Strain gauge and micrometer instrumentation of a truss structure

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Abstract

This project was performed as part of the Structural Analysis courses in order to compare theoretical results of the response of simple structures with the experimental results obtained at the Structures Engineering Laboratory of the Department of Civil Engineering. The paper presents a comparative analysis between theoretical and experimental response of a simple truss structure subjected to static loads. The variables selected to describe the response were displacements and strains.

The experimental results were obtained instrumenting a large scale truss with strain gauges and micrometers, and applying a sequence of static loads by means of hydraulic activators. The geometry of the truss and the cross sectional properties were measured in order to obtain the theoretical results by means of the virtual work analysis and by the stiffness method based on computer programs.

Sinopsis

Este proyecto se desarrolló para comparar la teoría enseñada en los cursos de análisis estructural con los resultados obtenidos en el Laboratorio de Estructuras. El artículo presenta un análisis comparativo entre la teoría y los resultados experimentales de una estructura que actúa como cercha. Las variables seleccionadas para el desarrollo del proyecto fueron desplazamientos y deformaciones.

Los valores experimentales se obtuvieron implementando una cercha a escala natural con deformadores y micrometros y aplicando una carga puntual en incrementos de 2kN por medio de un sistema hidráulico. Se midieron la geometría de la cercha y las propiedades seccionales para obtener los resultados teóricos por medio del análisis de trabajo virtual y por el método de rigidez a través de programas de computadora.

Introduction

One of the most difficult tasks of the engineer is to create a model that represents the real conditions of a system that he or she wants to reproduce. Civil engineers throughout the years have created a series of structures used in laboratories to recreate the real conditions. This work presents the analysis of one of the models used to recreate the conditions of a truss. This model was designed to act as a truss, however it is a frame because the joints are welded instead of pin connected. This truss was subjected to loads acting at the center of the structure. Joints A, C and E in figure 1 were selected as representative nodes of the structure, dial gauge readings were taken at these nodes. Members 6, 11 and 12 in figure 1 were also selected as representative of all the members of the structure. Strain gauges were placed at these members in order to obtain representative strains of the structure.

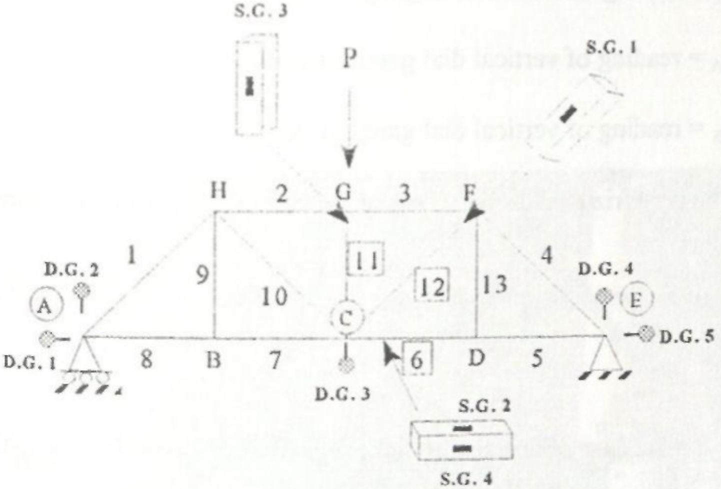

Figure 1. Planar truss

At nodes A and E dial gauges were located in order to measure horizontal and vertical displacements of the truss. A dial gauge was located at node C to measure the vertical displacement. The objective of these dial gauges was the measurement of the net vertical displacement of the node C and the net horizontal displacement at node A. The floor of the laboratory was the reference for all the dial gauges.

