New Tools for the Design and Analysis of Complex Springs

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ABSTRACT

The molded case circuit breaker industry, in general, produces great families of breakers, i.e., the bolt-on and the plug-in units. The bolt-on unit uses a screw to connect the unit to the bus bar of the panelboard. In the case of the plug-in breaker, the unit has a much higher chance to heat up due to the current flowing through the clip to the panelboard bus bars. This is due to several factors:

a- The spring action of the clip to the panelboard bus bars is reduced due to the heat produced by the electrical current. In this case, the connection loosens, producing more heat.
b- Poor handling of the unit during the process of plugging it up, affecting the shape of the clip.
c- Dirt, scale and rust in the panelboard bus bars or in the clip.
d- The shape of the clip is not the best for its application.

The design of the breaker clip is very complex due to its shape and application.

SINOPSIS

En la industria se producen varias familias de disyuntores, por ejemplo las unidades del tipo "bolt-on" y del tipo "plug-in". Las unidades del tipo "bolt-on" usan un tornillo para conectarse a la barra del panel, mientras que las unidades de tipo "plug-in" utilizan un sujetador. En el caso del tipo "plug-in", hay una mayor probabilidad de calentamiento debido a la corriente que fluye a través del sujetador hacia la barra del panel. Este calentamiento se debe a diferentes factores:

a- La acción del resorte del sujetador en la barra del panel se reduce debido al calentamiento producido por la corriente eléctrica. En este caso la conexión se hace más débil, lo que a su vez produce más calor.
b- Manejo inapropiado de la unidad durante el proceso de instalación, afectando la forma del sujetador.
c- Suciedad, sedimento y herrumbre en la barra del panel o en el sujetador.
d- La forma del sujetador no es la mejor para la aplicación.

El diseño del sujetador del disyuntor es muy complejo debido a su forma y aplicación.

I- INTRODUCTION

There have been several discussions about the effectiveness of some line clips for plug-in breakers. Of special interest for us has been the live part line clip for the C and the A breakers. Also the old C breaker line clip presents some weakness which has inclined us to think for a substitute.

Both C line clips depend upon a spring to maintain the required contact pressure. In the type A breaker the line clip contact pressure depends upon its own spring action. To bring some light into this problem it was thought advisable to study the theory behind the complex shaped spring.

The following work pretends to analyze the deflection and stresses of the C live part line clip spring and the A live part line clip [1].

II- ANALYSIS OF THE C LIVE PART LINE CLIP SPRING

To ease the analysis of this spring we broke it into three parts:

A - CIRCULAR WAVE SPRING

Figure 1 shows a circular wave spring. The moment along the curve up to C is:

$$M_C = P\, r \sin\theta \tag{1}$$

The moment from A to F along the arc is:

$$M_f = P * (\text{arm from A to D}) = P\,(r + r - r\cos\theta)$$

$$M_f = P\, r\,(2 - \cos\theta) \tag{2}$$

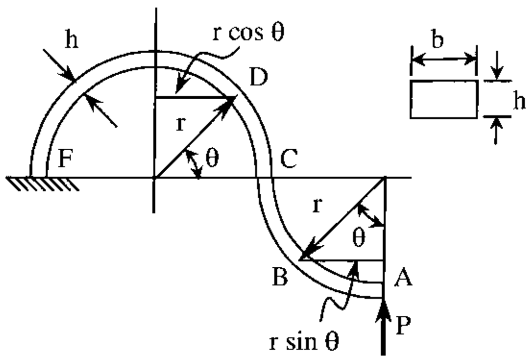


Figure 1: *Circular wave spring*

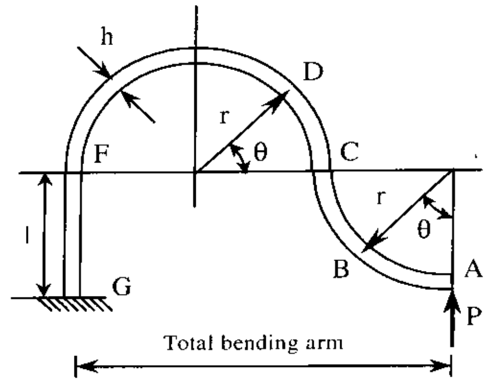


Figure 2: *Second part of the C clip spring*

By Castigliano principle [2], we have that (refer to the Appendix section):

$$Y = \frac{1}{EI}\int_0^{3\frac{\pi}{2}} M\left[\frac{d}{dP}M\right] r\,d\theta \qquad (3)$$

from which the deflection of A as a consequence of the moments at C and up to F may be computed with:

$$Y_C = \frac{1}{EI}\int_0^{\frac{\pi}{2}} P\,r\sin(\theta)\; r\,\sin(\theta)\;\; r\,d\theta \qquad (4)$$

$$Y_F = \frac{1}{EI}\int_0^{\pi} P\,r\,(2-\cos(\theta))\,r\,(2-\cos(\theta))\,r\,d\theta \qquad (5)$$

The summation of both integrals yields the total deflection at A:

$$Y_T = \frac{P\,r^3}{EI}\left[\frac{\pi}{4}+\left[4\pi+\frac{\pi}{2}\right]\right]$$

$$Y_T = \frac{19\;P\,\pi\,r^3}{4\,EI} \qquad (6)$$

B- Second Part of the Clip String: Straight Extension

The second part of the spring is shown in Figure 2.

The straight portion from F to G acts as a cantilever beam fixed at G. The maximum bending stress is not affected but the deflection includes now another element: the deflection of the straight portion.

