New technologies in mathematics

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"Those who scorn computer history are those who really don't grasp what is happening today and will never really shape tomorrow." (Don Congdon, 1996)

Abstract

Throughout the ages, the development of computational instruments runs parallel to the development of mathematics playing an important role in the dissemination of this subject matter and its applications. Nevertheless, from the abacus and the first calculating machine to the electronic computer and the graphing calculator, history shows that the introduction of new technologies tends to find an initial resistance in their use, although, in the end, all computational aids find their place in the process of transmission of knowledge and information.

After a brief description of some of the most relevant devices in the history of computation, this paper presents the use and applications of two modern mathematical tools: The TI-92 Graphing Calculator and the software package Derive. The study concludes with an analysis of the future impact of new technologies in mathematics and applied fields, mainly in engineering, based on personal experiences and the endorsements and critiques of national and international associations.

Sinopsis

Nuevas tecnologías en matemáticas

Históricamente la creación de instrumentos para la computación se desarrolla paralelamente al progreso de las matemáticas y juega un papel preponderante en la diseminación de esta materia y sus aplicaciones. Sin

embargo, desde el ábaco y la primera máquina calculadora hasta el ordenador electrónico y la calculadora gráfica, la historia demuestra que la introducción de nuevas tecnologías tiende a encontrar una resistencia inicial en su uso, aunque al final todas las ayudas computacionales hallan su lugar en el proceso de transmisión de conocimiento e información.

Después de una breve descripción de algunos de los instrumentos más relevantes en la historia de la computación, este artículo presenta el uso y la aplicación de dos herramientas modernas del cómputo matemático: la calculadora gráfica TI-92 y el programa Derive. El estudio concluye con un análisis del impacto futuro de las nuevas tecnologías en matemáticas y campos aplicados, principalmente el de la ingeniería, a base de experiencias personales y los endosos y críticas de asociaciones nacionales e internacionales.

Introduction

To understand the influence of new technologies in education, especially in the field of mathematics, we have to look back in the history of information. By knowing the past we can have a better understanding of the present, and perhaps predict the future. Some instruments and novel technologies had an immediate impact on education; this was the case of printing, the abacus, the logarithms or the slide rule. Others, such as the calculating machine, radio, television, the computer or the electronic calculator needed more time to get proper recognition as educational tools. As a matter of fact, the use of computers and graphing calculators in the classroom is a current, sometimes controversial, issue subject to daily debate. The resistance to use these tools in education can be attributed to two factors: one of technological and logistic character, which includes difficulties with their use, cost, accessibility and implementation; the other one related to the human nature of potential users and their fear of the unknown, fear to be displaced by technology, lack of knowledge and unwillingness to learn. In the end, though, history shows that new technologies always find their place in education; in fact, the radical changes in this process always coincide with the introduction of new instruments. Table 1 shows, in chronological order of appearance, some of the most relevant computational aids and mechanisms (other than oral) used by humankind to transmit information throughout the ages.

Table 1. Most revelant computational aids and mechanisms to transmit information

Year	Description
600 A.D.	Book printing begins in China.
1400	First calculating machine: the abacus.
1614	Logarithms and Napier Bones. John Napier.
1642	Pascal's calculating machine. The Pascaline.
1650	The slide rule.
1823-1843	Babbage's first calculating machines. Ada's first "computer program."
1896	Marconi invents the radiotelegraph.
1925	First television transmission.
1944	Aiken builds the first automatic computer. The Mark I.
1946	ENIAC (Electronic Integrator and Computer)
1950	Cable TV
1951	The IBM 701
1952	The UNIVAC (Universal Automatic Computer).
1956	Texas Instruments patents the integrated circuit.
1971	First use of the semiconductor chip. The pocket calculator.
1976	Apple II introduces the first microcomputer.
1981	The IBM PC.
1990's	The Information Age. Networking and the Internet.

The development of mathematics is directly or indirectly associated with these selected milestones and we can analyze it from different perspectives. Focusing on physical devices, we can look at two major groups: the mechanical and the electric. Instruments from both groups played an important role in mathematics. The abacus, the slide rule and the calculating machines (ancient and modern) for instance, are considered milestones in this history of calculating. Next I include a brief, but informative, description of some of these instruments.

The abacus.

The name abacus derives from the Greek word ABAX, meaning table or board covered with dust. Its origin is unknown. What we know is that in its 'modern' form it appeared in China in the 13th century AD. The Chinese abacus has 13 columns with two beads on top (heaven) and five beads bellow (hearth). Around the 17th century AD the Japanese copied the Chinese abacus and adapted it to their way of thinking. Their version has 21 columns with one bead on top and four beads below. The abacus is still part of the regular school training in the Far East and is commonly used in many places. In 1946 a contest between a Japanese abacist (Kiyoshu Matzukai) and an electronic computer lasted two days resulting in an unmistakable victory of the abacist. A third modern form of the abacus is Russian with 10 arched rows.

