Optimization of a SDOF Cantilever Beam Piezoelectric Energy Harvester with a Lump Mass at the End Tip

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Abstract *— With the development of low power electronics and energy harvesting technology, self-powered systems have become a research hotspot over the last decade. The main advantage of self-powered systems is that they require minimum maintenance which makes them to be deployed in large scale or previously inaccessible locations. Therefore, the target of energy harvesting is to power autonomous 'fit and forget' electronic system over their lifetime. Some possible alternative energy sources include photonic energy, thermal energy and mechanical energy. The source of mechanical energy can be a vibrating structure, a moving human body or air water flow induced vibration. The primary objective of this project was the quantification of the vibration spectrum of an AC fan machine, using an accelerometer to provide the amplitude for the frequency of the machine. The frequency (hz) and magnitude (m/s*^{2}*) of the signal was obtained using Fast Fourier Transform analysis from Matlab Signal Processing Tool (SPTOOL).[1] The secondary objective was to analyze a Measurement Specialties MiniSense 100 Vibration Sensor modeled as a single degree of freedom, SDOF, base excited unimorph piezoelectric film energy harvester using a basic cantilevered beam with mass at the tip configuration to calculate the maximum power output produced by the system and tuned the proposed design to reached a maximum power generation. The same analysis was performed on the data from the cantilever sensor and the frequency of vibrations was obtained. The sensors were installed on a rigid base in order to obtain a higher stiffness and force transmissibility therefore obtaining a strong vibration signal input acting on the cantilevered beam. A wide range of research and proposed models shows that the maximum power output of a resonant energy harvester subjected to an ambient vibration is reached when the natural angular frequency (*ω_n*) of the mass-spring structure is tuned to the natural frequency of ambient vibrations (*ω*).*

Key Terms *— Energy Harvester, Natural Frequency, Resonance, Vibration Spectrum.*

Introduction to Vibration and Free Response

The physical explanation of the phenomena of vibration concerns with the interplay between potential energy and kinetic energy. A vibrating system must have a component that stores potential energy and releases it as kinetic energy in the form of motion (vibration) of a mass. The motion of the mass then gives up kinetic energy to the potential-energy storing device.

The degree of freedom of a system is the minimum number of displacement coordinates needed to represent the position of the system's mass at any instant of time. Free response refers to analyzing the vibration of a system resulting from a nonzero initial displacement and/or velocity of the system with no external force or moment applied.

The fundamental kinematical quantities used to describe the motion of a particle are displacement, velocity, and acceleration vectors. The laws of physics state that the motion of a mass with changing velocities is determined by the net force acting on the mass.

In the simple case of a hanging mass and spring combination with a vertical orientation, and ignoring the mass of the spring, the forces acting on a mass consist of the forces of gravity pulling down (mg) where m is the hanging mass and g is the acceleration due to gravity, and the elastic restoring force of the spring pulling back up (f_k). The equation that describes the force applied by the spring (f_k) to the mass is a linear relationship,

$$f_k = ky \tag{1}$$

where k is the stiffness of the spring and y is the displacement of the mass in the y direction. For strength of material considerations, a linear spring of stiffness k stores potential energy of the amount $\frac{1}{2}ky^2$.

In the case of a mass and spring combination with a horizontal orientation and assuming that the mass moves on a frictionless surface, the sum of all the forces on the free body diagram acting along the x-direction yields to

$$f_k = kx \tag{2}$$

$$m\ddot{x}(t) = -kx(t) \ \ or \ \ m\ddot{x}(t) + kx(t) = 0 \tag{3}$$

where $x(t)$ denotes the second time derivative of the displacement (i.e. the acceleration).

The solution this periodic motion based on physical observation and experience from watching this mass and spring system is

$$x(t) = A\sin(\omega_n t + \phi) \tag{4}$$

where A is the amplitude of the displacement, ω_n is the angular natural frequency which determines the interval in time during which the function repeats itself, and ϕ is the phase which determines the initial value of the sine function. It is standard to measure t in seconds (s), the phase in radians (rad) and the frequency in radians per seconds (rad/s). The angular frequency ω_n is determined by the physical properties of the of the mass and stiffness of the spring (m and k),

$$\omega_n{}^2 = \frac{k}{m} \quad or \quad \omega_n = \sqrt{\frac{k}{m}} \tag{5}$$

and the amplitude and phase are determined by the initial positions and velocities, as well as the systems natural frequency measures in hertz (Hz) or cycles per seconds (cycles/s) denoted by

$$f_n = \frac{\omega_n}{2\pi} \tag{6}$$

By differentiating the equation of motion yields the velocity given by

$$\dot{x}(t) = \omega_n A\cos(\omega_n t + \phi) \tag{7}$$

and the acceleration is given by

$$\ddot{x}(t) = -\omega_n{}^2 A\sin(\omega_n t + \phi) \tag{8}$$

The two constants of integration that need to be evaluated are A and ϕ which are determined by the initial state of motion of the spring and mass system. When the mass is displaced a distance x_0 at time t_0 (t = 0) the force kx_0 in the spring will result in motion. Also, if the mass is given an initial velocity v_0 at time t_0, motion will result because of the induced change in momentum. By substituting the initial conditions into the equations of motion yields to

$$x_0 = x(0) = A\sin(\omega_n 0 + \phi) = A\sin(\phi) \tag{9}$$

$$v_0 = \dot{x}(0) = \omega_n A\cos(\omega_n 0 + \phi) = \omega_n A\cos(\phi) \tag{10}$$