By the method of superposition the total displacement will be:

$$M_F = 3\,P\,r, \qquad (7)$$

which is the moment at F from P.

$$\frac{d}{dP}M_F = 3\,r$$

$$Y_F = \frac{1}{EI}\int_0^{\textcircled{3}} M\left[\frac{d}{dP}M\right] d\textcircled{3}$$

$$Y_F = \frac{1}{EI}\left[3\,P\,r\,(3\,r)\int_0^{\textcircled{3}} d\textcircled{3}\right]$$

$$Y_F = \frac{9\,P\,r^2\,\textcircled{3}}{EI} \qquad (8)$$

The total deflection is Y_F plus Y_T:

$$Y_{TT} = \frac{1}{EI}\left[\frac{19\pi\,P\,r^3}{4} + 9\,P\,r^2\,\textcircled{3}\right]$$

$$Y_{TT} = \frac{P\,r^3}{EI}\left[\frac{19\pi}{4} + \frac{9\textcircled{3}}{r}\right] \qquad (9)$$

which is the deflection for half of the spring.

C - The Complete Clip Spring:

Figure 3 shows the complete spring.

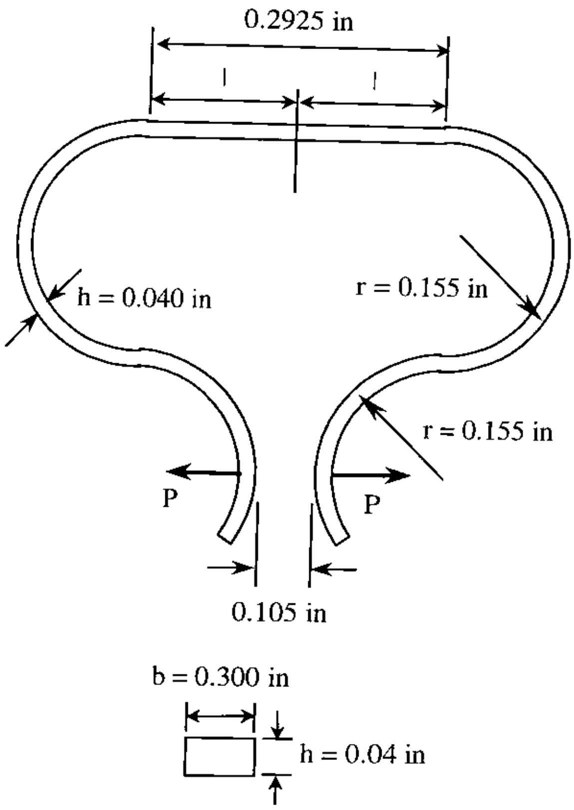


Figure 3: *The complete C clip spring*

III- ANALYSIS OF THE TYPE C CLIP SPRING

A- DEFLECTION AS A FUNCTION OF THE FORCE

r = 0.155 in. h = 0.04 in.

$③ = \frac{0.2925}{2}$ in ③ = 0.146 in

b = 0.3 in.

E = 30000000 psi

P = 1, 2, ..., 18 lbs.

$$f(P) = \frac{P r^3}{E \frac{b h^3}{12}} \left[\left[19 \frac{\pi}{4} \right] + \left[9 \frac{③}{r} \right] \right] \quad (10)$$

deflection (see Figure 4).

B- SPRING CONSTANT AS A FUNCTION OF THE WIDTH

$$K_1 = \frac{r^3}{E \frac{b h^3}{12}} \left[\left[19 \frac{\pi}{4} \right] + \left[9 \frac{③}{r} \right] \right] \quad (11)$$

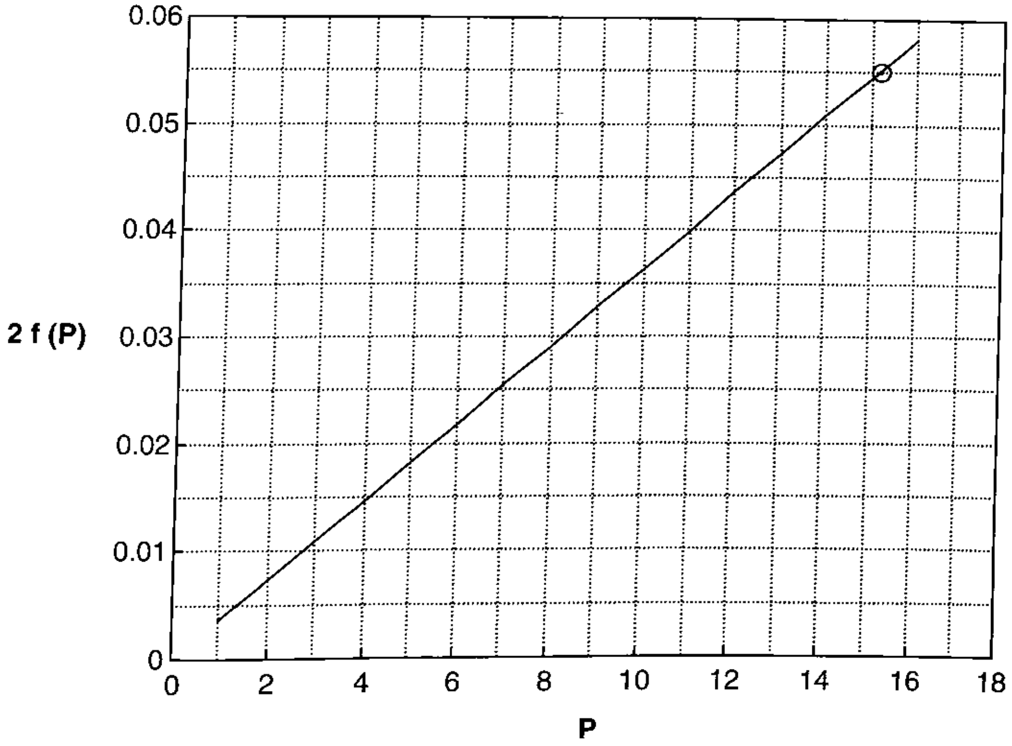


Figure 4: *Deflection as a function of the force.*

$K_1 = 0.0018165$

$b_1 = 0.2, 0.21, ..., 0.31$

$$f(b_1) = \frac{r^3}{E\frac{b_1 h^3}{12}}\left[\left[19\frac{\pi}{4}\right] + \left[9\frac{\textcircled{3}}{r}\right]\right] \quad (12)$$

