Time Confinement Simulation of a Charged Particle in a Mirror Magnetic Field

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Abstract — *Plasma is a state of matter formed of electrically charged particles. Those particles are greatly influenced by electromagnetic interactions, which have been abundantly studied since the 19 century. The Lorentz equation is used to analyze the force that acts over a particle. This equation depends on the evaluated magnetic and electric field in a specific position. The electric field is described using the Maxwell equation. This study has the purpose of presenting and discussing the results of a simulation done using the MatLab computer software, to measure the confinement time of a particle in a magnetic field. The confinement time is one of the statements that constitute the Lawson's criterion that establishes the conditions necessary for fusion.*

Key Terms — *Confinement Time, Lawson's Criterion, Lorentz Force, Plasma.*

INTRODUCTION

The research refers to the topic of confinement time in a charged particle in a mirror magnetic field. The confinement time is defined as "the minimum duration in which adequate conditions are maintained, and the fusion energy being released is higher than the energy employed in the plasma confinement and plasma heating" [1]. This is one of the main parameters for the Lawson Criterion [2]. Both are highly useful because they tell us the probability of creating fusion energy [3] of deuterium-tritium and energy.

With the purpose of achieving the objective, a simulation in MatLab will be done to insert a charged particle in a *mirror* magnetic field that will be governed by the Lorentz equation.

THEORETICAL

The single particle movement is subject to interactions of both the magnetic and electric fields. In this case, collisional interactions between particles and the collective effects both get despised [4]. The behavior of a charged particle that moves in a magnetic field and is determined by the Newton laws. The basic equation is stated by the Lorentz force [5] [6].

$$\boldsymbol{m}\,\frac{\boldsymbol{dv}}{\boldsymbol{dt}} = \boldsymbol{q}\,(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \tag{1}$$

Where × represents a vector product, m is the particle mass and $\boldsymbol{v}$ the velocity of the charge q that moves through the electric field vector $\boldsymbol{E}$ and the magnetic field $\boldsymbol{B}$. As an example it's considered that $\boldsymbol{B}$ and $\boldsymbol{E}$ are uniformed where $\boldsymbol{E} = 0$. As a result, the movement equation is going to be:

$$\boldsymbol{m}\,\frac{\boldsymbol{dv}}{\boldsymbol{dt}} = \boldsymbol{qv} \times \boldsymbol{B} \tag{2}$$

By differentiating this equation with respect to time, the simple harmonic movement equation is obtained were the cyclotron frequency [7] is described by:

$$\boldsymbol{\Omega_c} = \frac{\boldsymbol{qB}}{\boldsymbol{m}} \tag{3}$$

The rotation time scale [7] of a particle is:

$$\boldsymbol{T_o} = \frac{\boldsymbol{2\pi}}{\boldsymbol{\Omega_c}} \tag{4}$$

Then the time is calculated as a fraction of the particle's rotation time where the X variable is an adjustable parameter.

$$\boldsymbol{dT_o} = \mathbf{X} * \boldsymbol{T_o} \tag{5}$$

While the displacement velocity of the particle can be expressed as:

$$\boldsymbol{v_1} = \left[\left(\frac{q}{m}\right)\boldsymbol{v_0} \times \boldsymbol{B}\right] dT_o + \boldsymbol{v_o} \tag{6}$$

It is important to mention that $\boldsymbol{p_o}$ and $\boldsymbol{v_o}$ are the position and the initial velocity. Having obtained the values for the velocity of the particle, acceleration can then be calculated.

$$\boldsymbol{a_o} = \boldsymbol{m} * \frac{v_1 - v_o}{dT_o} \quad (7)$$

Lastly, the particle's new position is calculated.

$$\boldsymbol{p_1} = \boldsymbol{p_o} + \boldsymbol{v_o}\, \boldsymbol{dT_o} + \frac{a_o (dT_o)^2}{2} \quad (8)$$

Generally, the movement of a charged particle in a constant magnetic field creates a propeller shape. But if the velocity is perpendicular, the shape of the field will be circular.