Solving these two equations simultaneously for the two constants of integration, A and ϕ yields to

$$A = \frac{\sqrt{\omega_n{}^2 x_0{}^2 + v_0{}^2}}{\omega_n} \tag{11}$$

$$\phi = tan^{-1}\left(\frac{\omega_n x_0}{v_0}\right) \tag{12}$$

Substituting this constants into the equation of motion gives the solution of motion for the spring and mass system which is given by

$$x(t) = \frac{\sqrt{\omega_n{}^2 x_0{}^2 + v_0{}^2}}{\omega_n}\sin\left(\omega_n t + tan^{-1}\left(\frac{\omega_n x_0}{v_0}\right)\right) \tag{13}$$

This solution is called the free response of the system because no force external to the system is applied after t = 0. The motion of the spring-mass system is called simple harmonic motion or oscillatory motion. [2]

VISCOUS DAMPING

The response of the spring and mass models predict that the system will oscillate indefinitely. However, everyday observations indicate that freely oscillating systems eventually die out and reduce to zero motion. These observations suggest that the spring-and-mass model and the corresponding mathematical equations need to be modified to account for this decaying motion. The theory of differential equations suggest that adding a term to the equation of the form $c\dot{x}(t)$, where c is a constant, will result in a solution $x(t)$ that dies out. Physical observations agree fairly well with this model and are used successfully to model de damping, or decay, in a variety of mechanical systems. This type of damping is called viscous damping.

While the spring forms a physical model for storing potential energy and hence causing vibration, the dashpot, or damper, form the physical model for dissipating energy and thus damping the response of the mechanical system. The force is proportional to the velocity of the piston in the direction opposite that of the piston motion. The damping force is given by

$$f_c = c\dot{x}(t) \tag{14}$$

where, is a constant of proportionality to the oil viscosity is called the damping coefficient with units of force per velocity.

Using a simple force balance on the mass in the x-direction the equation of motion $x(t)$ becomes

$$m\ddot{x} = -f_c - f_k \tag{15}$$

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = 0 \tag{16}$$

In order to solve this differential equation of second order, let's assume a particular solution of $x(t) = a\lambda e^{\lambda t}$. Substituting it into the equation of motion yields to

$$(m\lambda^2 + c\lambda + k)ae^{\lambda t} = 0 \tag{17}$$

since $ae^{\lambda t} \neq 0$, then

$$m\lambda^2 + c\lambda + k = 0 \tag{18}$$

Using the quadratic equation to solve this second order differential equation yields to two solutions

$$\lambda_{1,2} = -\frac{c}{2m} \pm \frac{1}{2m}\sqrt{c^2 - 4km} \tag{19}$$

Examination of this expression indicates that the root λ will be real or complex depending on the value of the discriminant, $\sqrt{c^2 - 4km}$. As long as m, c, and k are positive real numbers, λ_1 and λ_2 will be distinct negative real numbers if $c^2 - 4km > 0$. On the other hand if the discriminant is negative, the roots will be a complex conjugate pair with negative real parts. If the discriminant is zero, the two roots λ_1 and λ_2 are equal negative real numbers. For these three cases, it is both convenient and useful to define the critical damping coefficient, c_{cr}, by

$$c_{cr} = 2m\omega_n = 2\sqrt{km} \tag{20}$$

The non-dimensional number ζ called the damping ratio is defined by

$$\zeta = \frac{c}{c_{cr}} = \frac{c}{2m\omega_n} = \frac{c}{2\sqrt{km}} \tag{21}$$

It is now clear that the damping ratio ζ determines whether the roots are complex or real. This in turn determines the nature of the response of the damped single-degree-of-freedom system.

Under Damped Motion

In the case of damping ratio ζ less than 1 ($0 < \zeta < 1$) and the discriminant $\sqrt{c^2 - 4km}$ is negative, the results in a complex conjugate pair of roots that become

$$\lambda_1 = -\zeta\omega_n - \omega_n\sqrt{1-\zeta^2}\, j \tag{22}$$

$$\lambda_2 = -\zeta\omega_n + \omega_n\sqrt{1-\zeta^2}\, j \tag{23}$$

where $j = \sqrt{-1}$. Following the same argument as that was made for the un-damped response equation, the solution is then in the form of

$$x(t) = e^{-\zeta\omega_n t}\left(a_1\, e^{j\sqrt{1-\zeta^2}\omega_n t} + a_2\, e^{-j\sqrt{1-\zeta^2}\omega_n t}\right) \tag{24}$$

where a_1 and a_2 are arbitrary complex values of integrations determined by the initial conditions. Using Euler's relations this equation can be simplified to

$$x(t) = A\, e^{-\zeta\omega_n t}\sin(\omega_d t + \phi) \tag{25}$$

where A and ϕ are constants of integration and ω_d is called the damped natural frequency which is given by

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \tag{26}$$

in units of rad/s. Differentiating $x(t)$ yields to

$$\dot{x}(t) = -\zeta\omega_n A\, e^{-\zeta\omega_n t}\sin(\omega_d t + \phi) + \omega_d A\, e^{-\zeta\omega_n t}\cos(\omega_d t + \phi) \quad (27)$$

and evaluation it with initial conditions of t = 0 and

$A = \frac{x_0}{\sin(\phi)}$,

$$\dot{x}(0) = v_0 = -\zeta\omega_n x_0 + x_0\omega_d \cot(\phi) \quad (28)$$

Solving for ϕ yields to ϕ,

$$tan\ (\phi) = \frac{x_0\omega_d}{v_0 + \zeta\omega_n x_0} \quad (29)$$

with this value of ϕ, the sine becomes

$$sin\ (\phi) = \frac{x_0\omega_d}{\sqrt{(v_0+\zeta\omega_n x_0)^2 + (x_0\omega_d)^2}} \quad (30)$$

thus the values of A and ϕ are determined by

$$A = \sqrt{\frac{(v_0+\zeta\omega_n x_0)^2 + (x_0\omega_d)^2}{\omega_d{}^2}} \quad (31)$$

$$\phi = \tan^{-1}\left(\frac{x_0\omega_d}{v_0 + \zeta\omega_n x_0}\right) \quad (32)$$

where x_0 and v_0 are the initial displacement and velocity respectively, and the damping ratio ζ determines the rate of decay.