Logarithms. John Napier, 1550-1617.

Napier played a key role in the history of computing. Besides being a clergyman and philosopher he was a gifted mathematician. In 1614 he published his work of logarithms in a book called "Rabdologia." His remarkable invention enabled the transformation of multiplications and divisions into simple additions and subtractions. His logarithmic tables soon became widespread and universally used. Another invention of his, nicknamed "Napier Bones," was a small instrument constructed of 10 rods with the multiplication table engraved on it. This device enabled a faster multiplication, provided one of the factors was one digit only.

The slide rule.

The first slide rule appeared in 1650 and was the result of a joint effort of two Englishmen: Edmund Gunter and William Oughtred. This rule, based on Napier's logarithms, was to become the first analog computer (of modern ages) since it figured out multiplication and subtraction by physical distance. This invention was dormant until 1850 when a French artillery officer, Amadee Mannheim, added the movable double sided cursor, which gave it today's appearance.

The calculating machine.

Despite all marvelous mechanical computational devices mentioned before, the glory goes to the Frenchman Blaise Pascal. In 1642, at the age of 21, Pascal invented the calculating machine, also known as the Pascaline. Blaise Pascal, who was a gifted lad from childhood, wanted to spend more time with his father, a tax collector. Since Dad was always busy at home adding columns of numbers, Blaise decided to invent a machine that would free his father from this tedious task. This machine, measuring 20" x 4" x 3", was of metal with eight dials manipulated by a stylus. Today there are 50 surviving Pascalines, manufactured by Blaise Pascal most of them, in the large science museums. He built these calculators for sale, but clerks and accountants refused to use them for fear it would do away with their jobs. As far as we know, there were two prior attempts to create a calculating machine. The first one was by Leonardo da Vinci. His notes, found in the National Museum of Spain, include a description of a machine bearing a certain resemblance to Pascal's machine. The second one was by Wilhelm Schickard, who invented a mechanical calculator in 1623. He built two prototypes, but their location is unknown. In 1820 the Frenchman Thomas de Colmar invented the 'Arithmometer' using a 'stepped drum' mechanism. This was the first calculating machine produced in large numbers. It was active until the late 1920s. The next generation of calculating machines came from the Swedish inventor Willgodt T. Odhner. His machine incorporated a 'pin wheel' mechanism. Based on mechanical principles, hundreds of manufacturers produced an amazing variety of calculating machines up to the late 1960s. Then they surrendered to the appearance of the electric calculator and the computer.

The computer

The first generation machines began with the ENIAÇ. Its designers were J. W. Mauchly and J. P. Eckert, Jr. from the University of Pennsylvania. They completed the project in three years, from 1942 to 1945. This machine weighed 30 tons, contained 18,000 vacuum tubes and had dimensions 100' x 10' x 3'. On average, a tube failed every 15 minutes. Programming required six thousand switches to be set, and it took 200 microseconds to add and 3 milliseconds to multiply by three. In 1951 the US Census Bureau received the first UNIVAC, and with it the Computer Age began. IBM started building and marketing the IBM 701, followed by the IBM 650. The second generation of computers (1958-1964) includes the IBMs 7090, 7070 and 1410. These machines used transistors instead of vacuum tubes, reducing the size and improving the performance. The third generation (1964-1970) includes the first true minicomputers. The use of integrated circuits, improved the reliability, size, cost and power. One of the most popular was the IBM 360. The fourth generation (1970-) marks the beginning of machines based on microprocessors. Early microcomputers include Altair, EMCII, Tandy TRS-80, Atari and Commodore. The industry, however, changed forever when S. Jobs and S. Wozniak built the first Apple computer. They did it in a garage, but by 1983, the company they founded, Apple, made the fortune 500 list. IBM announced its PC in 1981. Today, microcomputers like the IBM PC and the Apple Macintosh, are evolving with workstations into yet another generation of computer systems. The major innovation consists on sharing massive amounts of equipment and information resources throughout networks. This generation is bringing major changes in the way we work, teach and learn.

