Surface Response Modeling for Rotor-Dynamics Systems

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Abstract *— This paper presents the way to generate an equation that does not exist, to analyze different rotor-dynamics system. With the analysis and design software, the results desired were obtained in order to be able to create the equation needed. Using Matlab program and the Response Surface Methodology, an equation with four different variables was generated, in order to use it and obtain the critical speeds of a rotor without the necessity to analyze the rotor in a design simulator program.*

Key Terms *— Critical Speed Analysis, Finite Element (FE), Response Surface, Rotor-Dynamics.*

INTRODUCTION

The main reason of this report is to demonstrate how to create an equation for the critical speeds of the rotor-dynamics system and see how Matlab works as a design program, to obtain the different critical speeds by changing the inputs. The rotor-dynamic system was first used as the interplay between its theory and its practice, driven more by its practice than by its theory. Today is commonly used to analyze the behavior of structures with finite element systems ranging from jet engines and steam turbines to auto engines and computers disk storages.

Throughout the report, the rotor-dynamics system is explained and a brief history of the rotor-dynamics system principles is presented. Also, the different equations involved with a Rotor, the Design for Fitting Second-Order Models with the Class for Central Composite Design and the application explanation for both subjects are given. Also, a simulation in Matlab program [8] is presented and the data collected by the simulation it's used to create the equation and by changing the variables values in the equation, the different critical speeds can be obtained.

PROBLEM STATEMENT

It has come to attention that there is no way to know if a rotor-dynamic system is going to work adequately, unless there is an analysis with simulator program or all the analytical calculations is performed. With the equation created it is a closer way to face reality of knowing the critical speeds without the need of having to analyze the whole rotor. Using this equation will give an idea or a quick review of the critical speeds, taking in consideration that the system has the same properties.

PURPOSE AND SCOPE OF WORK

The main idea of this research is to learn about how an equation can be obtained, where the values can be changed like: the length of the disk 1, the length of the disk 2, the output diameter of disk 1 and the output diameter of disk 2. This project is addressed as a way to control the output critical speed of the rotor, to see if the variables combinations are going to work as expected.

Figure 1
Rotor-Dynamics [1]

WHAT IS A ROTOR DYNAMICS?

A rotor is a body supported through a set of bearings that allow it to rotate freely about an axis fixed in space. Engineering components concerned with the subject of rotor dynamics are rotors in machines, especially of turbines, generators, motors, compressors and blowers. The parts of the machine that do not rotate are referred to with general definition of stator. Rotors of machines have, while in operation, a great deal of rotational energy, and a small amount of vibrational energy. The purpose of rotor dynamics as a subject is to keep the vibrational energy as small as possible. In operational rotors undergoes the bending, axial and torsional vibrations [1].

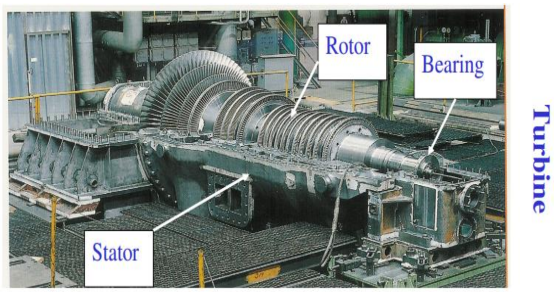

Figure 2
Rotor-Dynamics Example [1]

Rotor dynamics is the branch of the engineering that studies the lateral and torsional vibrations of rotating shaft, with the objective of predicting the rotor vibrations and containing the vibration level under an acceptable limit. The principal components of a rotor-dynamic system are the shaft or rotor with disk, the bearings, and the seals [2].

The shaft or rotor is the rotating component of the system. Many industrial applications have flexible rotors, where the shaft is designed in a relatively long and thin geometry to maximize the space available for components such as impellers and seals. Additionally, machines are operated at high rotor speeds in order to maximize the power output [2].

The other two of the main components of rotor-dynamics system are the bearings and the seals. The bearings support the rotating components of the system and provide the additional damping needed to stabilize the system and contain the rotor vibration. Seals, on the other hand, prevent undesired leakage flows inside the machines of the processing or lubricating fluids, however they have rotor-dynamic properties that can cause large rotor vibrations when interacting with the rotor [2].

Brief History of Rotor-Dynamic System

The first recorded supercritical machine (operating above first critical speed or resonance mode) was a steam turbine manufactured by Gustav de Laval in 1883 [2]. The credit for invention of the steam turbine is given both to the British engineer Sir Charles Parsons, for invention of the reaction turbine and to Swedish engineer Gustav de Laval, for invention of the impulse turbine [2].