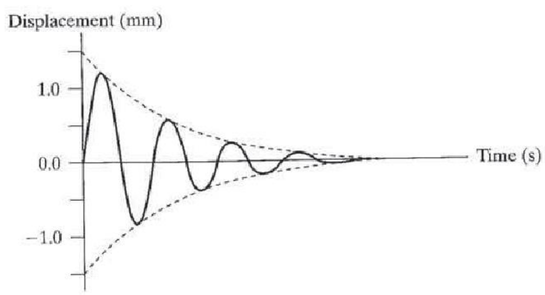


Figure 1
Response of an Underdamped System [2]

PRINCIPLE OF PIEZOELECTRIC EFFECT FOR ENERGY HARVESTING

Harvesting of mechanical energy is to convert it into electrical energy, which requires a mechanical system that couples motion or vibration to a transduction mechanism. The mechanical system should be designed to be able to maximize the coupling between the mechanical energy sources and the transduction mechanism, depending on the characteristics of the environmental motions. For example, energy due to vibration can be converted by using inertial generators, with the mechanical component attached to an inertial frame that acts as a fixed reference. The inertial frame transmits the vibrations to a suspended inertial mass to produce a relative displacement between them. System like this usually has a resonant frequency, which can be designed to match the characteristic frequency of the environmental motions.

These inertial-based generators can be well described as second-order spring-mass systems. For a system with a seismic mass of m on a spring with a stiffness of *k*, its energy loss, consisting of parasitic loss, *cp* and electric energy generated by the transduction mechanism, *ce*, can be represented by damping coefficient, *ct* . The system is excited by an external sinusoidal vibration, *y(t) = Ysin(xt)*. At resonant frequency, there is a net displacement, *z(t)*, between the mass and the frame. If the mass of the vibration source is greatly larger than that of the seismic mass, the latter can be ignored. If the external excitation is harmonic, the differential equation of the motion is given by equation (3). Energy conversion can be maximized when the excitation frequency matches the angular frequency of the system.

When there is sufficient acceleration, increased damping effects will lead to a response with broadened bandwidth, so that the generator will be less sensitive to frequency. An excessive device amplitude can lead to nonlinear behavior of the generator, which will make it difficult in keeping the generator working at resonance frequency. For specific applications, both the frequency of the generator and the level of damping should be specifically designed to maximize the power output. The power generation can also be maximized by maximizing the mass of the mechanical structure. [3]

Piezoelectric Energy Harvesting Devices

Piezoelectric materials can produce electrical charges when they are subject to external mechanical loads. Figure 2 shows working principle of a piece of piezoelectric material. The magnitude and direction of the electrical current are determined by the magnitude and direction of the external mechanical stress/strain applied to the materials.

There have been various modes of vibration that can be used to construct piezoelectric harvesting devices.

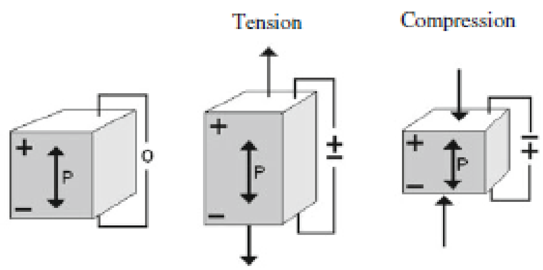


Figure 2
Schematic showing the Response of a Piece of Piezoelectric Ceramics to External Mechanical Stimulation [3]

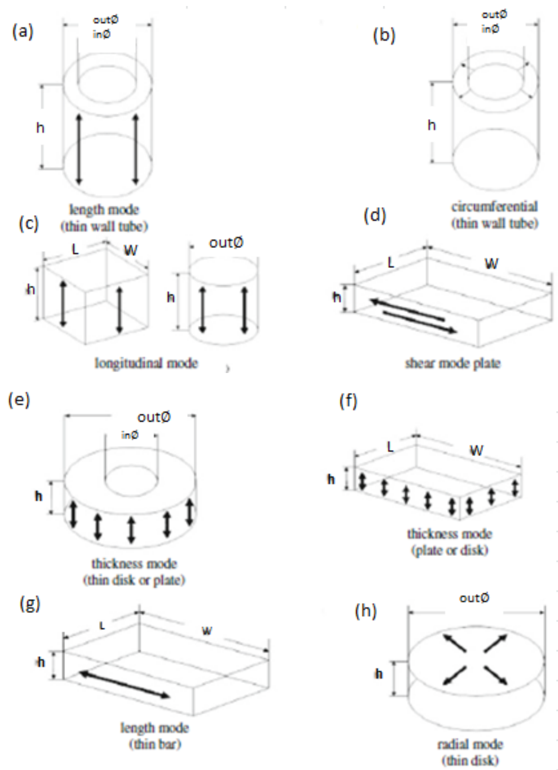


Figure 3
Common Modes of Vibration [3]