The electronic calculator

The first electronic calculators, such as the IBM Selective Sequence EC (48K) with three levels of memory, electronic, relay and paper tape, were constructed in the 1940s. In 1964 a Japanese firm introduced the first hand-transistor desktop calculator; it weighed 55 pounds and cost $2,500. However, the program that resulted in the development of the hand-held calculator was written in 1965. The authors were three Texas Instruments engineers, J. S. Kilby, who had invented the integrated circuit in the late 1950s, J. D. Merryman and J. H. van Tassel. They envisioned a calculator, based on integrated circuits and battery powered, that could add, subtract, multiply and divide and fit in the palm of the hand. After two years of development Kilby, Merryman and van Tassel completed their calculator at the Dallas headquarters of Texas Instruments. It could handle up to six-digit numbers, perform the four basic operations and print results on a thermal printer. The calculator had an aluminum case, measured 4.25" x 6.125" x 1.75" and weighed 45 ounces. In 1975 the prototype device became part of the permanent collection of the Smithsonian Museum. Since then electronic calculators have evolved continuously at a dramatic pace. The latest generations include programable, scientific and graphing calculators.

In contrast with earlier technologies, pocket calculators did not have a very warm reception in mathematics education. It took about 10 years, since their arrival to the markets in the early 1970s, to partially recognize them as teaching tools in the mathematics classrooms. Different reasons, some academic, some administrative, and others perhaps more personal as it happened with the Pascaline, contributed to this delay. They finally became common teaching tools in the early 1980s when their cost dropped considerably and the academic institutions agreed to revise the curriculum to allow their use. Nevertheless, the controversy about the use of calculators in the classroom did not stop there and it continues today as new generations of machines become available. With the introduction of more sophisticated and powerful graphing calculators, the old issue about their efficiency in the education process has resuscitated. While some institutions ban or restrict their use, others strongly recommend or require them. A similar situation occurs with individual instructors when the institution is indifferent to this matter. In all instances, though, there is an increasing tendency toward the use of new technologies at all levels, mainly in applied fields. According to the Curriculum and Evaluation Standards for School Mathematics of the National Council of Teachers of Mathematics (NCTM), calculators should be available always to all students. Lynn Arthur Steen, executive director of the Mathematical Sciences Education Board, says that the MSEB and the NCTM share a general vision for technology in math education. Although Steen discounts the importance of using computers at the Kindergarten to 12th grade levels, he acknowledges the role technology has had in shaping the new math curriculum when he says: "The curriculum is a mix of what we teach and how we teach it. Some topics used to be very important to teach. Now, because of computers and calculators, other topics are suddenly important." Others, like Judah Schwartz, professor of mathematics and physics at Harvard and MIT, passionately disagree. "That is a narrow view" he says, "I think computers are necessary tools for all math curriculum starting at age zero." James Landherr, mathematics coordinator for the East Hartford (Conn.) Public Schools, strongly believes that students need access to computers. Without them, he says, "they are deprived. Those who use programs like Geometric Supposer are more empowered to become better problem solvers. Students need to be exposed to technology because they need to be exposed to change to become flexible problem solvers with reasoning skills. In this day and age, you are handicapping those who don't get to use technology." Some like Jim Kaput, professor of mathematics at the University of Massachusetts, go even farther when they claim that "you cannot really achieve what the Standards suggest without technology." (The standards include number sense, algebra, data analysis, geometry, measurement and computation.) Important advances made in mathematics in the last 20 years, which include fractals, tessellations, and graph theory, are also accessible from kindergarten to high school.

The truth is that the revolution in technology is affecting routine mathematical and scientific work, just as the industrial revolution affected manual labor. Graphing calculators, like the family of TI, HP, SHARP or CASIO, are already having a significant impact in the teaching of mathematics at secondary and college levels. They bring changes, not only in the way we teach, but also in the subject content. Furthermore, they have certain advantages over microcomputers, like portability, low cost and user friendliness. The symbolic capabilities of the calculators, however, are restricted because of their small memory and slow processing speed. Consequently, sometimes, it is convenient to supplement them with microcomputer software packages such as DERIVE, MAPLE, MATLAB or MATHEMATICA. Nonetheless, graphing calculators are very powerful computational and instructional tools. With these calculators the students can do multiple symbolic manipulations, numerical and graphical analysis, avoiding the tedious pencil-and-paper process. This, however, does not mean that we have to delete those topics from the curriculum. It would be a mistake to think that with new technologies there is no need to teach basic computations, theory of equations, curve sketching, differentiation or integration. If we look back, the adoption of the slide rule, for instance, did not replace the teaching of arithmetical, algebraic or trigonometric operations. On the contrary, instructors had to teach more (and consequently students had to learn more) although with different emphasis. A similar situation occurs with the use of graphing calculators. Their incorporation to the teaching process implies the concurrent development of two interdependent tasks, learning the mathematical theories, and the functions of the calculator. How to combine these tasks is a challenge instructors and students face and, to a certain extent, it is the core of the controversy about the use of graphing calculators. Personally, I favor the use of new technologies, including graphing calculators, in the classroom. However, based on my experience with calculators and computers over the past 10 years, I must say that:

1. No machine can replace the instructor to carry out the most difficult part of teaching mathematics, which is getting the students to understand the concepts. A conceptual approach is crucial to understand the subject matter and its applications. For example, something as basic and important as estimations requires proper knowledge and understanding of the concepts involved in the procedures. Graphing calculators and computers, however, can help students to understand certain concepts like oscillatory or harmonic motions, through the graphical analysis of motion pictures (figs. 1 and 2)

2. The students need to develop the necessary critical-thinking skills to analyze problems, develop solving strategies, interpret and judge the reasonableness of the results. At this point, computers and calculators must remain one step behind the academic process. Their main role, along with the graphical analysis, will be, as it was in Pascal's mind, to speed processes and not to replace fundamental knowledge. This role is by itself crucial because it replaces not only the tedious and time consuming computational process, but it also gives students and instructors more time to explore new concepts.

How can we keep up with the astonishing progress of human knowledge without shortening the learning process? At least, in applied fields of mathematics, computers and calculators must be part of the answer.

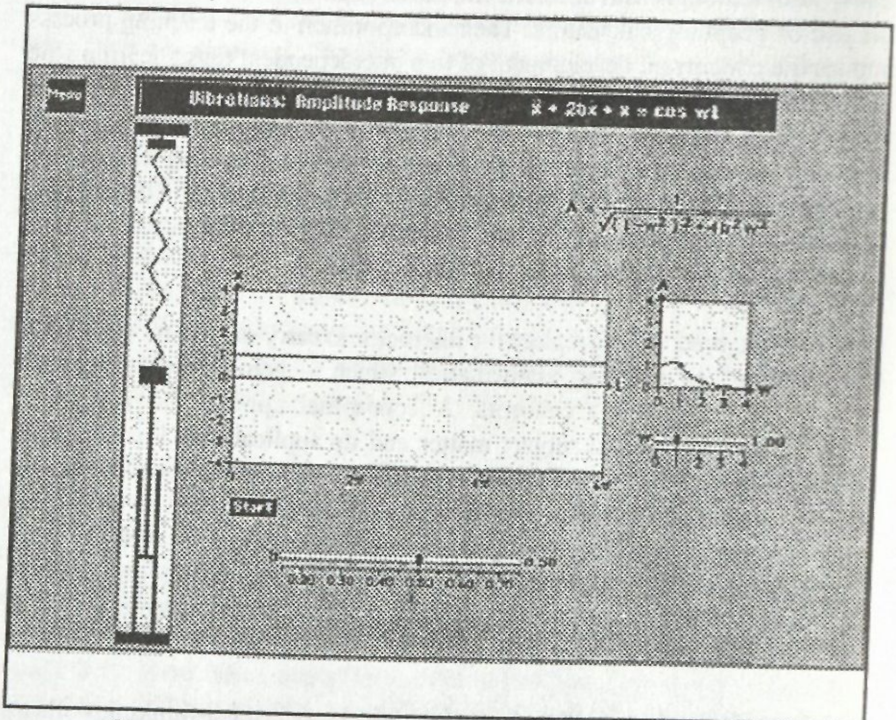

Figure 1. Vibrations: An example of amplitude response (IDE Addison-Wesley Interactive.)

3. A distinction has to be made when we teach applied mathematics to fields like engineering, and when we teach pure mathematics. While in applied fields computers and calculators could be used without practically any restriction, in pure mathematics their role would be, in general, less relevant.