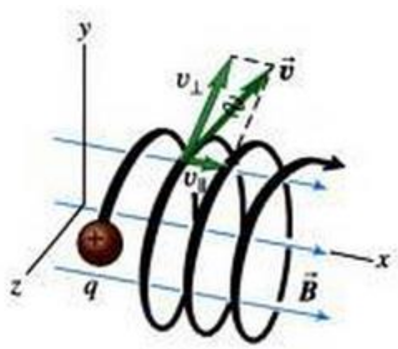

Figure 1
When a charged particle has velocity components in both perpendicular and parallel movement in a uniform magnetic field. The particle this way shows a helical trajectory. **[8]**

The relative movement of a particle in a magnetic field will depend on the symbol of the charged particle. [9].

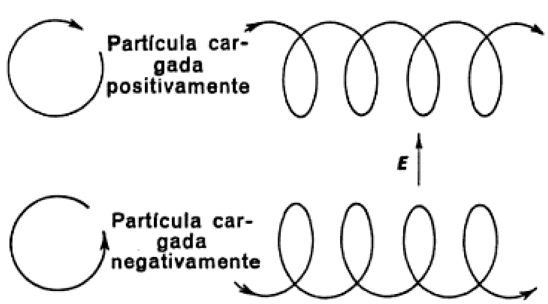

Figure 2
Particle rotation in a uniform magnetic field [9]

The magnetic field $\boldsymbol{B}$, can also be created using two coils. This can be summarized by the Biot-Savart law (9) that can be derived using the Maxwell equations [10]. The J variable represents the current density, c is the velocity of the light in the vacuum, the subtraction $\boldsymbol{r_2} - \boldsymbol{r_1}$ represents the distance of the *loop* conductor $\boldsymbol{r_{12}}$ and $dl * ds$ is the basic volume dv.

$$\boldsymbol{B(p)} = \frac{1}{c} \iint_{l\ s} \boldsymbol{J}\, \frac{dl_x\,(r_2 - r_1)}{|r_2 - r_1|^3}\, \boldsymbol{ds} \quad (9)$$

Figure 3 shows a representation of the variables mentioned before (9).

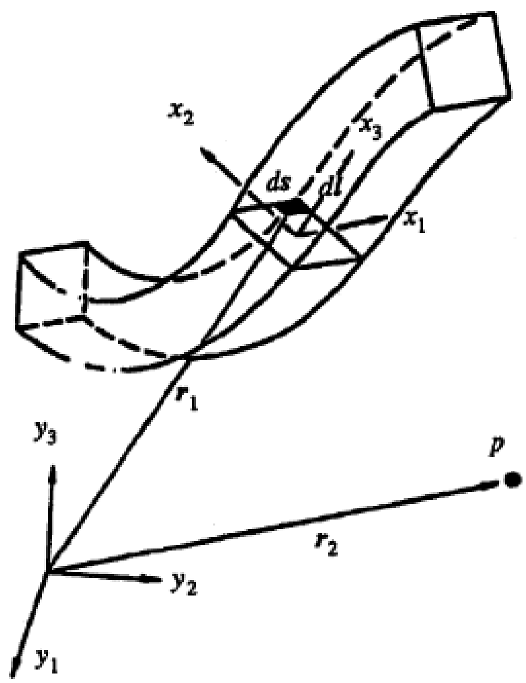

Figure 3
Variable coil elements in (9). [10]

The magnetic confinement is one of the main techniques used to create fusion [11] and it consist of creating a magnetic field with the purpose of confining the ionized plasma in the interior of a reactor.

The Lawson criterion [12] determines the minimal conditions necessary for creating fusion. This criterion establishes a relationship between the density (η: concentration of the particles by cubic centimeters) [13], temperature (T_p: in electron-volt) and the confinement time (τ: in seconds) and it is expressed with the product of three terms. The product of these three factors needs to outperform the value $6 * 10^{21}$. The confinement time for a particle is calculated by dividing the distance between its speed.