The common modes of vibration are summarized in Figure 3 [3]. With given modes of vibrations, there are different piezoelectric structures. Figure 4 [3] shows typical piezoelectric structures that can be found in open literature. Among the various piezoelectric structures for energy harvesters, the cantilevered beams with one or two piezoelectric ceramic thin sheets, which are named unimorph and bimorph Figure 4a, respectively, are the simplest ones. As discussed above, the harvester beam is positioned onto a vibrating host, where the dynamic strain induced in the piezoceramic layer(s) results in an alternating voltage output across their electrodes. Figure 5 shows a schematic of a cantilever tested under base excitation. When a harmonic base motion is applied to the structure, an alternating voltage output is produced. Cantilevered piezoelectric energy harvesters can work in two modes: d33 mode and d31 mode, as shown in Figure 6. In d31 mode, a lateral force is applied in the direction perpendicular to the polarization direction. In this case, the bending beam has electrodes on its top and bottom surfaces, as in Figure 6a. In d33 mode, forces are applied in

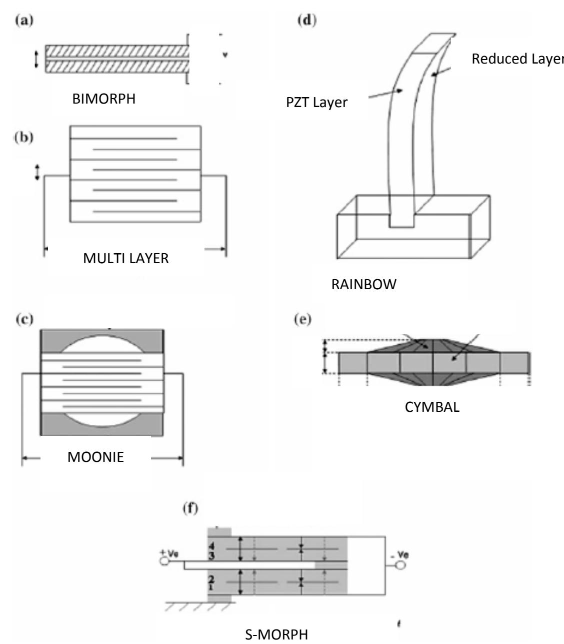


Figure 4
Piezoelectric Structures: (a) Bimorph, (b) Multilayer, (c) Moonie, (d) Rainbow, (e) Cymbal and (f) S-morph [3]

Although piezoelectric materials working in d31 mode normally have lower coupling coefficients than in d33 mode, d31 mode is more commonly used. This is because when a single-layer cantilever or a double-clamped beam bends, more lateral stress is produced than vertical stress, which makes it easier to couple in d31 mode. Similar principle can be applied to the harvesters with other structures. Beam structures are usually used for low stress levels, whereas at high stress levels, another type of device, ceramic–metal composites, is preferred. Ceramic–metal composites generally have a simple design with a metal faceplate, called shell or cap, which couples to both the ceramic and the surrounding medium.

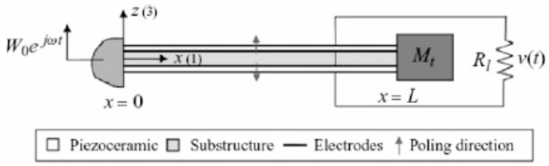


Figure 5
Schematic Cantilevered Piezoelectric Energy Harvester Tested under Base Excitation [3]

The metal component transfers the incident stress to the ceramic or the displacement to the medium. Flextensional transducers are good examples of ceramic–metal composites.

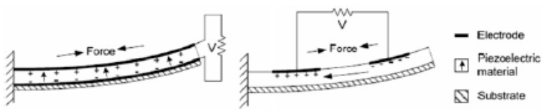


Figure 6
Two Types of Piezoelectric Energy Harvesters a d_{31} Mode and b d_{33} Mode [3]

To theoretically study the mechanics of piezoelectric energy harvesting and experimentally evaluate the performances of alternative current (AC) power generation, the devices are usually considered to be subject to a resistive load in the electrical domain. To use the electricity produced by a piezoelectric energy harvester, the alternating voltage output should be converted to a stable rectified voltage. This can be realized by using a rectifier bridge and a smoothing capacitor to form an AC–DC converter. The energy can be used to charge small batteries or stored in capacitors. To maximize the power transfer to the energy storage devices, it is also necessary to use a DC–DC converter to regulate the voltage outputs of the rectifier. These electrical circuit and power electronics are very important for practical applications of the energies harvested [3]. Electrical circuits or proposed power applications are beyond the scope of this project.

CANTILEVER BEAM: DEFLECTION, NATURAL FREQUENCY AND STIFFNESS

In engineering systems, it is often necessary to know the magnitude of beam deflections under various loading conditions. Often the maximum allowable beam deflection y_A is defined in terms of span *L* of the beam [4]. For the case of a cantilever beam, Figure 7, with uniform cross section loaded at the tip with a load *P*, the deflection at the end is obtained by:

$$y_A = \frac{PL^3}{3EI} \quad (33)$$

Where *E*, the Modulus of Elasticity and *I* is the moment of inertia. The formula for the natural frequency f_n of a single-degree-of-freedom system is:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{3EI}{mL^3}} \quad (34)$$

Where *m*, is the mass of the tip load. Other important property of the system is the stiffness *k* at the end of the beam is:

$$k = \frac{P}{y_A} = \frac{3EI}{L^3} \quad (35)$$

The stiffness is the ratio of the load to the deflection. Notice that the stiffness is inversely proportional to L^3; the beam's stiffness decreases rapidly with length. [4]

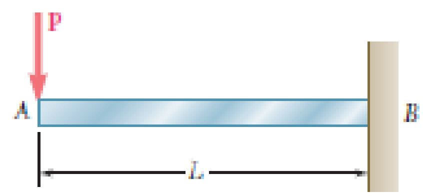


Figure 7
Cantilever Beam AB with a Load P at its Free End [5]

THE ENERGY HARVESTER

The two main objectives of this project were to measure the vibration spectrum from a household conventional air conditioning unit and quantify the amount of energy produced by the vibration with a currently available piezoelectric sensor.