Modern steam turbines frequently employ both reaction and impulse in the same unit, typically varying the degree of reaction and impulse from the blade root its periphery [2].

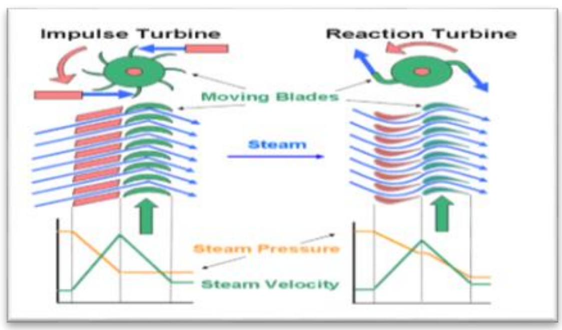

Figure 3
Turbine's Reaction and Impulse [7]

Modern high performance machines normally operates above the first critical speed margin of 15% between the operating speed and the nearest critical speed is a common practice in industrial applications [2].

Campbell Diagram

The Campbell diagram had been invented by W.E. Campbell in 1924 and widely adopted in the design and operation of rotating machines. According to Campbell, the diagram plots the natural frequencies against the rotational speed, along with the force order lines. The intersections of the force order lines and the natural frequencies indicate, not the actual, but the potential resonances, which ought to be avoided in actual operation of the machines [3].

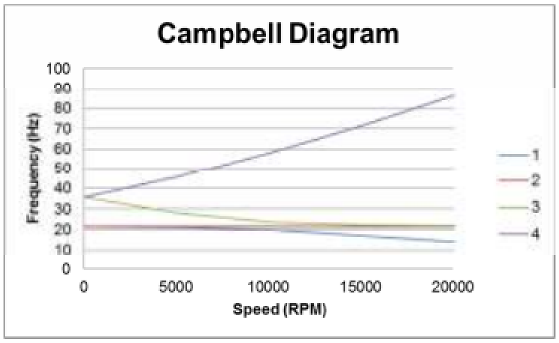

Figure 4
Campbell Diagram Example [4]

The obvious drawback of the Campbell diagram is that it is very conservative in prediction of possible resonances, because it only takes the information of the rotational speed-dependent natural frequencies, which are the imaginary part of eigenvalues, and the order of all possible excitation sources. It lacks the information on stability, which is related to the real part of eigenvalues, and modal and adjoint vectors which can be available even before the manufacture or installation of the machines. In other words, the Campbell diagram does not differentiate the importance or the potential severity of modes, equally treating all modes [3].

The Campbell diagram is helpful for design and practice engineers to judge on the margin of safe operation in the design as well as field operation processes. The Campbell diagram has been popularly adopted in the design of rotors with bladed disks such as turbines and fans. However, its usage is limited in the sense that it does not provide engineers with the essential information such as the stability and forced response of the actual rotor system. In other words, it does not tell us about which critical speed have to be considered seriously in design and operation [3].

Critical Speed

A critical speed occurs when the excitation frequency coincides with a natural frequency, and can lead to excessive vibration amplitudes [1]. All rotating shaft, even in the absence of external load, deflect during rotation. The combined weight of a shaft and wheel can cause deflection that will create resonant vibration at certain speeds. Critical speed depend upon the magnitude or location of the load or lad carried by the shaft, the length of the shaft, its diameter and the kind of bearing support [4].

Rotor Equations

The rotor's system behavior looks very much like another vibration problem. A vibrating beam system can be described by the equation;

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = F \qquad (1)$$

The rotor system can be described by the following differential equation in a stationary reference frame;

$$[M]\{\ddot{x}\} + ([C] + [C_{gyr}])\{\dot{x}\} + [K]\{x\} = F \qquad (2)$$

Using the Finite Element Method, [M], represents the mass matrix created for each discrete element. Similarly, [C], represents the damping matrix; [Cgyr], represents the gyroscopic matrix, and [K], represents the stiffness matrix. The additional gyroscopic term shows how the spin's speed of the rotor will directly affect: the natural frequencies of the rotor vibration and the mode shape or orbit of the system [5].

The equations system can be solved as an Eigen-value problem; the corresponding Eigen-values represents, the natural frequencies of the system, and the Eigen-vectors describes the mode shape and orbit of the rotor system. It is important in the system's solution, to understand the shape and rotational direction of the modes [5].

For isotropic bearings, the rotor will move in a circular orbit, however, if the bearings are not isotropic the orbits will be elliptical. The direction of rotation depends on the phase difference between the two directions of lateral motion. A mode is considered forward whirling if it is rotating in the same direction as the rotor rotation and a backward whirling if it is rotating in the opposite direction [5].