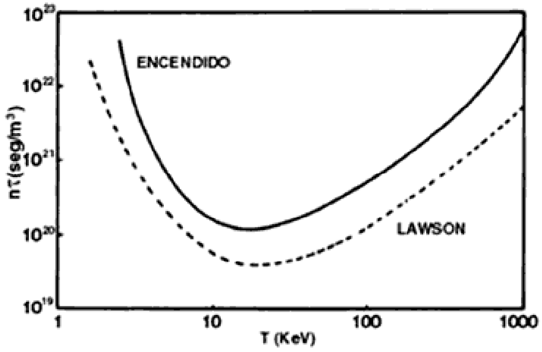

Figure 4
Product value of $\eta\tau$ in function of T_p necessary for the Lawson criterion and the start for the deuterium-tritium reaction. [11]

J. Thompson discovered that the speed of the electrons depends of the *V* [8] potential accelerator. Deducing that a relationship exists between kinetic energy and the loss of electrical potential energy *qV*, where *q* is the charge magnitude.

$$v = \sqrt{\frac{2|q|V}{m}} \quad (10)$$

The unit used to represent the energy is the electron-volt (eV), that is equal to 11,600 ℃. The energy receives an electron after being accelerated in a vacuum through a potential difference of one volt [14] [15].

Methodology

The simulation is based in the equations described in the previous section. But in this case the magnetic field can change depending on the position. The program to compute this tasks was given, and it is based on the Biot-Savart law (9). During this simulation, a mirror magnetic field was created as shown in **figure 5**. Of the given program we understand that the separation of the coils was 0.17 cm, with a radius of 0.27305 cm and 340 turns.

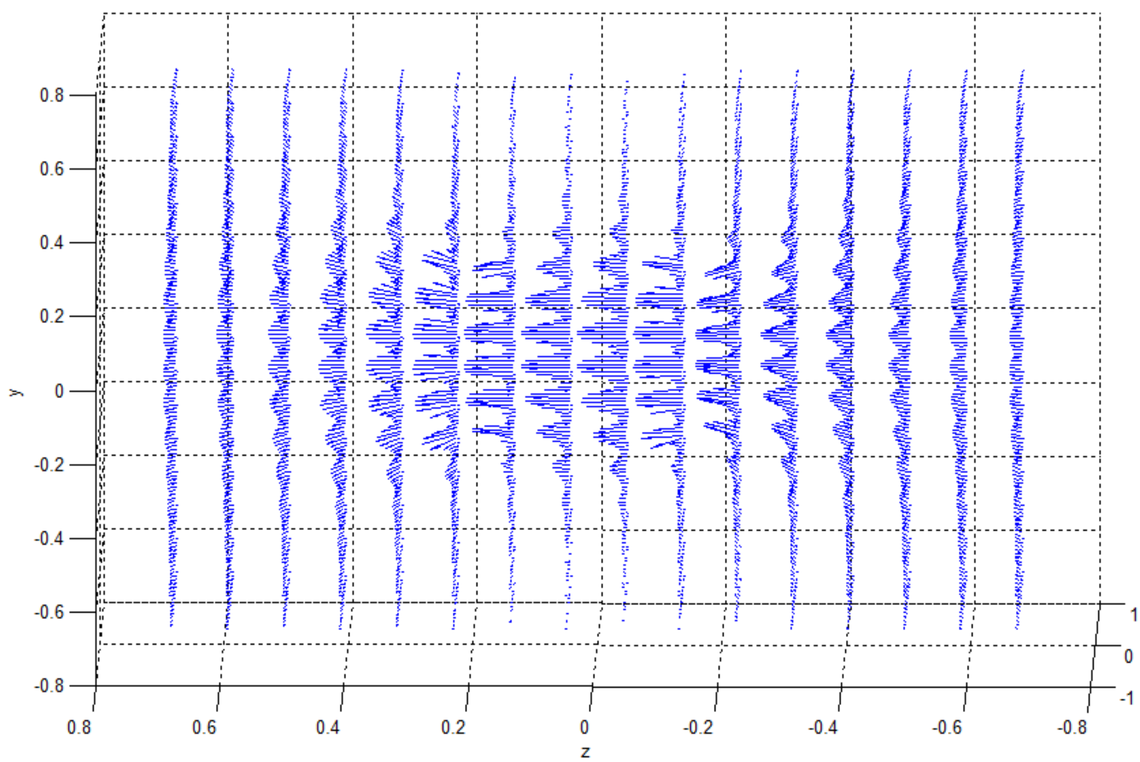

Figure 5
Mirror Magnetic Field

As an experimental configuration, arbitrary values were given to the energy which ranged between 1eV - 10keV and the adjustable parameter, X, that was $1\,x\,10^{-3}$. **Figure 6** shows the initial position ($\boldsymbol{p_o}$) given the particle in the space.