(see Figure 5)

C- Spring Constant as a Function of the Thickness

$h_1 = 0.030, 0.031, ..., 0.042$

$$f(h_1) = \frac{r^3}{E\frac{b h_1^3}{12}}\left[\left[19\frac{\pi}{4}\right] + \left[9\frac{\textcircled{3}}{r}\right]\right] \quad (13)$$

(see Figure 6)

D- Spring Force as a Function of the Thickness for Two Different Deflections:

1- *Live part deflection which is 0.052 inches.*

Let Q be:

$$Q = 19\frac{\pi}{4} + 9\frac{\textcircled{3}}{r} \quad (14)$$

and h:

$h = 0.030, 0.031, ..., 0.040$

For the live part breaker the deflection ylp = 0.052. Then:

$$f(h) = \frac{(ylp)\, E\frac{b h^3}{12}}{r^3 Q}(0.5) \quad (15)$$

(see Figure 7)

2- *Old C deflection which is 0.090 inches.*

For the old C breaker the spring has to stretch 0.090 inches to fit a 0.095 in. We have to remember that the new clip for the old C is 0.050 in thickness and the spring comes with the free ends separated by 0.105 in.

If the total deflection (y_{oldc}) is given by

$y_{oldc} = 0.090,$

Then the force in pounds as a function of the thickness of the spring is:

$$f(h) = \frac{(y_{oldc})\, E\frac{b h^3}{12}}{r^3 Q}(0.5) \quad (16)$$

(see Figure 8)

IV- ANALYSIS OF THE TYPE A CLIP

A- Deflection in Inches as a Function of the Force in Pounds

$$K = \frac{\textcircled{3}}{r} \quad (17)$$

$r = 0.080,$ $\textcircled{3} = 0.485,$

$K = 6.063,$

$h = 0.050,$ $\beta = 94.0349\frac{\pi}{180}$

$b = 0.144,$ $\alpha = 4.0349\frac{\pi}{180},$

$y = 0.02575,$ $E_{175} = 19000000$

$E_{194} = 17500000,$ $E_{197} = 17700000$

$E_{162} = 17000000,$ $E_{170} = 18500000$

$$G = \frac{K^3}{3} + K^2\beta + 2K(1-\cos(\beta)) + \frac{\beta}{2} - \frac{\sin(2\beta)}{4} \quad (18)$$

$G = 148.429$

$$P_{194} = y\, E_{194}\frac{b\frac{h^3}{12}}{2 G r^3 (\cos(\alpha))^2} \quad (19)$$

$P_{194} = 4.469$ (Force developed by the E_{194} material clip when deflected 0.02575 inches. See derivation of the above formulas in the Appendix section.)

$$I = b\frac{h^3}{12} \quad (20)$$

$I = 1.5\ 10^{-6},$ $P = 1, 2, ..., 10$

$$f(P) = \frac{2 P r^3 G}{E_{194}\ I} \quad (21)$$

(see Figure 9)