Modes typically come in pairs, with the forward whirling modes increasing in frequency with rotor speed, while the backward whirling modes decrease in frequency with rotor speed. A Campbell diagram is a very useful tool for understanding the interaction between rotor rotating speed and natural frequencies [5].

EXPERIMENTAL DESIGNS FOR FITTING RESPONSE SURFACES

Designs for Fitting Second-Order Models

The purpose of the experimental design is one that should allow the user to fit the second-order model that is;

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_1 + \sum_{i=1}^{k} \beta_{ii} {x_1}^2 + \sum \sum_{i<j=2}^{k} \beta_{ij} x_i x_j + \varepsilon \quad (3)$$

The minimum conditions for fitting a second-order model are:

1. At least three levels of each design variables.
2. At least 1+2k+k(k-1)/2 distinct design points.

In case of first-order designs (or fires-order-with-interaction designs), the dominant property is orthogonality. In case of second-order designs, orthogonality cases are an important issue, and estimation of individual coefficients, while still important, becomes secondary to the scaled prediction variance N Var[y(x)]/σ^2 [6].

There are a set of properties that should be taken into account when the choice of a response surface design is made. Some of the important characteristics are as follows [6]:

1. It results as a good fit model.
2. Give sufficient information to allow a test for lack of fit.
3. Allow models of increasing order to be constructed sequentially.
4. Provide an estimate of “pure” experimental error.
5. Be insensitive (robust) to the presence of outliers in the data.
6. Be robust to errors in control of design levels.
7. Be cost-effective.
8. Allow for experiments to be done in blocks.
9. Provide a check on the homogeneous variance assumption.
10. Provide a good distribution of Var[y(x)]/σ^2.

Most of them should be given serious consideration on each occasion in which one design experiments. Most of the properties are self-explanatory. Like a primary importance, designing an experiment is not necessarily easy and should involve balancing multiple objectives, not just focusing on a single characteristic.

The Class of Central Composite Designs

The central composite designs (CCDs) are without a doubt the most popular class of second-order designs. It was introduced by Box and Wilson (1951). Much of the motivation of the CCD evolves from its use in sequential experimentation. It involves the use of a two-level factorial or fraction (resolution V) combined with the 2k axial or star points [6].

Table 1
CCD Sequential Experimentation [6]

x_1	x_2	…	x_k
-α	0	…	0
α	0	…	0
0	-α	…	0
0	α	…	0
⋮	⋮		⋮
0	0	…	-α
0	0	…	α

The factorial points represent a variance-optimal design for a first-order model or a first-order + two-factor interaction model. Center runs clearly provide information about the existence of curvature in the system. If curvature is found in the system, the addition of axial points allow for efficient estimation of the pure quadratic terms [6].

The three components of the design play important and somewhat different roles:

- The resolution V fraction contributes substantially to the estimation of linear terms and two-factor interactions.
- The axial points contribute in a large way to estimation of quadratic terms. The axial points do not contribute to the estimation of interaction terms.
- The center runs provide an internal estimate of error (pure error) and contribute toward the estimation of quadratic terms.

The areas of flexibility in the use of the central composite design reside in the selection of α, the axial distance, and n_c, the number of center runs. The choice of α depends to a great extent on the region of operability and region of interest. The choice of n_c often has an influence on the distribution of N Var[y(x)]/σ^2 in the region of interest [6].

ROTOR-DYNAMIC SYSTEM USING MATLAB AND RESPONSE SURFACE METHODOLOGY

This is the Rotor dynamic created in Matlab [8] to obtain the critical speeds that are needed. This program runs 30 times, every run with a different combination of variables, obtaining seven critical velocities per each run.

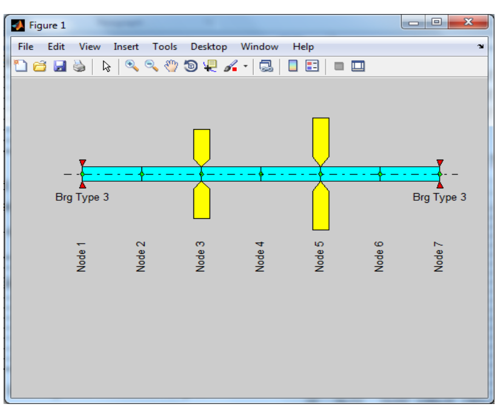

Figure 5
Rotor-Dynamic with Bearings, Disks and Shaft

Here we have seven equal spaced points at; $node\ 1 = 0.00$ m, $node\ 2 = 0.25$ m, $node\ 3 = 0.50$ m, $node\ 4 = 0.75$ m, $node\ 5 = 1.00$ m, $node\ 6 = 1.25$ m and $node\ 7 = 1.50$ m.