The Measurement Specialties MiniSense 100 Vibration Sensor is a low-cost cantilever-type vibration sensor loaded by a mass to offer high sensitivity at low frequencies. [6]

Figure 8
MiniSense 100 Vibration Sensor [6]

The pins are designed for easy installation and are solderable. Horizontal and vertical mounting options are offered as well as a reduced height version. The active sensor area is shielded for improved RFI/EMI rejection. Rugged, flexible PVDF sensing element withstands high shock overload. Sensor has excellent linearity and dynamic range, and may be used for detecting either continuous vibration or impacts. The mass may be modified to obtain alternative frequency response and sensitivity selection.

The MiniSense 100 acts as a cantilever-beam accelerometer. When the beam is mounted horizontally, acceleration in the vertical plane creates bending in the beam, due to the inertia of the mass at the tip of the beam. Strain in the beam creates a piezoelectric response, which may be detected as a charge or voltage output across the electrodes of the sensor.

The sensor may be used to detect either continuous or impulsive vibration or impacts. For excitation frequencies below the resonant frequency of the sensor, 75Hertz (Hz), the device produces a linear output governed by the “baseline” sensitivity. The sensitivity at resonance is significantly higher. Impacts containing high-frequency components will excite the resonance frequency which is the response of the MiniSense 100 to a single half-sine impulse at 100Hz, of amplitude 0.9 g. [6]

Mechanical Properties

From the energy harvesting theory, the power output by the harvester is a maximum when the frequency of the excited system reaches resonance. The MiniSense 100 specifications states that the resonance frequency of the system is reached at 75Hz. In order to obtain viable results which lead to a good analysis and optimization of the system, certain properties like the modulus of elasticity *E* is needed. Since this is a property not provided by manufacturer, firstly we have to obtain the modulus of elasticity. To do this, the values of vertical displacement were obtained experimentally for different values of the concentrated load *P* applied at the free end of the beam.

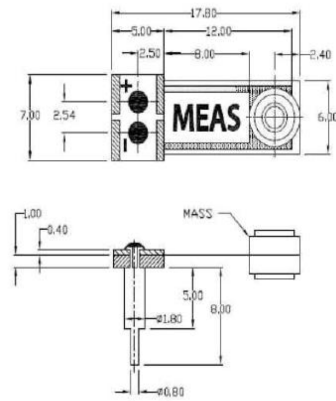


Figure 9
Harvester Beam Dimensions [6]

The loads to compute the displacement are: (2.94, 5.14, 7.34, 9.53, 11.73 and 13.93) x10^{-3}N.

Table 1
Experimental Beam Displacement

P (N)	Experimental YA(m)
2.94E-03	0.00002
5.14E-03	0.00010
7.34E-03	0.00032
9.53E-03	0.00068
1.17E-02	0.00116
1.39E-02	0.00176

Inserting the experimental displacement results into (33) and solve for *E*. To obtain a constant value the Root Mean Square of the vector matrix from the previous results gives, E = 2.21GPa. From (35), the stiffness *k* is: 64.83N/m^2. Next the natural frequencies when the beam is subjected to a load at the end of the beam can be obtained from (34), experimental natural frequencies in Table 2 with respect to the load applied to the system. The experimental data validates the data from the manufacturer by limiting the harvester to a resonance frequency of 75Hz.

Table 2
Natural Frequency with Respect to the Load P Applied to the Beam

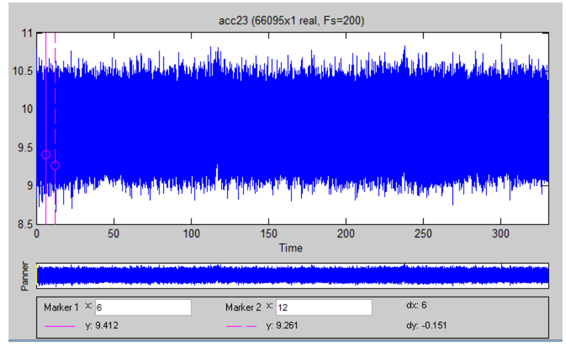
	Experimental
P (N)	fn (Hz)
2.94E-03	74.0
5.14E-03	56.0
7.34E-03	49.6
9.53E-03	41.1
1.17E-02	37.1
1.39E-02	34.0

VIBRATION SPECTRUM AND RESULTS

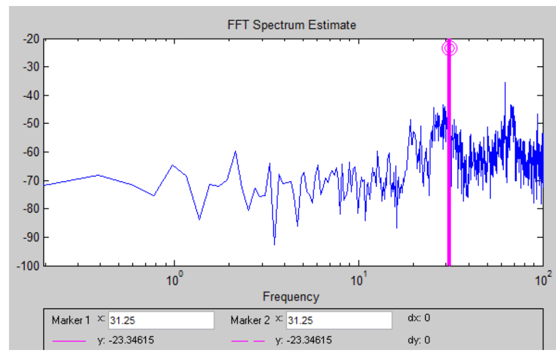
The source of the input vibrational signal comes from the fan component of a household air conditioning unit. The fan model as shown in Figure 10 operates from 1690/1900 rpm at Hi (Cool/Heat).