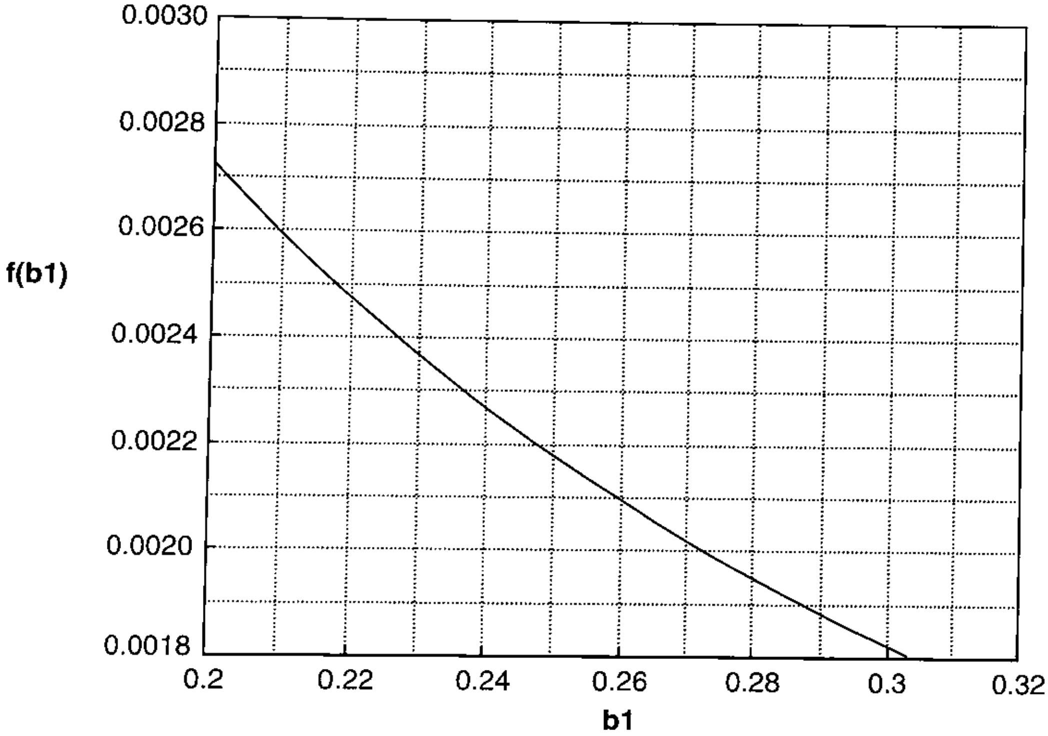


Figure 5: *Spring constant as a function of the width.*

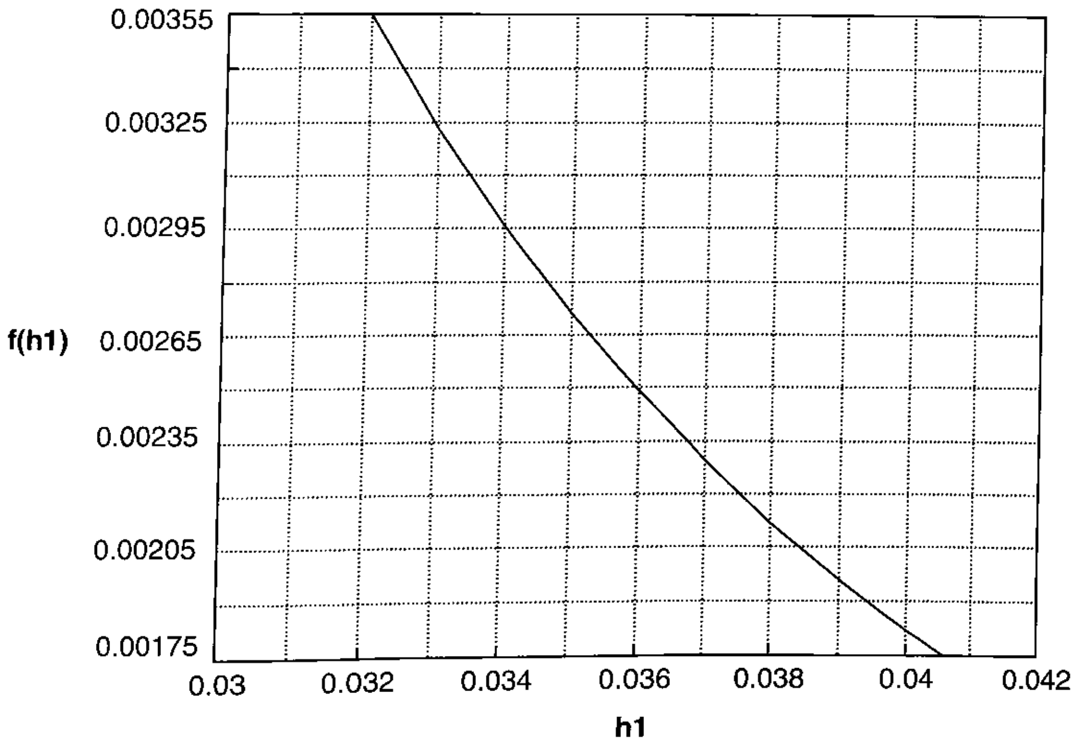