To establish the particle limits, a cylinder was used and which represents the walls of the plasma machine. The particle kept moving until it exceeded the radius or length of the figure. This is one of the conditions necessary to stop the simulation.

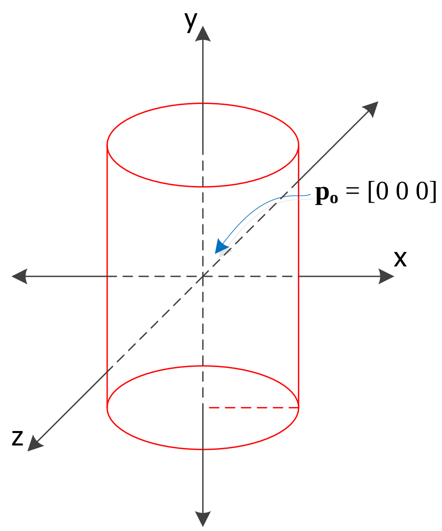

Figure 6
The cylinder represents the walls of the plasma machine.

The equations have to be calculated in a specific order because it depends one of the other. **Figure 7** shows flowchart of the program.

Once the simulations end, all the gathered data was analyzed and the confinement time was calculated.

Note:

- The units of the computations made were given using the international system (IS).
- The vectors represent the values for i, j, k
- The mass value (*m*) of the particle is 9.10938188 x 10^{-3} Kg
- The value of the charge (*q*) of the particle is -1.60217646 x 10^{-19} C

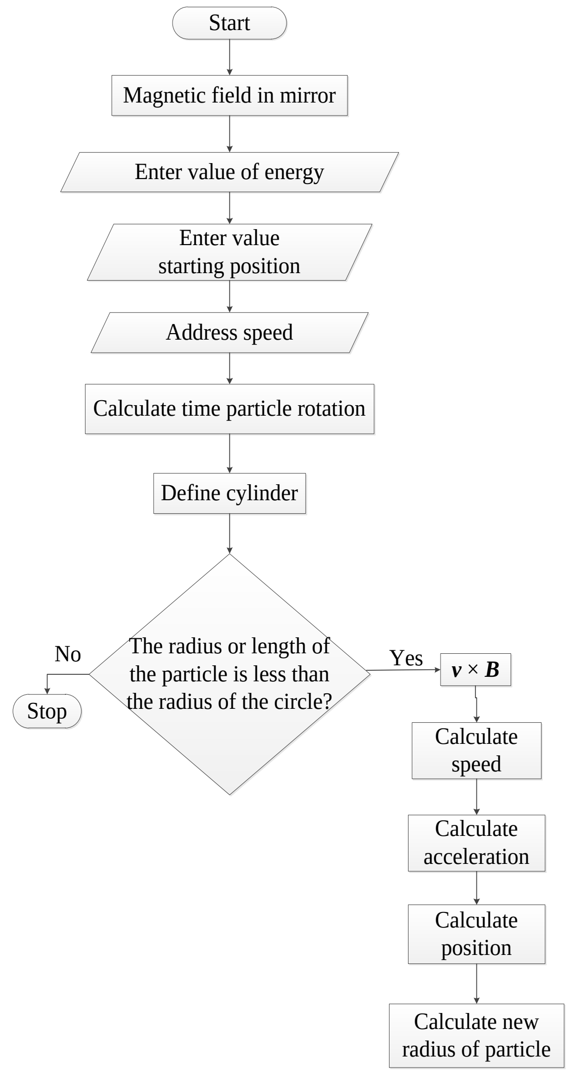

Figure 7
Flowchart

DATA AND RESULTS

Figure 8 and 9 shows the trajectory of a particle that was named P_R. This particle was introduced in a mirror magnetic field with 100eV and the radial direction. Its initial position in the space, $\boldsymbol{p_o}$, was [0 0 0].

Figure 10 and 11 show particle P_A with the same conditions as P_R with the only difference of the axial direction.