The net vertical displacement of the node C (Δ_{vc}) is found using equation (1)

$$\Delta_{vc} = \delta_c - \frac{\delta_A - \delta_E}{2} \tag{1}$$

where:

δ_c = reading of vertical dial gauge at node C

δ_A = reading of vertical dial gauge at node A

δ_E = reading of vertical dial gauge at node E

The net horizontal displacement of the node A is given by equation (2)

$$\Delta_{HA} \equiv \delta_{HA} - \delta_{HE} \qquad (2)$$

where:

δ_{HA} = reading of horizontal dial gauge at node A (positive at left)

δ_{HE} = reading of horizontal dial gauge at node E (positive at left)

In the experiment δ_{HA} was at left and δ_{HE} was at right, then;

$$\Delta_{HA} \equiv \left| \delta_{HA} \right| + \left| \delta_{HE} \right| \qquad (3)$$

At members 11 and 12 one strain gauge was located (S.G. 1 and S.G.3, respectively) in the neutral axis of each section, and at the center of the bar (fig. 1). In member 6, two strain gauges were located (S.G. 2 and S.G. 4). The S.G. 4 was located at neutral axis of the section and S.G. 2 at the top fiber of the same section (fig. 1).

The lectures of strain gauges were done directly with Data Acquisition Equipment shown in Figure 2. The vertical load was applied at node G with a hydraulic jack, using a frame with a capacity of 300 kN. The loads were applied up to 40 kN with an increment of 2 kN.

Figure 2. Planar truss with dial and strain gauges attached

The horizontal and vertical displacement of joints A and E and vertical displacement at C and strain of members 6, 11 and 12 of the planar truss in figure 1 were determined for an increment load of 2kN each time until reaching 40kN. The virtual work method [1] was used to obtain the joint displacement (table 1). The strain gauges readings were compared with the

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[1] Hibbeler, R.C., 1995, "Structural Analysis", Prentice Hall, 3rd. Ed.

theoretical strains obtained with the joint method. The theoretical strain of member 6 was obtained by combining the strains caused by axial forces and the bending moment obtained using "FRAME"[2] computer program. To model with "FRAME" the joints were considered as rigid, according to the real situation.

Table 1. Virtual work method: P=40kN, AE=95530 kN

Member		n[kN]	N [kN]	L [mm]	nNL (kN^{2}mm)
AB	8	0.499	20	900	8982
AH	1	-0.707	-28.28	1273	25452
GF	3	-0.999	-40	900	35964
HG	2	-0.999	-40	900	35964
FE	4	-0.707	-28.28	1273	25452
ED	5	0.499	20	900	8982
DF	13	0	0	900	0
DC	6	0.499	20	900	8982
DB	7	0.499	20	900	8982
BH	9	0	0	900	0
GC	11	0	40	900	0
HC	10	0.707	28.28	1273	25452
CF	12	0.707	28.28	1273	25452
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Equation (3) was used with virtual work method

$$1 \cdot \Delta \equiv \sum \frac{n\,N\,L}{A\,E} \qquad (4)$$

where:

1 = external virtual unit load acting on the truss joint in the stated

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[2] Pesquera, C.I., 1992, "Frame Software", V 1.80

direction of D

Δ = external joint displacement caused by the real loads on the truss

n = internal virtual normal force in a truss member caused by the external virtual unit load applied at the location and the direction of the desired displacement

N = internal normal force in a truss member caused by the real loads

L = length of a member

A = cross-sectional area of a member

E = modulus of elasticity of a member obtained from HPM. 6/1[3]

Equation (5) was used to compute the strain of members 1, 3 and 4

$$\varepsilon \equiv \frac{P}{A\,E} \tag{5}$$

where:

ε = normal strain of a member

P = internal resultant normal force acting on the centroid of the cross-sectional area of a member

A = cross-sectional area of a member

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[3] HI-TECH Instruction Manual HPM. 6/1, Issue 1, *Plane Frames*, January

E = modulus of elasticity of a member obtained from HPM. 6/1^3

The theoretical results of strain gauge 2 at the member 6 was obtained by using the following equation where the displacements caused by axial forces and the displacements from the bending moment were combined:

$$\varepsilon = \frac{P}{AE} + \frac{MY}{IE} \tag{6}$$

where:

M = the resultant internal moment, obtained from "FRAME"

Y = the perpendicular distance from the neutral axis to the end of a member

I = the moment of inertia of the cross-sectional area obtained from HPM. 6/1^3

Instrumentation

Figure 2 shows the planar truss with the equipment used to measure the strains and displacements of the truss properly indicated. The strain gauges were placed at several members of the truss to measure the strain of said members; the readings were taken with the use of the Data Acquisition Equipment. The vertical and horizontal displacements of the truss were measured with dial gauges that were placed on the lower nodes of the truss. The external load was applied with a jack and a hydraulic hand pump. Figure 3 shows a a student team applying load and taking readings.

Discussion of results

Figure 4 shows the structural analysis of the frame, considering the welded joints. Table 2 shows the results from the dial readings and table 3 shows the strain gauge readings and the error percentages. Figure 5 shows the behaviour of load versus displacement of joint C and figures 6 to 9 show the plots of load versus strain for the different strain gauges.

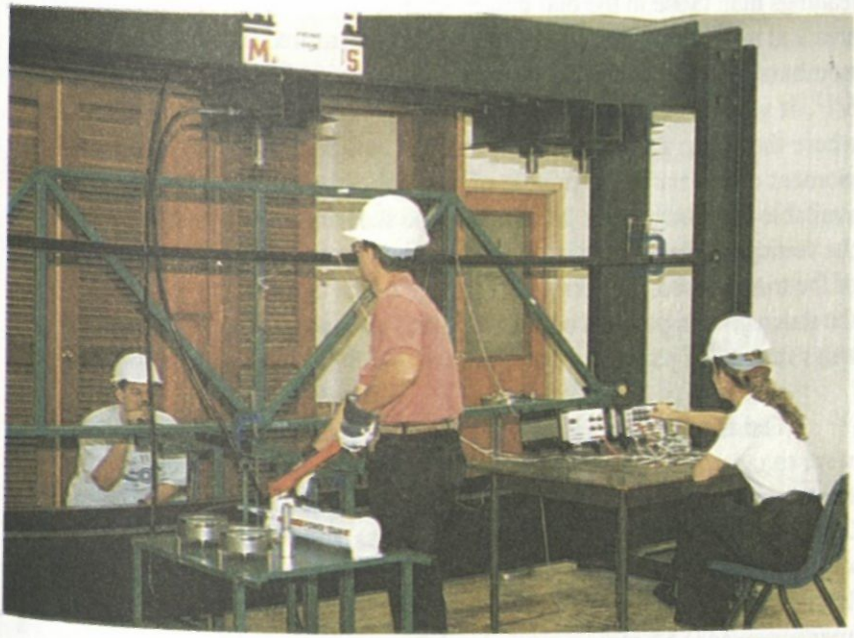

Figure 3. Student team applying load and taking readings.

The error percentages observed at strain gauge 3 in table 3 and figure 7 were so low because of the direct load applied to member 11 shown in figure 1. Strain gauge 4 was placed on member 6, high error percentages were observed at this member up to a load of 10 kN due to an initial adjustment of the truss members. After the load of 10 kN the error percentages decreased (table 3). Strains at member 6, obtained from strain gauge 2, were slightly higher than the deflections obtained at other members. Member 6 of the truss was subjected to bending stress at the face of the member where strain gauge 2 was placed.

Tables 2 and 3 show that the errors are higher in the strain gauge readings than those in the dial gauge readings. The strain are sensitive to the area and moment of inertia, and this was not measured directly from the truss members. The sectional properties were obtained from the manual HPM 6/1[3]. It was difficult to calculate the moment of inertia at the exact place where the strain gauges were placed, the equipment needed to calculate the moment of inertia at the middle of any hollow member of the truss was not available in the structures laboratory. The theoretical dial gauge readings for the vertical displacements were calculated using equation 1, in which the area of the truss is used. The error percentages of the dial gauges were lower than the strain gauges perhaps because the dial gauges use an average area of the truss instead of using the exact area in which the dial gauges were placed.