Figure 6: *Spring constant as a function of the thickness.*

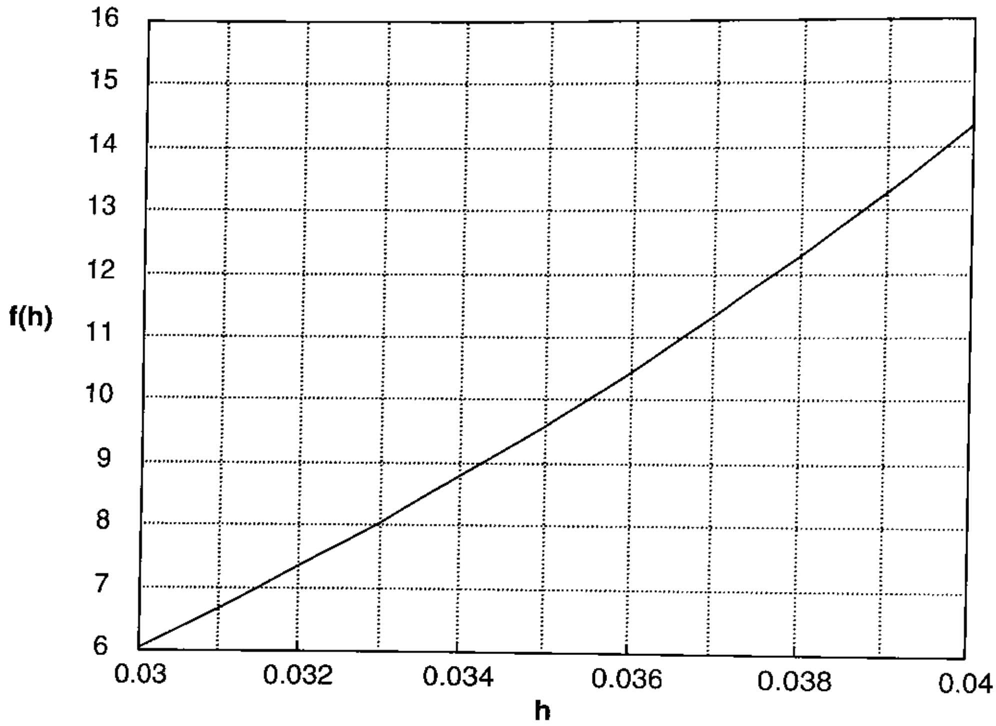
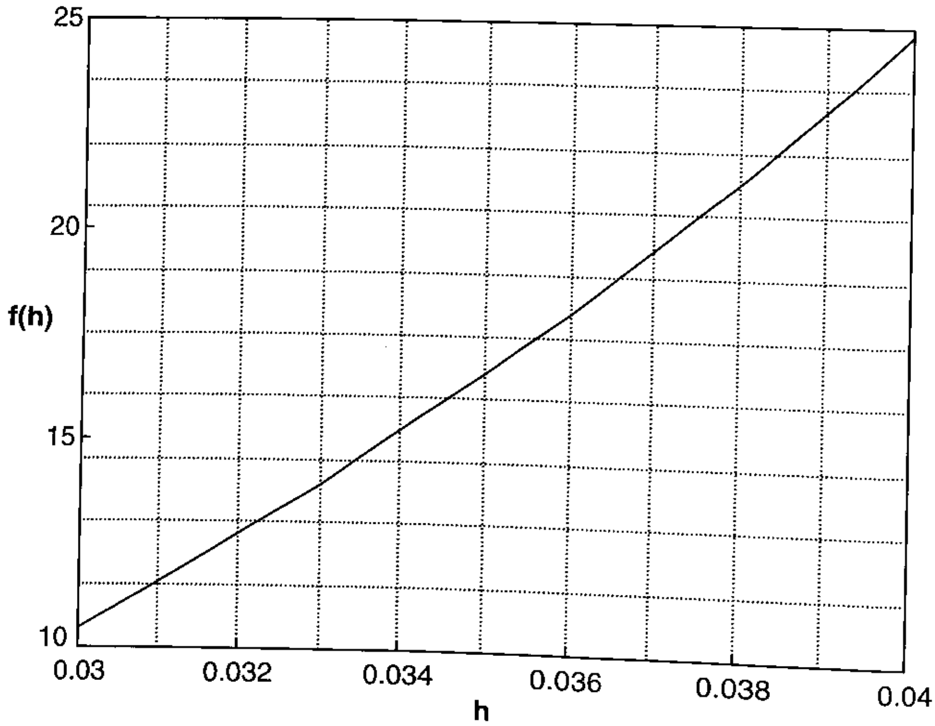