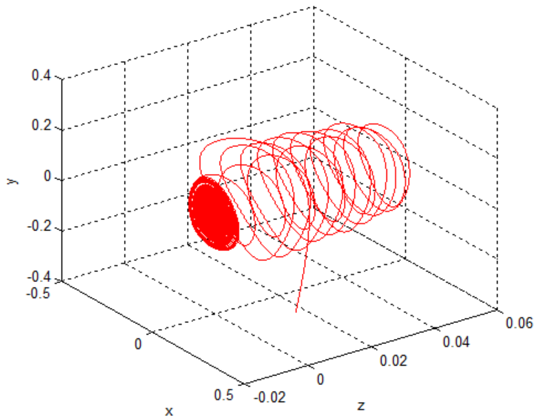

Figure 8
3D trajectory visualization of a particle with 100eV and radial. Its initial position p_o is [0 0 0].

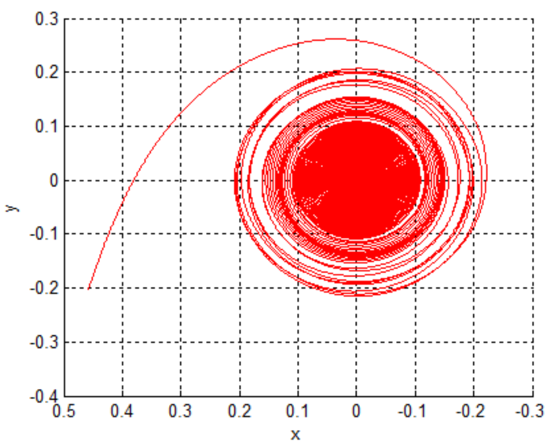

Figure 9
2D trajectory visualization of a particle with 100eV y radial. Its initial position p_o is [0 0 0].

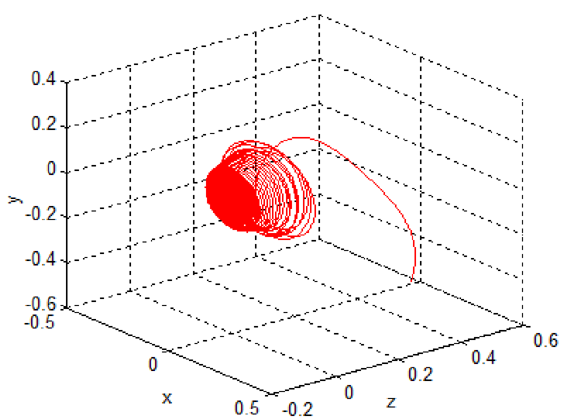

Figure 10
3D trajectory visualization of a particle with 100eV and axial. Its initial position p_o is [0 0 0].

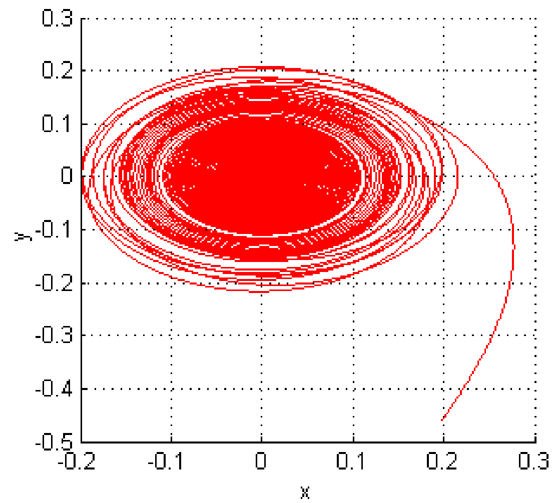

Figure 11
2D trajectory visualization of the particle with 100eV and axial. Its initial position p_o is [0 0 0].

Table 1 shows the confinement times obtained when the energy of the particle increased. It shows that when the energy increases in the particle, the confinement time decreases. Looking at the data table we understand the minimal variation of the confinement time in axial and radial. At the same time the density value, η, was calculated to be able to reach the $6 * 10^{21}$.

Table 1
Confinement time with different energies in radial and axial.

