COEFICIENT DISTRIBUTIONS FOR CL = 1.520
Item       CL        CDI        CPD         CD         CM
    1    1.46188   0.160288   0.010000   0.170288   -0.030000
    2    1.49511   0.153480   0.010000   0.163480   -0.030000
    3    1.53148   0.145728   0.010000   0.155728   -0.030000
    4    1.57160   0.136802   0.010000   0.146802   -0.030000
    5    1.61627   0.126389   0.010000   0.136389   -0.030000
    6    1.62807   0.121684   0.010000   0.131684   -0.030000
    7    1.63289   0.118263   0.010000   0.128263   -0.030000
    8    1.63552   0.115246   0.010000   0.125246   -0.030000
    9    1.63559   0.112723   0.010000   0.122723   -0.030000
   10    1.63265   0.110796   0.010000   0.120796   -0.030000
   11    1.62610   0.109587   0.010000   0.119587   -0.030000
   12    1.61028   0.107629   0.010000   0.117629   -0.030000
   13    1.58616   0.105772   0.010000   0.115772   -0.030000
   14    1.55533   0.105172   0.010000   0.115172   -0.030000
   15    1.51590   0.106041   0.010000   0.116041   -0.030000
   16    1.46515   0.108601   0.010000   0.118601   -0.030000
   17    1.39883   0.113053   0.010000   0.123053   -0.030000
   18    1.30965   0.119477   0.010000   0.129477   -0.030000
   19    1.18251   0.127474   0.010000   0.137474   -0.030000
   20    0.96917   0.134099   0.010000   0.144099   -0.030000

Angle from WL to Wing Aero Lift line = 3.988147
Angle from relative wind to wing zero Lift Line = 18.9154
Angle from relative wind to Waterline = 14.927250
Cl(wing) = 1.5209
CD(wing) = 0.1344
CM(wing) = -0.0300
```

(a)



(b)

Figure 23

Sample – Aerodynamic Coefficients

Input Data

----

true velocity = 117.400, CL = 1.520

waterline of 25 percent from root chord = 78.500

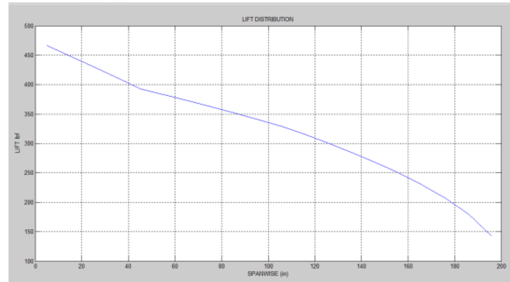
Dihedral angle = 6.000

Output Data

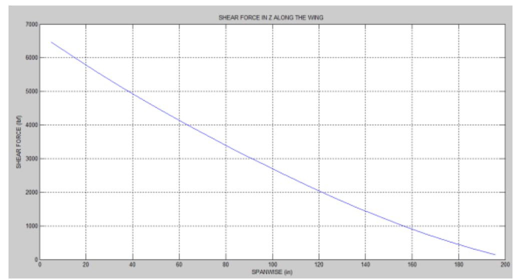
WS	Y	X	L	D	M
1	5.025	71.628	466	-68	-943
2	15.075	74.383	448	-69	-837
3	25.125	77.139	430	-69	-737
4	35.175	79.895	412	-70	-644
5	45.225	82.650	393	-70	-557
6	55.275	83.000	383	-70	-521
7	65.325	83.000	373	-69	-493
8	75.375	83.000	363	-68	-466
9	85.425	83.000	352	-66	-439
10	95.475	83.000	341	-64	-413
11	105.525	83.000	329	-62	-388
12	115.575	83.000	316	-60	-364
13	125.625	83.000	301	-57	-341
14	135.675	83.000	285	-54	-318
15	145.725	83.000	268	-50	-296
16	155.775	83.000	250	-45	-275
17	165.825	83.000	230	-40	-254
18	175.875	83.000	208	-34	-235
19	185.925	83.000	181	-26	-216
20	195.975	83.000	143	-16	-198

WS	x	y	z	Fx	Fz	Sx	Sz	Mxx	Myy	Mzz