Figure 7: *Force in pounds as a function of the thickness of the spring. The deflection was assumed to be 0.052.*



Figrue 8: *Force in pounds as a function of the thickness of the spring. A deflection of 0.090 in corresponding to the old C breaker was used for this analysis.*

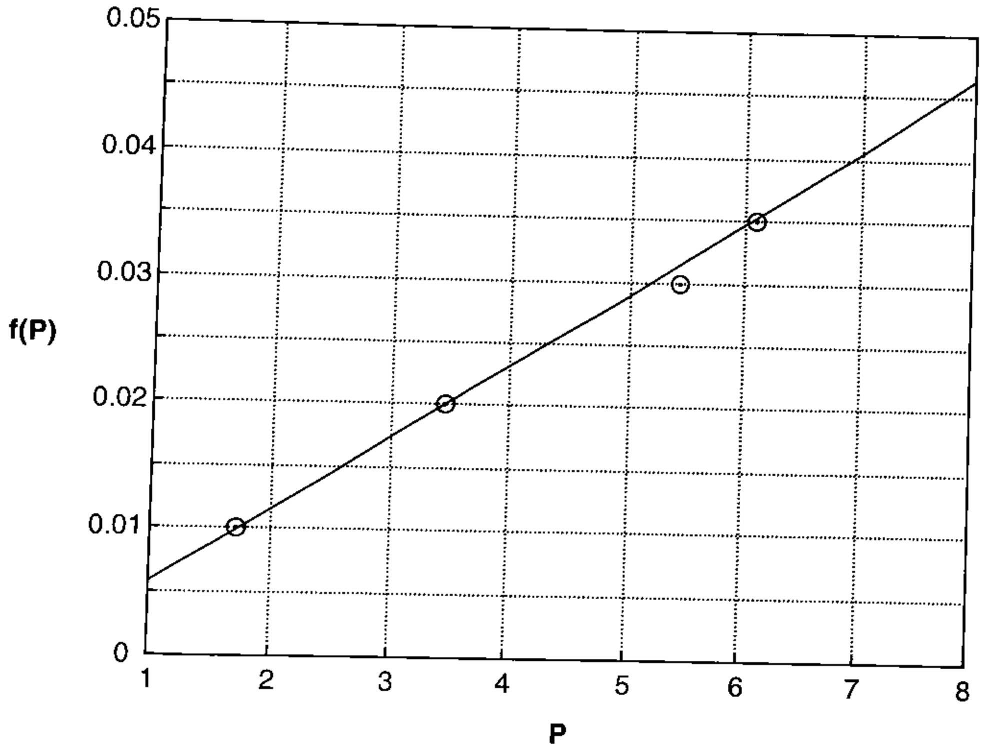


Figure 9: *Deflection in inches as a function of the force in pounds.*

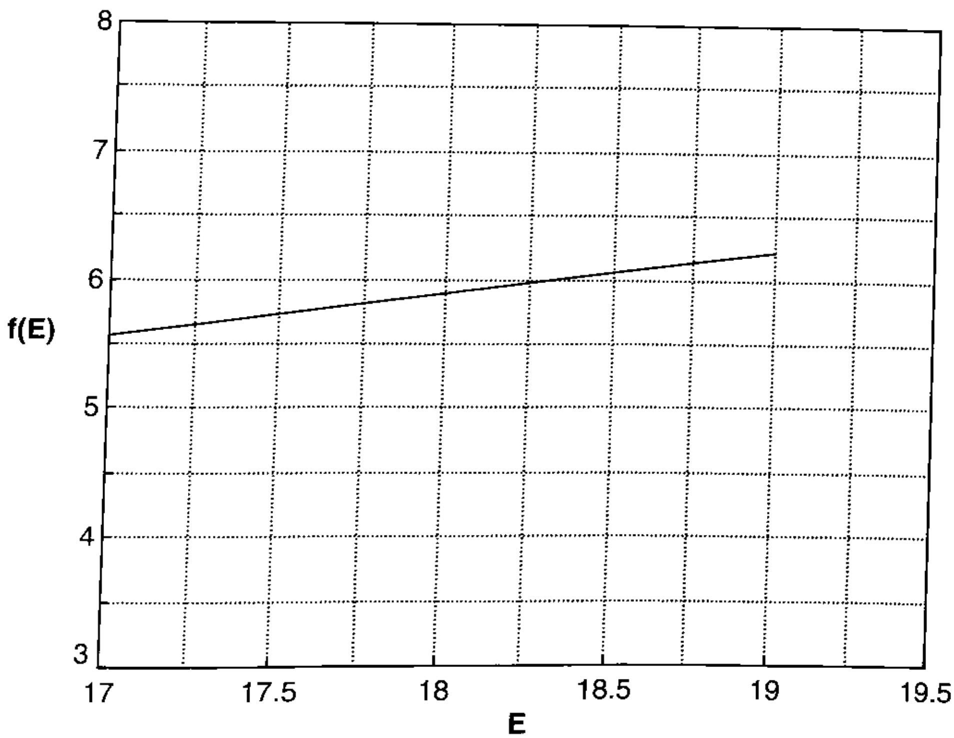


Figure 10: *Force in pounds as a function of the modulus of elasticity.*

B- Deflection in Pounds as a Function of the Modulus of Elasticity

Lets determine for a particular deflection how the force varies as a function of the modulus of elasticity of the various materials proposed. If the deflection is 0.033, which is the displacement needed by a 0.062 inch clip to fit a 0.095 inch thickness stab, then E = Modulus of elasticity divided by a million may be expressed by:

E = 17, 17.5, ..., 19

$$f(E) = \frac{0.033\, I\, E}{2\, G\, r^3 \left(\cos(\alpha)\right)^2}\, 10^6 \qquad (22)$$

(see Figure 10)

C- Deflection in Pounds as a Function of the Thickness

Lets determine how the force varies as a function of the thickness of the material. For a displacement of 0.033 inches and a modulus of elasticity of 17500000 psi, we have:

h = 0.040, 0.042, ..., 0.060

$$f(h) = \frac{0.033\, E_{194}\, \dfrac{b\, h^3}{12}}{2\, G\, r^3 \left(\cos(\alpha)\right)^2} \qquad (23)$$

(see Figure 11)

V- CONCLUSION

The theoretical formulas derived give very good results compared to true experimental data. In both "Deflection vs. Force" graphs for the A and the C types breakers (see Figures 4 and 9), the experimental data was plotted with a circle around.

The formulas presented may be used to anticipate the effect of varying any of the parameters of the two springs. There is no doubt that the results obtained with these formulas are much more accurate than those to be obtained using the approximate cantilever formula together with the concept of developed length.

VI- REFERENCES

1- Alexander Blake, "Deflection and Stresses of Complex Shaped Springs".

2- Arthur P. Boresi, Richard J. Schmidt and Omar M. Sidebottom, "Advanced Mechanics of Materials". John Wiley and Sons, Inc. 1993, pages 169-179, 385-391.