Figure 10
FAN Unit Model with Harvester and Measurement Equipment on Top

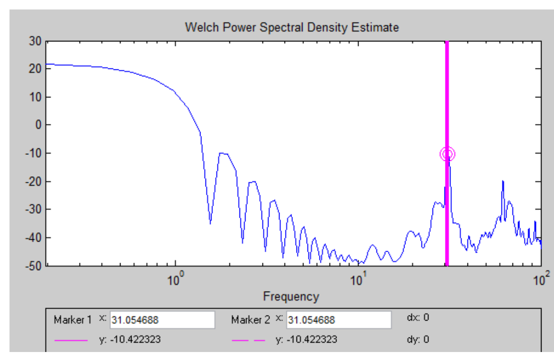
The vibration spectrum from the source is obtained from an accelerometer hardware/software. The accelerometer was located on top of the fan metal casing axially aligned with the center of the rotating fan shaft. Figure 11a, shows the data obtained by the accelerometer and plotted using MATLAB Signal Processing Tool (SPTOOL).



(a)



(b)



(c)

Figure 11
FAN Vibrational Signal Spectrum (a) and Operating Frequency (b) from FFT Analysis (c) Welch Power Spectral Density Analysis

The frequency of the fan from Matlab Sptool, using the Fast Fourier Transformation (FFT) and Welch spectral density estimate analysis is 31.25Hz and 31.05Hz respectively with a magnitude of 0.07m/s^2, Figure 11b, c, or translated into rotational speed 1875rpm, which is close to the fan equipment, HI (Heat) operation from manufacturer specifications.

The Energy Harvester and the Data Acquisition Setup

A single energy harvester was tested with the objective to obtain the frequency at which the cantilevered beam system operates when subjected to a vibrational source as received from the manufacturer. The MiniSense 100, Figure 8, is equipped with a 0.3g mass located at the end tip. The harvester was installed over a rigid base and wired at the two connecting poles (+,-), Figure 12 and connected to a data logging multi-meter, Figure 13.

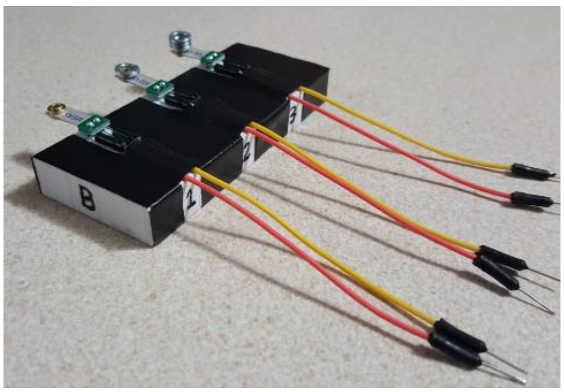


Figure 12
Harvester Configuration

The data was measured while the fan was operating for a lapse of five minutes. FFT analysis of the data shows that the fan is exciting the harvester at 15.19Hz.

Figure 13
Testing Setup

Harvester #1, with a 0.3g mass at the end tip is excited and vibrates at 15.19Hz. Figure 14. Shows the FFT power spectrum estimates for the test.

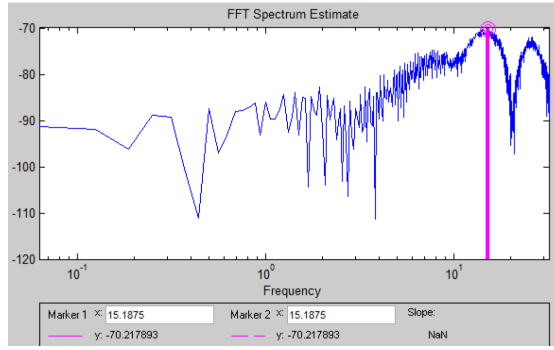


Figure 14
Harvester #1 Frequency of Vibration from FFT Analysis

MODEL OPTIMIZATION

By modifying the harvester structure we can increased the frequency at which the harvester is excited. Among all the possible modification to the system, there are two ways which can lead to a good optimization of the harvester. One is to increase or decrease the length of the beam and two by tuning the tip mass. The later was selected since the integrity of the piezoelectric coupling with the protective shield and the mechanical properties of the system were too maintained intact. A second harvester is equipped with an extra 0.224g mass and a third with 0.672g. Total mass acting on the harvesters beam end tip is: (0.3g, 0.524g and 0.972g). The three harvesters were tested for five minutes and the frequency of excitation obtained. Figure 15, shows the FFT spectrum estimate for harvester #2.

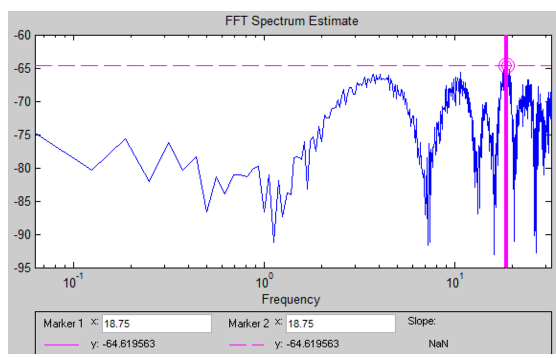


Figure 15
Harvester #2 Frequency of Vibration from FFT Analysis

The results from the harvester #2 test shows that by modifying the end tip mass increases the frequency response of the system. The third harvester was tested with similar behavior on the results, see Figure (16). The response of the harvester is that the excitation frequency is again increased.