The theoretical dial gauge readings for the horizontal displacements were calculated by using equation 2; these readings were slightly higher than the experimental readings, producing an error of 25 % for loads higher than 20 kN. The existing friction on the truss supports held the truss from displacing freely for loads lower than 20 kN. The theoretical reading for the horizontal displacement at 40 kN was 0.75 mm and the experimental measure was 0.55 mm.

Possible sources of errors

The sectional properties of the members were obtained directly from the manual HPM. 6/1 provided by the supplier of the structures laboratory equipment. This affects the comparison between theoretical and experimental values. The moment of inertia at the strain gauges was approximated and must be obtained in order to adjust the error deviation up to ± 5%. It is difficult to calculate the moment of inertia at the middle of the hollow member because lack of adequate equipment in the structures laboratory.

The hydraulic loading system presented an error of ± 5% as shown in its reference manual HPM. 6/1.

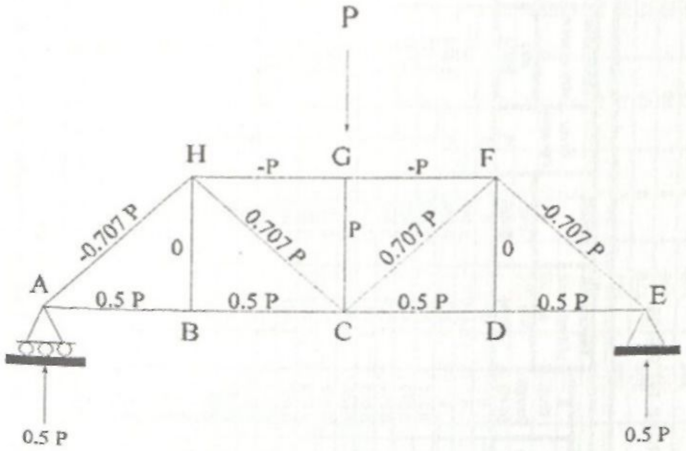

Figure 4. Structural analysis of the frame considering welded

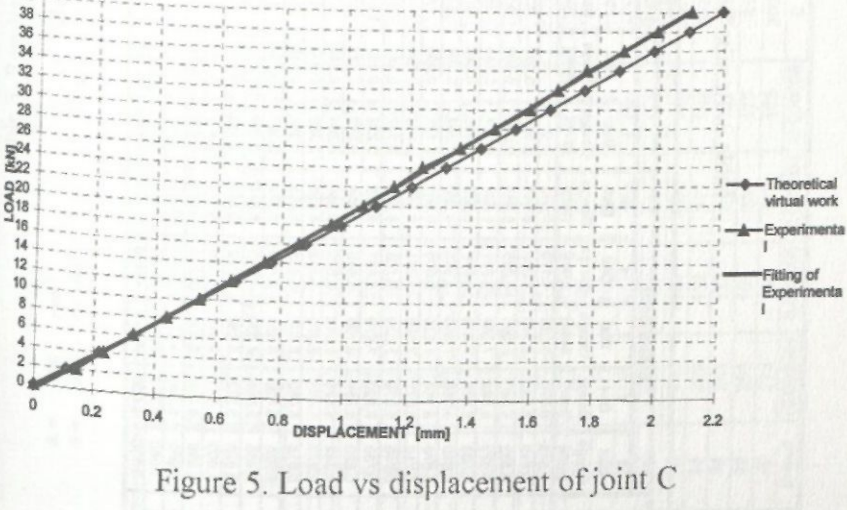

Figure 5. Load vs displacement of joint C

Table 2. Results from the gauge readings

Load	Reading															
	Experimental					Experimental					Theoretical virtual work Δ	Experi. Δ	Error Percent	Theoretical virtual work Δ	Experi. Δ	Error Percent
[kN]	1 Dial Gauge [mm]	2 Dial Gauge [mm]	3 Dial Gauge [mm]	4 Dial Gauge [mm]	5 Dial Gauge [mm]	1 Dial Gauge [mm]	2 Dial Gauge [mm]	3 Dial Gauge [mm]	4 Dial Gauge [mm]	5 Dial Gauge [mm]	(VC) [mm]	(VC) [mm]	%	(HA) [mm]	(HA) [mm]	%
0	0.548	6.070	6.520	4.176	8.541	0	0	0	0	0	0	0	0	0	0	
2	0.549	6.010	6.740	4.090	8.541	0.001	0.060	0.220	0.086	0	0.110	0.147	-	0.037	0.001	-
4	0.552	5.940	6.900	4.015	8.541	0.004	0.130	0.380	0.161	0	0.219	0.235	7	0.075	0.004	-