20	83.000	195.975	99.096	-16	143	-16	143	0	-198	0
19	83.000	185.925	98.040	-26	181	-43	323	1436	-431	-163
18	83.000	175.875	96.984	-34	208	-76	531	4686	-711	-590
17	83.000	165.825	95.928	-40	230	-117	761	10022	-1046	-1358
16	83.000	155.775	94.871	-45	250	-162	1011	17669	-1444	-2530
15	83.000	145.725	93.815	-50	268	-212	1279	27829	-1911	-4158
14	83.000	135.675	92.759	-54	285	-266	1564	40682	-2453	-6287
13	83.000	125.625	91.703	-57	301	-323	1864	56398	-3074	-8957
12	83.000	115.575	90.647	-60	316	-383	2180	75136	-3779	-12201
11	83.000	105.525	89.590	-62	329	-445	2509	97044	-4571	-16047
10	83.000	95.475	88.534	-64	341	-509	2850	122259	-5455	-20519
9	83.000	85.425	87.478	-66	352	-575	3202	150901	-6432	-25638
8	83.000	75.375	86.422	-68	363	-643	3565	183083	-7506	-31421
7	83.000	65.325	85.365	-69	373	-712	3938	218911	-8678	-37884
6	83.000	55.275	84.309	-70	383	-781	4320	258487	-9951	-45036
5	82.650	45.225	83.253	-70	393	-851	4713	301907	-12844	-52888
4	79.895	35.175	82.197	-70	412	-921	5125	349276	-27375	-61443
3	77.139	25.125	81.141	-69	430	-990	5555	400784	-43208	-70697
2	74.383	15.075	80.084	-69	448	-1058	6004	456616	-60398	-80646
1	71.628	5.025	79.028	-68	466	-1126	6470	516955	-79003	-91283

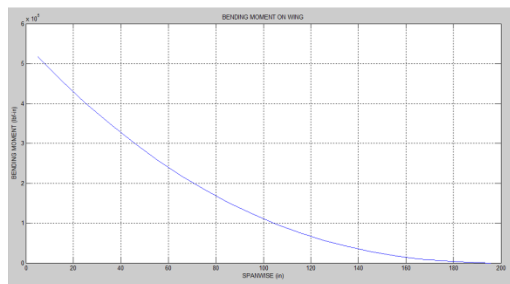
(a)



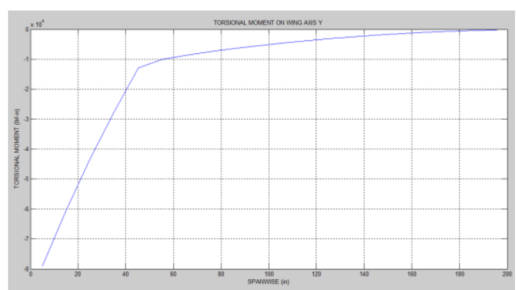
(b)



(c)



(d)



(e)

Figure 24

Sample – Air-load Distribution

CONCLUSIONS

The principal propose this work, obtained the Air-loads distribution along the wing for the Schrenk method and using the Matlab program, it was satisfactory results shown as Figure 24 b-d and data output in the Figure 24a. For the compare the output data see reference [6].

As shown in the windows of this tool, it's easy to use, so that, achieve the other goal.

This tool was thinking in the class of structural design from Polytechnic University of Puerto Rico, but any student the other university or Aircraft designer pioneer can be to use with reliability their analysis.

For the future work, this tool can be expanded to tail load or fuselage analysis, also can be implemented for the Code of Federal Regulation and build to V-n diagram. In addition, this tool can be implemented for structural design, since the wing geometry drawing to obtain the structural wing.

Finally, same this tool can be development other program using other method of the approximation for obtain the load distribution in aircraft.

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