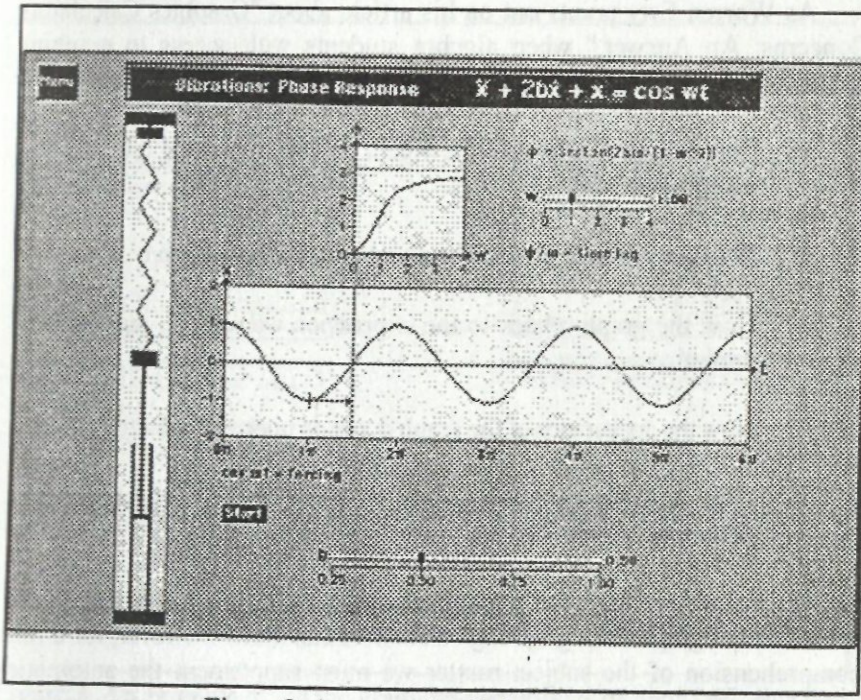

Figure 2. An example of phase response
(IDE Addison-Wesley Interactive.)

4. The integration of new technologies to the teaching and testing process requires appropriate modification of all educational materials, including examinations. With the availability of computers and graphing calculators, the emphasis on computations, equation solving or function graphing will no longer be same. The students will have to demonstrate the necessary skills to solve basic problems. Then they can take advantage of the new technologies to solve more and more complex cases. They can also use them as instrumental aids to compare, interpret and analyze the results.

As Warren Esty points out on his article about "Graphics Calculators Concerns: An Answer," when algebra students with access to graphing calculators plot a graph, they should learn how to read it. This includes:

1. How the graph relates to the algebraic and numerical relationships it expresses

2. How the graph relates to the equation we want to solve

3. How the graph relates to the expression we want to maximize or minimize

4. How the appearance of the graph depends upon the selected window

5. How to recognize and interpret common types of expressions and equations

As we will see next, the students can get most of this learning with the usual sorts of graphs that graphing calculators can draw. However, for a full comprehension of the subject matter we must supplement the automatic graphing and solving processes with some manual work. This may include simple problems with hand-drawn graphs that do not have an associated equation, and various questions about them, without particular numbers. The testing process should also combine both skills.

The following examples show some capabilities of the TI-92 graphing calculator used here for illustrative purposes. This versatile calculator has a QWERTY keyboard and employs a friendly graphic user interface (GUI). A set of function keys provides access to pull down menus, similar to a PC. With optional GRAPH LINK and cable software, we can transfer data and programs between the calculator and the computer, store information on disk or print it. Figures 3 and 4 show some of these graphing capabilities of the TI-92.

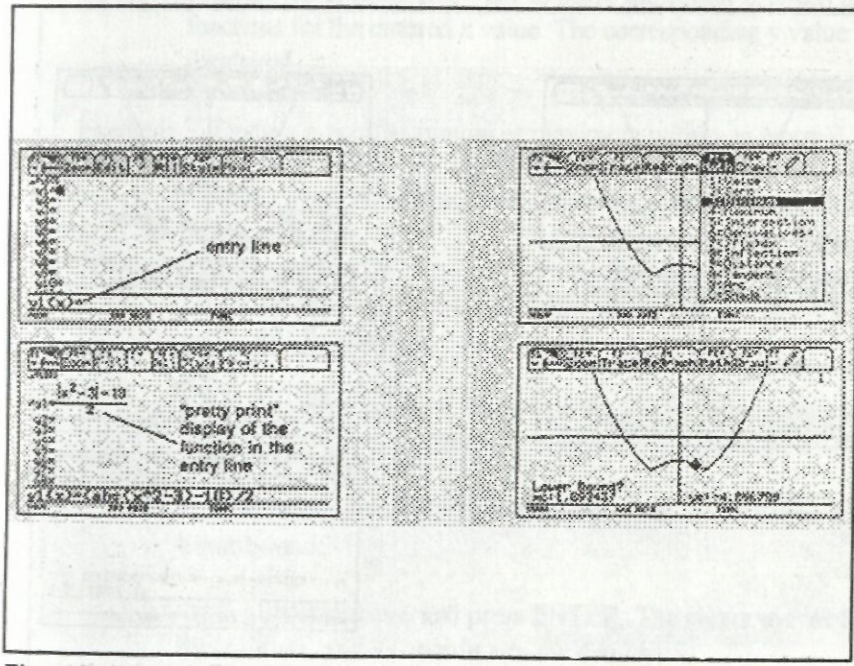

Figure 3. An example of the the graphing capabilities of the TI-92: function graphing.