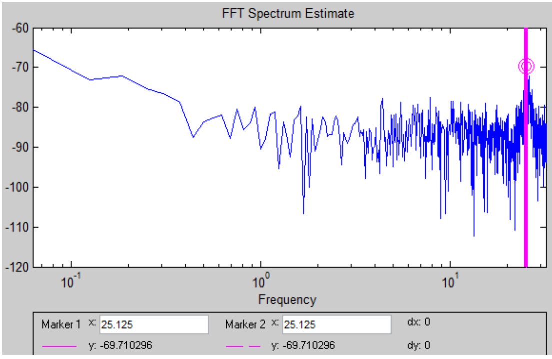


Figure 16
Harvester #6 Frequency of Vibration from FFT Analysis

Figure 17, shows a comparison of the natural frequency of the harvesters, the frequency of the operating fan and the frequency of the excited harvesters in Hz. From the graph it can be observed that the increased in the mass at the end of the beam had the effect to increase the frequency of excitation of the harvester.

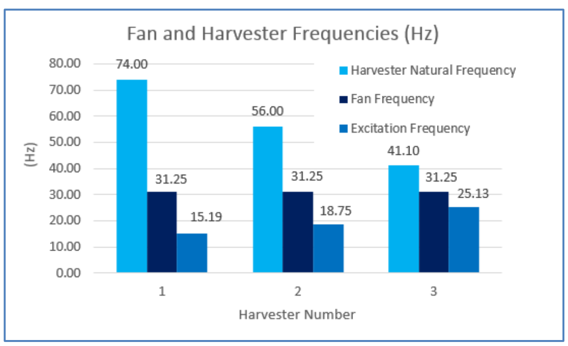


Figure 17
Harvester #6 Frequency of Vibration from FFT Analysis

By inspecting the excitation frequencies of the three harvester it is expected that the system with the frequency closest to the fan frequency, (31.25Hz), will produced the highest power output. To obtain the power produced by the harvester, the magnitude of the signal needs to be calculated from the spectrum estimate results. The FFT spectrum not only estimates the frequency response of the vibration signal, but also the magnitude of the vibration in Decibels (dB). The time-domain magnitude of the data in in volts (V). Equation (35) converts from the frequency-domain magnitude into the time-domain magnitude. The magnitude in (V) the three harvesters are: ($3.08x10^{-4}$, $5.88x10^{-4}$, and $3.27x10^{-4}$).

$$mag(volts) = 20 * Log_{10}(mag(dB)) \quad (35)$$

One of the most common use of Fast Fourier transform is to find the frequency components of a signal buried in the noisy time domain. It is from the FFT analysis that a single dominant signal is extracted from the original vibrational spectrum, (36) represents the dominant signal of the excitation frequency from the air conditioning fan, and (37), (38) and (39) the response of the energy harvesters to that of the excitation frequency in a simple sinusoidal wave form. From the resulting sinusoidal wave signals the point of interest is the magnitude of the signal, which allows to obtained the highest power output. Frequency at this point is secondary since circuit optimization is beyond the project's scope. Considering the sinusoidal wave form, Figure 18, represents the plot of the amplitude A and frequency ω_n of the inputted signal acting on the energy harvesters and Figure 19 the response of the harvesters to the inputted signal. Substituting amplitudes and frequencies into (4) we obtained the displacement functions obtained from the FFT spectrum estimate for a phase shift set to 0.

$$y(t) = 0.07 \sin(31.25t) \quad (36)$$

$$y(t) = 3.08x10^{-4} \sin(15.19t) \quad (37)$$

$$y(t) = 5.88x10^{-4} \sin(18.75t) \quad (38)$$

$$y(t) = 3.27x10^{-4} \sin(25.13t) \quad (39)$$

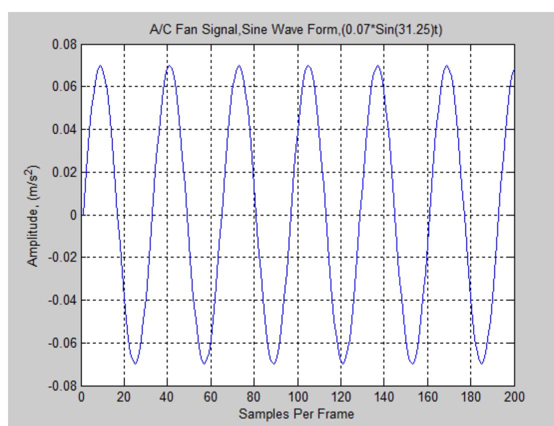
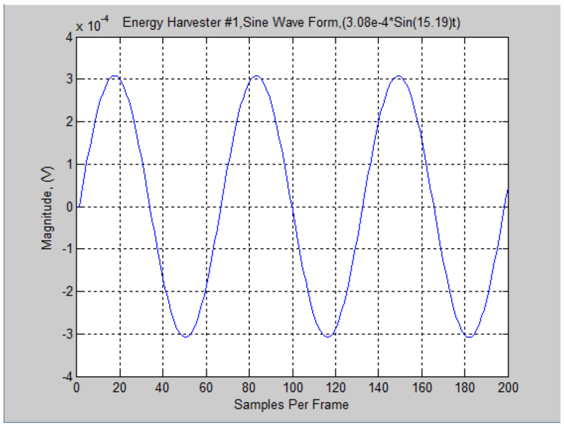
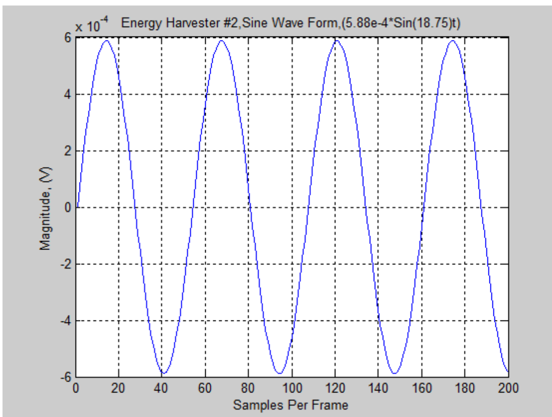