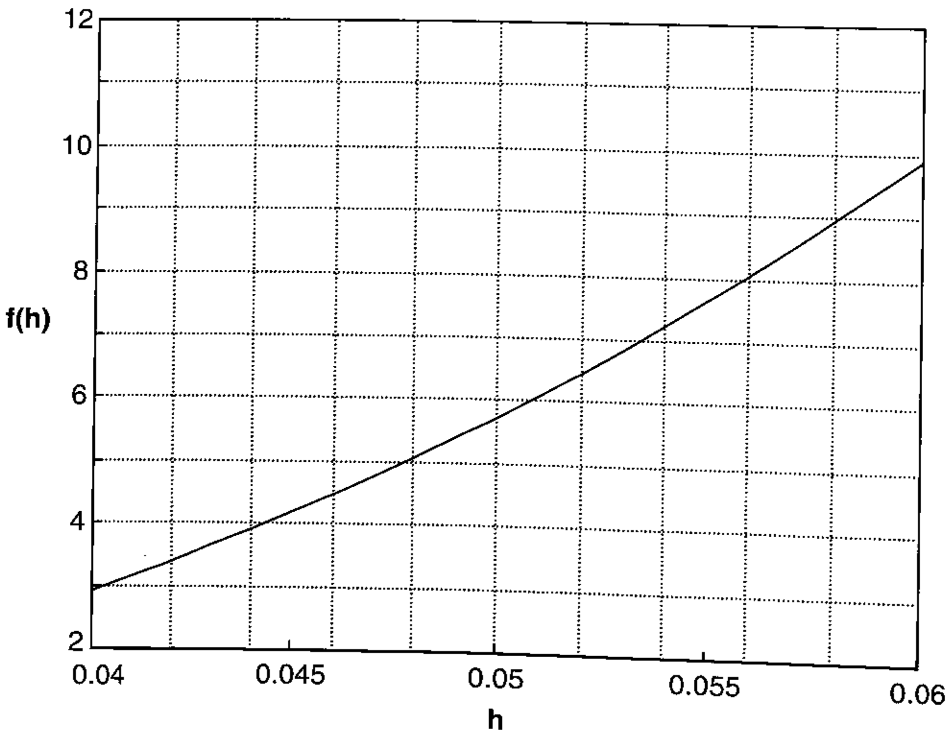


Figure 11: *Force in pounds as a function of the thickness of the clip.*

VI- APPENDIX: FORMULA DERIVATIONS

A- C LIVE PART LINE CLIP SPRING

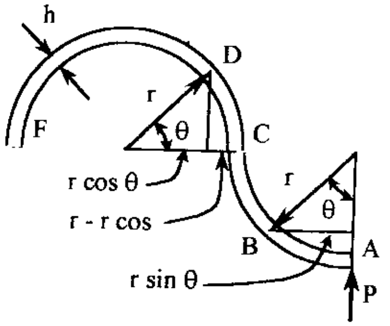


Figure 12: *C live part line clip*

Moment of P along the curve up to C: (see Figure 12)

$$M_{AC} = P(r \sin(\theta)) \quad (24)$$

Moment of P along the curve up to F:

$$M_{CF} = M_{AC} + M_{CF} \quad (25)$$

$$M_{CF} = P\,r + P\,(r - r\cos(\theta))$$

$$M_{CF} = P\,(r + r - r\cos(\theta))$$

$$M_{CF} = P\,(2r - r\cos(\theta))$$

$$M_{CF} = P\,r\,(2 - \cos(\theta)) \quad (26)$$

Castigliano Principle:

$$\text{Deflection } Y = \frac{1}{EI}\int M\left(\frac{\partial M}{\partial P}\right) r\,d\theta \quad (27)$$

Where:

E = Modulus of Elasticity,

$$I = \text{Moment of Inertia} = \frac{b\,h^3}{12}, \quad (28)$$

M = Moment of Force P,

$\frac{\partial M}{\partial P}$ = Partial derivative of the moment with respect to P, and

$$M_{AC} = P(r \sin(\theta))$$

$$\therefore \frac{\partial M}{\partial P} = r\sin(\theta) \quad (29)$$

Therefore, the deflection

$$Y_{AC} = \frac{1}{EI}\int M_{AC}\left(\frac{\partial M_{AC}}{\partial P}\right) r\,d\theta \quad (30)$$

$$Y_{AC} = \frac{1}{EI}\int P\,r\sin\theta\,(r\sin\theta)\,r\,d\theta$$

$$Y_{AC} = \frac{P\,r^3}{EI}\int_0^{\frac{\pi}{2}} \sin^2\theta\,d\theta \quad (31)$$

$$M_{CF} = P\,r\,(2 - \cos(\theta)) \quad (32)$$

$$\frac{\partial M_{CF}}{\partial P} = r\,(2 - \cos(\theta))$$

$$\therefore Y_{CF} = \frac{1}{EI}\int P\,r\,(2-\cos\theta)(r)(2-\cos\theta)\,r\,d\theta \quad (33)$$

$$Y_{CF} = \frac{P\,r^3}{EI}\int (2-\cos\theta)^2\,d\theta$$

$$Y_{CF} = \frac{P\,r^3}{EI}\int_0^{\pi}\left(4 - 4\cos\theta + \cos^2\theta\right)d\theta \quad (34)$$

But $Y_{AF} = Y_{AC} + Y_{CF}$ (35)

$$Y_{AF} = \frac{P\,r^3}{EI}\int_0^{\frac{\pi}{2}} \sin^2\theta\,d\theta + \frac{P\,r^3}{EI}\int_0^{\pi}\left(4 - 4\cos\theta + \cos^2\theta\right)d\theta$$

$$Y_{AF} = \frac{P\,r^3}{EI}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\frac{\pi}{2}} + \frac{P\,r^3}{EI}\left[4\theta - 4\sin\theta + \frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_0^{\pi}$$

$$Y_{AF} = \frac{P\,r^3}{E\,I}\left[\frac{\pi}{4} + \left[4\pi + \frac{\pi}{2}\right]\right] = \frac{P\,r^3}{E\,I}\left[\frac{19\pi}{4}\right]$$

$$Y_{AF} = \frac{19 P\,r^3 \pi}{4\,E\,I} \qquad (36)$$

B- Second part of the Clip Spring: Straight Extension

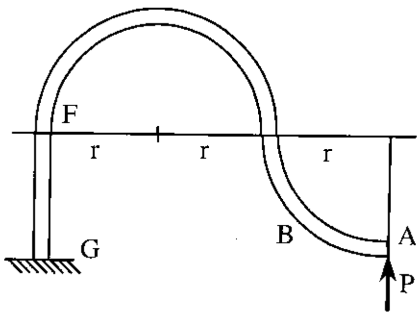


Figure 13: *Straight extension of the clip spring*

Moment from A to G due to P: (see Figure 13)

$$M_G = P(3\,r) \qquad (37)$$

$$\frac{\partial M_G}{\partial P} = 3\,r$$

$$Y_{FG} = \int_0^{③} \frac{P\,(3\,r)}{E\,I}(3\,r)\,d③ \qquad (38)$$

$$Y_{FG} = \frac{9\,P\,r^2}{E\,I}\int_0^{③} d③$$

$$Y_{FG} = \frac{9\,P\,r^2}{E\,I}③ \qquad (39)$$

∴ The total deflection for half of the spring is

$$Y_{AG} = Y_{AF} + Y_{FG} \qquad (40)$$

$$Y_{AG} = \frac{19\,P\,r^3\pi}{4\,E\,I} + \frac{9\,P\,r^2\,③}{E\,I}$$

$$Y_{AG} = \frac{P\,r^3}{E\,I}\left[\frac{19\pi}{4} + \frac{9\,③}{r}\right], \qquad (41)$$

which is the formula used in the text.