<u>Example 1</u>. Graphing a function.

1. Display the Y=Editor.

2. Enter the function or functions.

3. Select 6:ZoomStd (or any other appropriate window) and press ENTER.

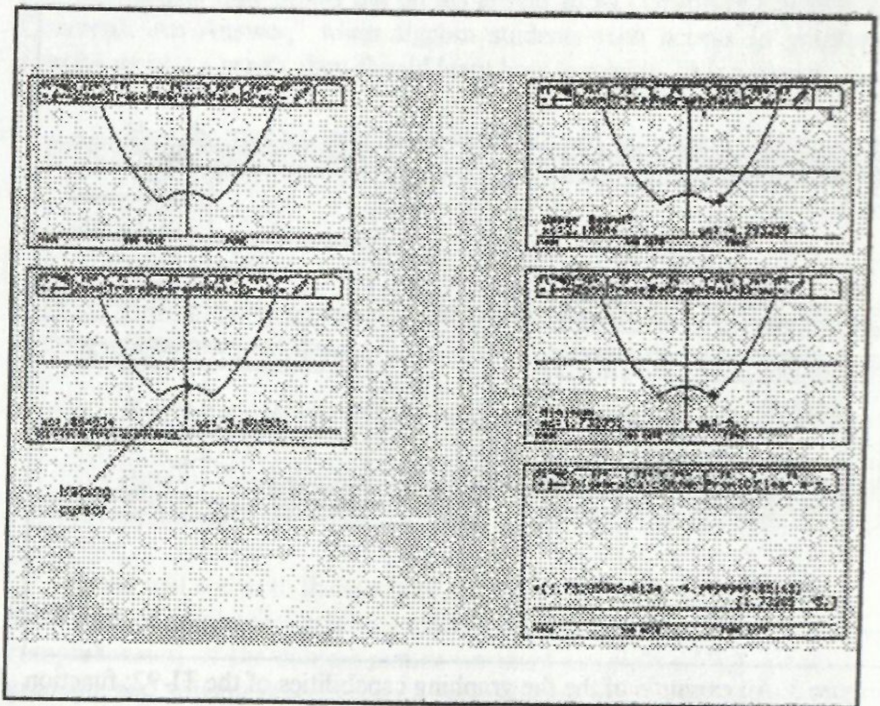

Figure 4. Another example of the the graphing capabilities of the TI-92: relative extrema.

<u>Example 2</u>. Finding y(x) at a specified point.

1. From the Graph screen, press F5 and select 1:Value.

2. Type the x value, which must be a real number or expression, between xmin and xmax.

3. Press ENTER. The cursor moves to that x value on the first function selected in the Y=Editor, and its coordinates are displayed.

4. Press the up or down arrow key to move the cursor between the functions for the entered x value. The corresponding y value is displayed.

Example 3. Finding a zero, minimum or maximum within an interval.

1. From the graph screen press F5 and select 2: zero, 3:minimum, or 4:maximum.

2. As necessary, use the up or down arrow key to select the applicable function.

3. Set the lower bound for x. Either use the left or right arrow key to move the cursor to the lower bound or type its x value.

4. Press ENTER. An arrow head at the top of the screen marks the lower bound.

5. Set the upper bound, and press ENTER. The cursor moves to the solution, and its coordinates are displayed.

Example 4. Finding the Intersection of Two Functions within an Interval.

1. From the graph screen, press F5 and select 5:intersection.

2. Select the first function, using the up or down arrow key as necessary, and press ENTER. The cursor moves to the next graphed function.

3. Select the second function and press ENTER.

4. Set the lower bound for x. Either use the left or right arrow key to move the cursor to the lower bound or type its value.

5. Press ENTER. An arrow head at the top of the screen marks the

lower bound.

6. Set the upper bound and press ENTER. The cursor moves to the intersection, and its coordinates are displayed.

Similarly we can find the derivative of a function at a point, definite integrals, inflection points, the distance between two points, the equations of a tangent line, the length of an arc, or use Math Tools to analyze functions. Through the INTERNET, a supplementary library of functions and subroutines for the TI-92 is also available. The current edition of this library is down-loadable at http:/www.derive.com. It consists of an ASCII text version of the document README.TXT, and three TI-GRAPH LINK(tm) group files, the UNIT.92G, the ELEM.92G, and the ADV.92G. UNIT.92G implements units algebra and automatic conversions. ELEM.92G includes pre-calculus mathematics capabilities such as simultaneous nonlinear equations, regression, contour plots and plots of implicit functions. ADV.92G contains more advanced capabilities such as:

1. ImpDifN(equation, independent Var, dependent Var, n), which returns the nth derivative of the implicit function defined by "equation."