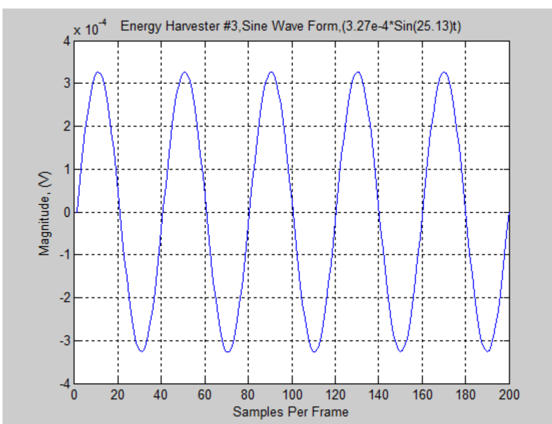
Figure 18
Sine Wave Form of the A/C Fan Signal Acting on the Energy Harvesters



(a)



(b)



(c)

Figure 19
Energy Harvester's Response to the Vibrational Spectrum Produced by the a/c fan. Energy Harvester 1(a), 2(b) and 3(c)

Power Generated by the Harvester

The system configuration is set as an open circuit. The voltage obtained from the vibrating harvester is measured, but in order to obtain the power that can be harvest with the system a current flow is needed. The harvesters acting as a voltage source, were connected to a 100Ω resistor, Figure 20, to obtain the current flowing thru the electrical component and therefore calculate the power that can be generated from the vibration of the harvesting system.

When the current is flowing thru the resistor, electrical damping acts directly to the systems by decreasing the excitation frequency. When the system is connected to the resistor the excitation frequency decreases substantially. Harvester #1 excitation frequency drops from 15.18Hz to 0.25Hz, Harvester #2 from 18.75Hz to 0.1875Hz and Harvester #3 from 25.13 to 0.096Hz.

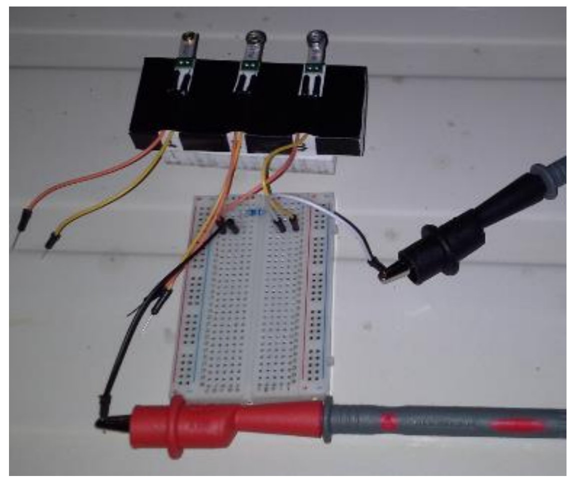


Figure 20
Harvester Acting as a Voltage Source to Measure the Current Flowing thru the Resistor

The drop in frequency shows two main facts, 1) Electrical damping is acting on the system and 2) Power is been generated across the circuit. The current in Amps (A) flowing thru the resistor from the three harvesters is: ($2.95x10^{-6}$, $2.07x10^{-5}$, and $7.63x10^{-5}$). From basic electrical principles Power (watts, W) is equal to the voltage (Volts, V) times the current (Amps, A). The energy harvesters generates a power output of: ($9.07x10^{-10}$, $1.88x10^{-8}$ and $2.49x10^{-8}$). Figure 21, shows a comparison of the power generated from the three harvesters. The results shows that when optimizing the system by tuning the mass at the end tip, the power generated by the system is increased. The small amount generated by the system may not be sufficiently enough to energize some of the daily electronic devices that we use, but the results prove that power is generated and ultimately increased.

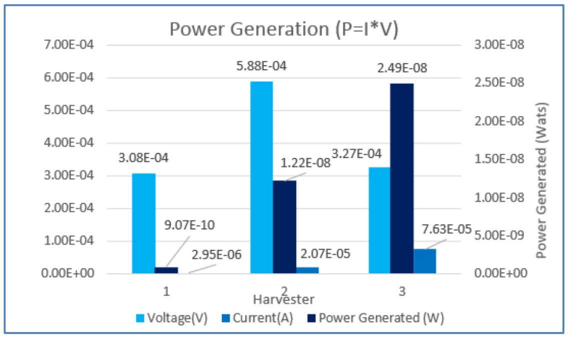


Figure 21
Final Power Generated Comparison

CONCLUSION

By placing a mass at the end tip of the cantilevered energy harvester, not only the frequency of the harvesters is excited to a higher frequency, but the increase in the ability of the piezoelectric material to generate 1,345% from energy harvester #2 and 2,745% for energy harvester #3, more power output compared to the original energy harvester generation. The amount of power generated by the piezoelectric harvesters stills significantly small, if compared with today's power demand, but the reduction in size and power consumption of micro electro mechanical systems gives this source of energy a fertile ground for extensive research.

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