C- Type A Breaker Clip

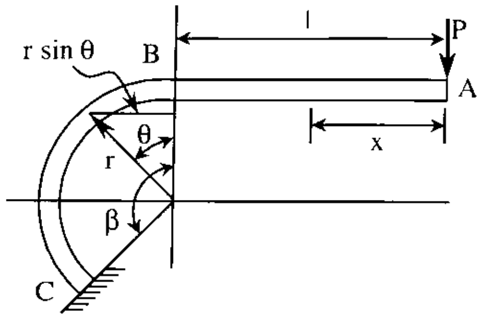


Figure 14: *Type A breaker clip*

Moment from A to B (see Figure 14):

$$M_B = P\,x \qquad (42)$$

$$\frac{\partial M_B}{\partial P} = x$$

$$\therefore Y_{AB} = \frac{1}{E\,I}\int_0^{③}(P\,x)(x)\,dx \qquad (43)$$

$$Y_{AB} = \frac{P}{E\,I}\left[\frac{x^3}{3}\right]_0^{③}$$

$$Y_{AB} = \frac{P\,③^3}{3\,E\,I} \qquad (44)$$

Moment from A to C:

$$M_C = P\,(l + r\sin\theta) \qquad (45)$$

$$\text{If } K = \frac{③}{r}\text{, then } M_C = P\,r\,(K + \sin\theta) \qquad (46)$$

$$\frac{\partial M_C}{\partial P} = r\,(K + \sin\theta)$$

$$Y_{BC} = \frac{1}{E\,I}\int P\,r(K + \sin\theta)(r)(K + \sin\theta)\,r\,d\theta$$

$$Y_{BC} = \frac{1}{E\,I}P\,r^3\int (K + \sin\theta)^2\,d\theta$$

$$Y_{BC} = \frac{P\,r^3}{E\,I}\int_0^{\beta}\left(K^2 + 2\,K\sin\theta + \sin^2\theta\right)d\theta$$

$$Y_{BC} = \frac{P\,r^3}{E\,I}\left[K^2\theta - 2\,K\cos\theta + \frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\beta}$$

$$Y_{BC} = \frac{P r^3}{E I}\left[K^2\beta - (2K\cos\beta - 2K) + \frac{\beta}{2} - \frac{\sin 2\beta}{4}\right]$$

$$Y_{BC} = \frac{P r^3}{E I}\left[K^2\beta + 2K(1 - \cos\beta) + \frac{\beta}{2} - \frac{\sin 2\beta}{4}\right] \tag{47}$$

The total deflection is $Y_{AC} = Y_{AB} + Y_{BC}$ (48)

$$Y_{AC} = \frac{P \ell^3}{3 E I} + \frac{P r^3}{E I}\left[K^2\beta + 2K(1 - \cos\beta) + \frac{\beta}{2} - \frac{\sin 2\beta}{4}\right]$$

$$Y_{AC} = \frac{\Pr^3}{EI}\left[\frac{\ell^3}{3r^3} + K^2\beta + 2K(1 - \cos\beta) + \frac{\beta}{2} - \frac{\sin 2\beta}{4}\right]$$

$$Y_{AC} = \frac{P r^3}{E I}\left[\frac{K^3}{3} + K^2\beta + 2K(1 - \cos\beta) + \frac{\beta}{2} - \frac{\sin 2\beta}{4}\right]$$

Simplifying,

$$Y_{AC} = \frac{P r^3}{E I} G \tag{49}$$

where

$$G = \frac{K^3}{3} + K^2\beta + 2K(1 - \cos\beta) + \frac{\beta}{2} - \frac{\sin 2\beta}{4} \tag{50}$$

The type A Breaker Clip has a shape as shown in Figure 15. Note that we are presenting that part of the clip that deflects when inserted in a load center stab.

$$P_1 = P\cos\alpha \tag{51}$$

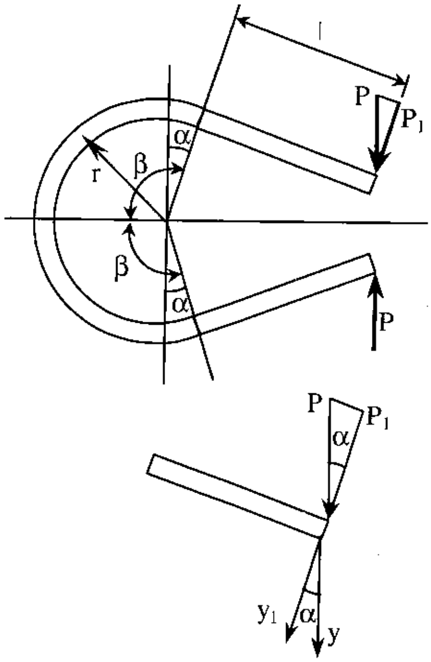


Figure 15: *Type A breaker clip*

P_1 deflects the clip in the direction of y_1, but the deflection of our interest is in the direction of y.

Therefore,

$y = y_1 \cos\alpha$, but y is half of the deflection or

$$Y = 2y_1\cos\alpha \tag{52}$$

But from above (half spring)

$$y_1 = \frac{P_1 r^3}{E I} G \tag{53}$$

$$\therefore Y = 2\left(\frac{P_1 r^3}{E I} G\right)\cos\alpha \tag{54}$$

also $P_1 = P\cos\alpha$

$$\text{Therefore } Y = \frac{2 P r^3 (\cos^2\alpha) G}{E I}, \tag{55}$$

which is the equation used in the text.