2. LapTran(expression, timeVar). If the global variable s has no stored value, returns a symbolic expression for the Laplace transform of the expression in terms of s. If s has a particular numerical value, returns the Laplace transform for that particular value.

3. FourierCf(expression, var, lowerLimit, upperLimit, n). Returns the truncated Fourier series of "expression" for "var" from "lowerLimit" to "upperLimit," through the nth harmonic. With the split-screen feature we can get a simultaneous graphical representation.

4. Ode1IV(expressionForDeriv, indepVar, depVar, initValIndepVar, initValDepVar). Returns a symbolic solution of the 1st-order differential equation. Otherwise returns false.

Second-order differential equations, surface integrals, centroids, moments of inertia, characteristic polynomials, Eigenvalues and Eigenvectors, gradients and divergencies, curl, gamma function, Chevychev polynomials, normal probabilities, mean and standard deviations, are among other topics included in the library.

Along with the TI-92, a software package that I frequently use is DERIVE. This is a powerful system, yet affordable and easy to use for simplifying mathematical expressions, solving equations and graphing functions. It runs on PCs based on the Intel 8086 or compatible microprocessors. DERIVE's Utility Files contain multiple instructional commands that we can use as pedagogical tools in courses like calculus, differential equations and linear algebra. They offer students and instructors an opportunity to reform the curricula by applying the subject matter to complex real data. We can also employ them to analyze algebraic and graphical solutions of equations that model physical phenomena. The next four examples will give us a concise overview of DERIVE's capability. Figures 5 and 6 show graphing capabilities of the DERIVE software package.

Example 5. Plotting the Direction Field of a first order differential equation $y'=f(x,y)$.

1. Author the following command:

   DIRECTION-FIELD $(f(x,y),x,x_0,x_m,y,y_0,y_n,n)$,

   where (x_0,x_m) and (y_0, y_n) are intervals on the x and y axes, and m and n are the corresponding step sizes.

2. Press ENTER, type approXimate, and press ENTER again. The result is a matrix of 2-component vectors which create the direction field through their graphical representation.

3. Type Plot twice from window 1 for a plot of the direction field.

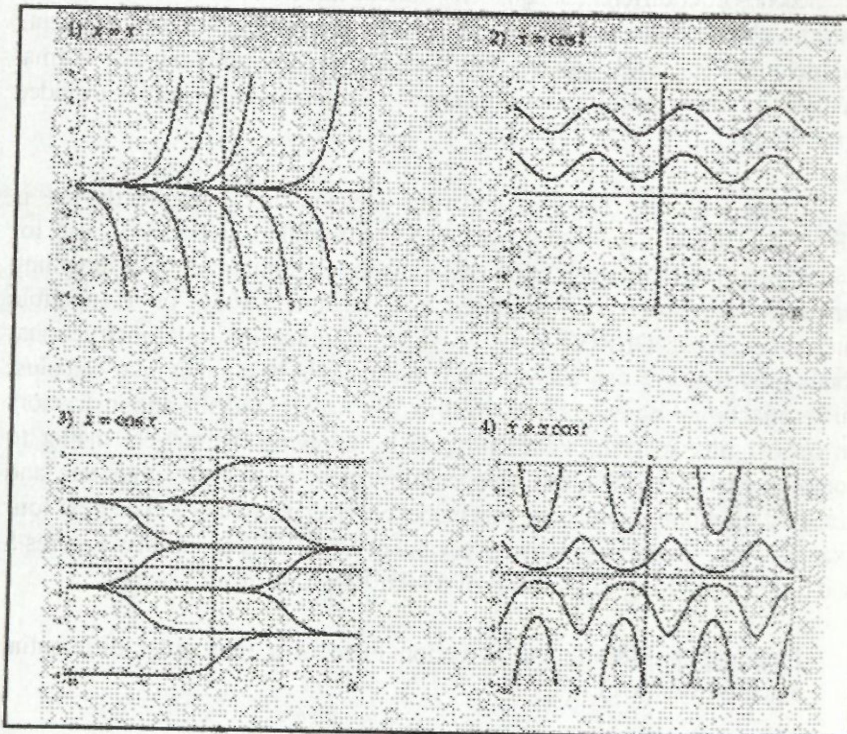

Figure 5. Examples of direction, or slope, fields

<u>Example 6</u>. Solving an Exact equation of the form p(x,y)+q(x,y)y'=0.