6	0.572	5.882	7.056	3.951	8.551	0.024	0.188	0.535	0.225	0.010	0.329	0.329	0	0.113	0.034	-
8	0.603	5.821	7.220	3.899	8.559	0.055	0.249	0.700	0.277	0.018	0.439	0.437	0	0.150	0.073	-
10	0.640	5.766	7.380	3.845	8.560	0.092	0.304	0.860	0.331	0.019	0.549	0.543	1	0.188	0.111	-
12	0.670	5.714	7.530	3.800	8.560	0.122	0.356	1.010	0.376	0.019	0.658	0.644	2	0.226	0.141	-
14	0.710	5.671	7.680	3.751	8.559	0.162	0.399	1.160	0.425	0.018	0.768	0.748	3	0.263	0.180	-
16	0.740	5.629	7.830	3.705	8.559	0.192	0.441	1.310	0.471	0.018	0.878	0.854	3	0.301	0.210	-
18	0.780	5.590	7.979	3.651	8.559	0.232	0.480	1.459	0.525	0.018	0.988	0.957	3	0.339	0.250	-
20	0.820	5.550	8.120	3.610	8.558	0.272	0.520	1.600	0.566	0.017	1.097	1.057	-4	0.376	0.289	23
22	0.842	5.525	8.243	3.575	8.558	0.294	0.545	1.723	0.601	0.017	1.207	1.150	5	0.414	0.311	25
24	0.870	5.495	8.370	3.532	8.558	0.322	0.575	1.850	0.644	0.017	1.317	1.241	6	0.452	0.339	25
26	0.910	5.460	8.530	3.488	8.558	0.362	0.610	2.010	0.688	0.017	1.427	1.361	5	0.489	0.379	22
28	0.935	5.426	8.680	3.439	8.558	0.387	0.644	2.160	0.737	0.017	1.536	1.470	4	0.527	0.404	23
30	0.955	5.395	8.830	3.396	8.559	0.407	0.675	2.310	0.780	0.018	1.646	1.583	4	0.565	0.425	25
32	0.978	5.372	8.950	3.352	8.559	0.430	0.698	2.430	0.824	0.018	1.756	1.669	5	0.603	0.448	26
34	1.000	5.342	9.072	3.320	8.558	0.452	0.728	2.552	0.856	0.017	1.866	1.760	6	0.640	0.469	27
36	1.020	5.310	9.230	3.272	8.558	0.472	0.760	2.710	0.904	0.017	1.975	1.878	5	0.678	0.489	28
38	1.050	5.290	9.372	3.221	8.558	0.502	0.780	2.852	0.955	0.017	2.085	1.985	5	0.716	0.519	28
40	1.080	5.270	9.510	3.180	8.558	0.532	0.800	2.990	0.996	0.017	2.195	2.092	5	0.753	0.549	27
Unload	0.530	6.050	6.540	4.125	8.491	-0.018	0.020	0.020	0.051	-0.050	0	0.015	0	0.000	0.068	0

Table 3. Results from the strain gauge readings

A = **466** **mm²** **I =** **135691.3** **mm⁴**
E = **205** **kN.mm²** **M =** **33.91** **kN.mm** **This bending moment was obtained from "FRAME" (1992), it was used for the calculations of the theoretical values of strain gauge 2.**