Energy (eV)	***τ (seconds)***		***η (particles/cm³)***
	Radial	axial	Radial
1	$1.38x10^{-4}$	$1.35x10^{-4}$	$4.35x\ 10^{25}$
10	$4.20x10^{-5}$	$4.33x10^{-5}$	$1.43x\ 10^{25}$
25	$2.85x10^{-5}$	$2.69x10^{-5}$	$8.42x\ 10^{24}$
50	$1.90x10^{-5}$	$1.94x10^{-5}$	$6.32x\ 10^{24}$
75	$1.62x10^{-5}$	$1.59x10^{-5}$	$4.94x\ 10^{24}$
100	$1.39x10^{-5}$	$1.46x10^{-5}$	$4.32x\ 10^{24}$
150	$1.10x10^{-5}$	$1.20x10^{-5}$	$3.63x\ 10^{24}$
200	$9.69x10^{-6}$	$1.02x10^{-5}$	$3.10x\ 10^{24}$
250	$8.70x10^{-6}$	$9.04x10^{-6}$	$2.48x\ 10^{24}$
300	$7.92x10^{-6}$	$7.91x10^{-6}$	$2.53x\ 10^{24}$
350	$6.92x10^{-6}$	$7.33x10^{-6}$	$2.48x\ 10^{24}$
400	$6.77x10^{-6}$	$6.86x10^{-6}$	$2.21x\ 10^{24}$
450	$6.46x10^{-6}$	$6.47x10^{-6}$	$2.06x\ 10^{24}$

500	$5.77x10^{-6}$	$6.13x10^{-6}$	$2.07x\ 10^{24}$
550	$5.84x10^{-6}$	$5.84x10^{-6}$	$1.87x\ 10^{24}$
750	$5.01x10^{-6}$	$5.00x10^{-6}$	$1.60x\ 10^{24}$
$1k$	$4.26x10^{-6}$	$4.34x10^{-6}$	$1.41x\ 10^{24}$
$2k$	$3.06x10^{-6}$	$3.06x10^{-6}$	$9.80x\ 10^{23}$
$3k$	$2.45x10^{-6}$	$2.45x10^{-6}$	$8.16x\ 10^{23}$
$4k$	$2.12x10^{-6}$	$2.12x10^{-6}$	$7.08x\ 10^{23}$
$5k$	$1.90x10^{-6}$	$1.89x10^{-6}$	$6.32x\ 10^{23}$
$6k$	$1.74x10^{-6}$	$1.76x10^{-6}$	$5.75x\ 10^{23}$
$7k$	$1.60x10^{-6}$	$1.64x10^{-6}$	$5.36x\ 10^{23}$
$8k$	$1.52x10^{-6}$	$1.50x10^{-6}$	$4.93x\ 10^{23}$
$9k$	$1.44x10^{-6}$	$1.44x10^{-6}$	$4.63x\ 10^{23}$
$10k$	$1.36\ x10^{-6}$	$1.37x10^{-6}$	$4.41x\ 10^{23}$

The equation that describes the graph behavior in **figure 12** is $y = .000138x^{-0.5029}$. This information was obtained using the ezyfit toolbox.

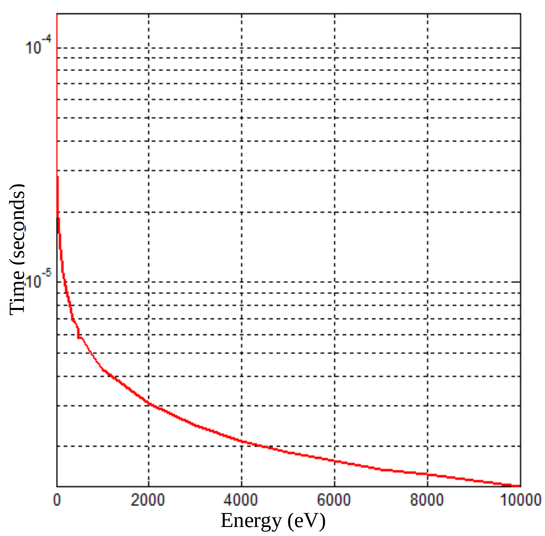

Figure 12

Confinement time of the particles in a radial mirror field

CONCLUSIONS AND RECOMMENDATIONS

The obtained results demonstrate that the confinement time, the density and the temperature are related. Getting into the conclusion that higher energy will get you less confinement time of the particle in the magnetic. The same occurs with the density and the confinement time.

This work leaves the door open to any undergraduate and graduate students for ideas for future research. The next step would be to include collisions and see how this change can affect the confinement time in the particle. It is also possible to modify the simulation to add the electric field. Also, it is also an idea to create various magnetic fields to observe and compare the confinement times.

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