1. Author the command EXACT_TEST(p,q,x,y) and Simplify. A result of 0 guarantees that the equation is exact.

2. Author the command EXACT_TEST(p,q,x,y,x_0,y_0)) and Simplify.

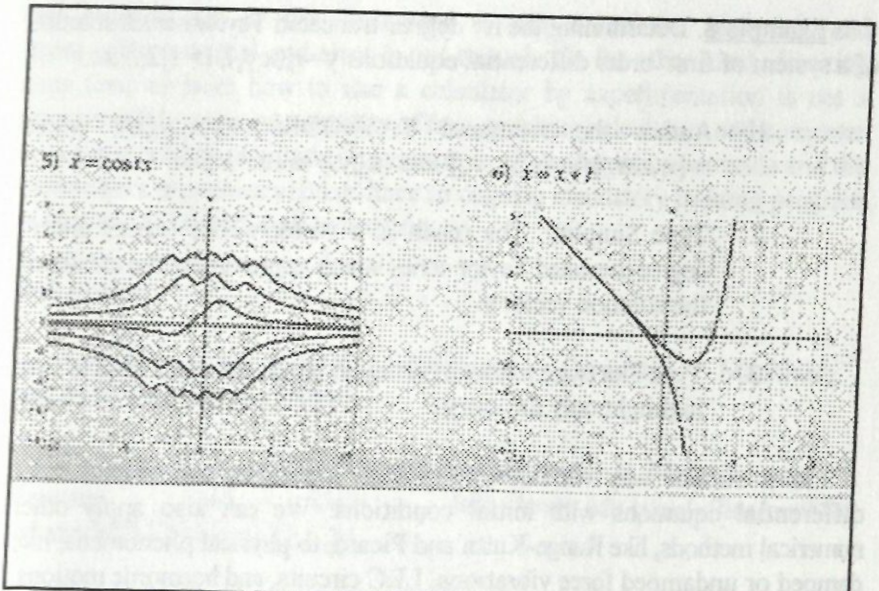

Figure 6. More examples of direction, or slope, fields.

Example 7. Using Euler's Numerical Method to obtain an approximation to the solution of the Initial Value problem $y'=r(x,y)$, $y(x_0)=y_0$.

1. Author the command EULER(r,x,y,x_0,y_0,h,n), where h is the step size and n the number of steps.
2. Press ENTER, type approXimate, and press ENTER again. The result is a vector of n-coordinate pairs which are an approximation for the points in the solution curve.
3. Type Plot twice from window 1 for a plot of the approximate solution.

<u>Example 8</u>. Determining the n^{th} degree truncated Taylor series solution of a system of first order differential equations $y'=r_k(x,y_k)$, $k=1,2,...,m$.

1. Author the command TAY_ODES(r,x,y,x_0,y_0,n), where $r=[r_1,...,r_m]$, $y=[y_1,...,y_m]$ and (x_0,y_0)=initial value.

2. Type Simplify. The result is a vector whose entries are n^{th} degree truncated Taylor series which approximate the entries of the solution vector y.

3. Type Plot twice to plot a particular entry, but make sure to first highlight only the entry.

In a similar way we can solve linear, separable, and second order differential equations with initial conditions. We can also apply other numerical methods, like Runge-Kutta and Picard, to physical phenomena, like damped or undamped force vibrations, LRC circuits, and harmonic motions.

As can be observed, graphing calculators, like the TI-92, and software packages, like DERIVE, are not just calculating machines. Appropriately used, they can be very valuable conceptual aids and effective tools in both the teaching-learning and the testing process. Professional mathematics organizations, such as the National Council of Teachers of Mathematics (NCTM), the Mathematical Association of America (MAA), and the Mathematical Education Board of the National Academy of Sciences (MSEB-NAS), have strongly endorsed their use in mathematics instruction and have emphasized that, consequently, they must be used in the testing process. So far all indicators point toward a complete integration of new technologies to the math curriculum. New computer systems, interactive technology, and graphing calculators, will mark the beginning of the 21st century in mathematics. To make the transition viable, math instructors should know what these machines can do and what topics will benefit the students most. They would also have to learn how to test the students and not the devices. Thus, all current mathematics teachers should get formal and, most importantly, frequent training in the use of computers systems and calculators. Rejecting the use of new technologies in the classroom is not a realistic approach, and improvisation or trial-and-error is not enough. On the other hand, diverting class time to learn how to use a calculator by experimentation is not a recommended practice. As a matter of fact, graphing calculators and computer systems should be required components in all mathematics curricula and the mathematics departments should have an ongoing mandatory training program in new technologies for all instructors.

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