Load [kN]	Reading								1 Error Percent %	2 Error Percent %	3 Error Percent %	4 Error Percent %
	Experimental				Theoretical							
	1 Strain Gauge [µ]	2 Strain Gauge [µ]	3 Strain Gauge [µ]	4 Strain Gauge [µ]	1 Strain Gauge [µ]	2 Strain Gauge [µ]	3 Strain Gauge [µ]	4 Strain Gauge [µ]				
0	0	0	0	0	0	0	0	0	0	0	0	0
2	11	12	9	10	15	9	20.9	10.5	27	33	57	5
4	23	15	44	16	30	19	41.9	20.9	23	21	5	23
6	34	22	59	24	44	28	62.8	31.4	23	21	6	24
8	53	30	84	32	59	37	83.7	41.9	10	19	0	24
10	65	43	93	50	74	47	105	52.3	12	9	11	4
12	75	48	127	57	89	56	126	62.8	16	14	1	9
14	88	62	141	65	104	65	147	73.3	15	5	4	11
16	118	68	161	74	118	74	167	83.7	0	8	4	12
18	130	70	178	86	133	84	188	94.2	2	17	5	9
20	133	84	200	94	148	93	209	105	10	10	4	10
22	145	85	206	101	163	102	230	115	11	17	10	12
24	157	96	231	108	178	112	251	126	12	14	8	14
26	172	103	246	148	192	121	272	136	10	15	10	9
28	189	124	270	150	207	130	293	147	9	5	8	2
30	203	119	274	140	222	140	314	157	9	15	13	11
32	216	122	302	145	237	149	335	167	9	18	10	13
34	229	132	312	152	252	158	356	178	9	16	12	15
36	244	137	318	161	266	168	377	188	8	18	16	14
38	258	153	351	175	281	177	398	199	8	14	12	12
40	273	154	372	183	296	186	419	209	8	17	11	12
Unload	-0.000004	-0.000008	-0.000008	-0.000006	0	0	0	0	0	0	0	0

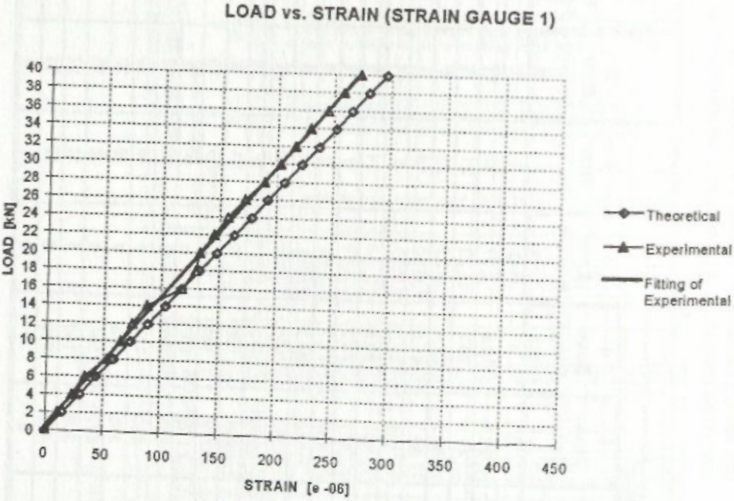

Figure 6. Load vs strain gauge #1

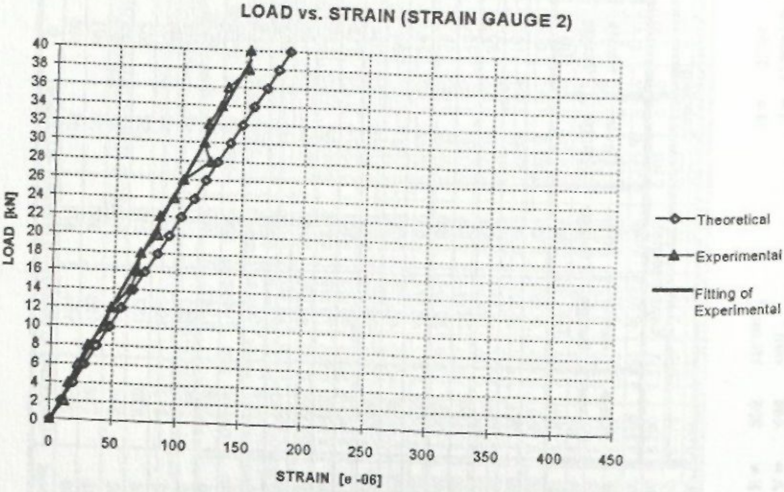

Figure 7. Load vs strain gauge #2

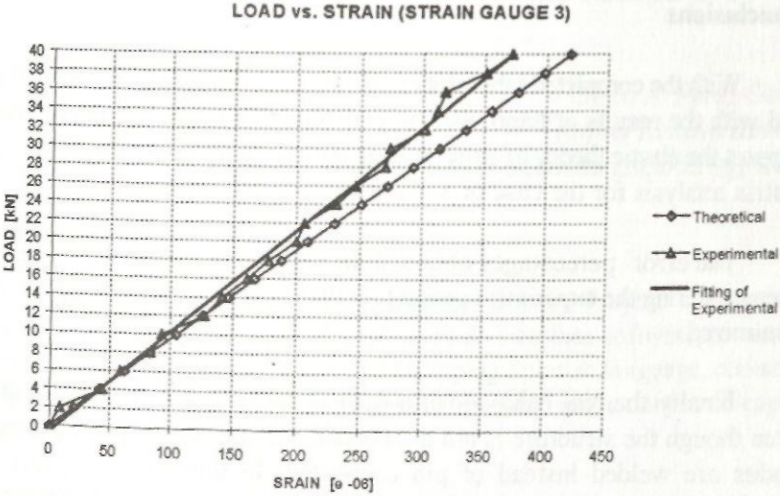

Figure 8. Load vs strain gauge #3

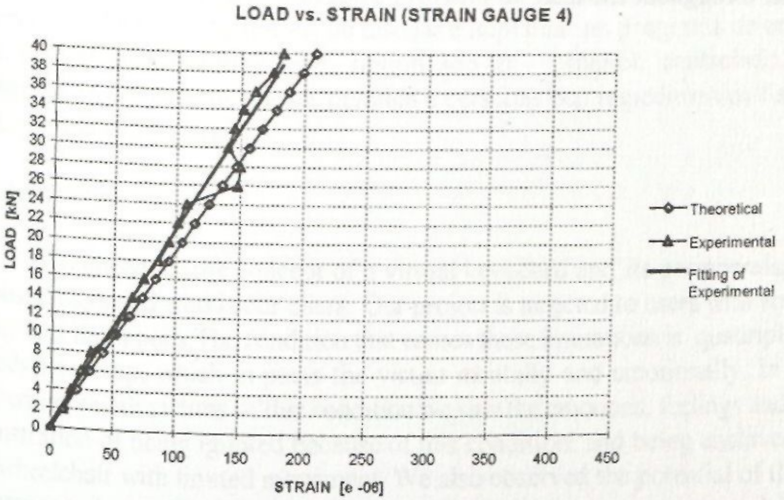

Figure 9. Load vs strain gauge #4

Conclusions

With the completed laboratory experience of analyzing the plane frame and with the results at hand, we can conclude that the experimental results support the elastic theory of structures analysis, such as Virtual Work, and matrix analysis for the case of a real truss.

The error percentages obtained throughout the experiment were low because during the experiment process all the possible sources of error were minimized.

Finally the real behavior of a natural scaled truss can be visualized, even though the structure is not a theoretical truss, but a frame because its nodes are welded instead of pin connected. In this case the previously mentioned effect disappeared due to the members length. This behavior was verified by comparing the theoretical behavior of a truss with the collected data throughout the test.