Properties and Data

Modulus of elasticity, $E = 211 \times 10^9\ Pa$
Shear Modulus, $G = 81.2 \times 10^9\ Pa$
Density, $\rho = 7810\ kg/m^3$
Disk 1 & Disk 2 Thickness = 0.07 m
Bearing Stiffness = 1×10^6
Rotor Spin Speed = 10000 RPM

These are the variables designed and the ones that are going to be changed in Matlab program [8] to obtain the desirable output.

1. *Length of Disk 1*
   Disk 1 Length, min = 0.40 m
   Disk 1 Length, max = 0.60 m
2. *Length of Disk 2*
   Disk 2 Length, min = 0.90 m
   Disk 2 Length, max = 1.10 m
3. *Outer Diameter of Disk 1*
   Disk 1 OD, min = 0.27 m
   Disk 1 OD, max = 0.29 m
4. *Outer Diameter of Disk 2*
   Disk 2 OD, min = 0.34 m
   Disk 2 OD, max = 0.36 m

This is the table obtained from the Design Expert Program [9] of the design values using the maximum and minimum from each variable.

Table 2
Design of Variables using Maximum and Minimum

	Length Disk1	Length Disk2	Disk1 OD	Disk2 OD
1	0.60	1.10	0.29	0.36
2	0.50	1.00	0.28	0.37
3	0.60	0.90	0.29	0.34
4	0.60	0.90	0.29	0.36
5	0.50	1.00	0.28	0.33
6	0.60	1.10	0.27	0.36

7	0.50	1.00	0.26	0.35
8	0.50	1.00	0.28	0.35
9	0.60	1.10	0.29	0.36
⁞	⁞	⁞	⁞	⁞
30	0.60	0.90	0.27	0.34

Last run, out of thirty runs, Campbell Diagram from Matlab program [8];

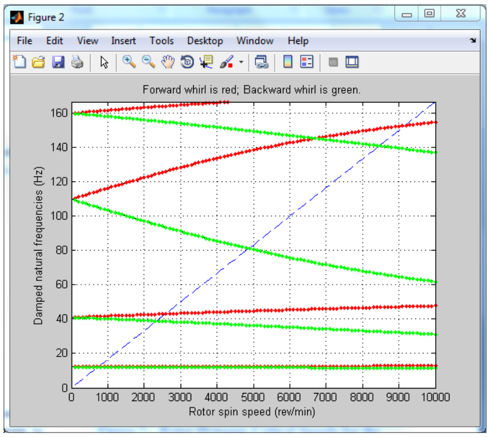

Figure 6
Rotor-Dynamic Critical Speeds for the Last Run

These are the modes and orbits for the last run, out of thirty runs, from Matlab program [8];

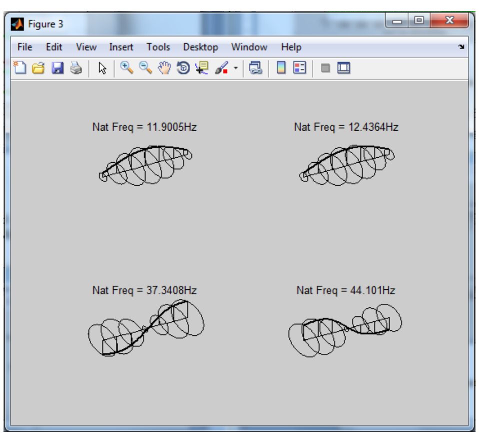

Figure 7
Rotor-Dynamic Modes and Orbits for the Last Run

After obtaining the critical speeds of the Rotor from Matlab, these values are going to be used at the Response, or **(y),** to generate the fitted regression model equation in the Design Expert analysis program.

Table 3
Rotor-Dynamics Critical Speeds

The Critical Speeds	
1	722.234
2	705.2498
3	691.1889
4	670.1276
5	751.7959
6	741.7897
7	745.4636
8	728.1712
9	742.8322
10	730.2939
11	728.1712
12	700.2781
13	766.8693
14	778.0921
15	691.0669
16	728.1712
17	719.8595
18	686.3806
19	728.1712
⁞	⁞
30	709.1222

Coefficients for the Critical Speed Equation

Using the Center Composite design study method in the Design Expert Program [9], the user can obtain the coefficients for the critical speed equations applied to four different transformations;

The first transformation is the ***Standard Quadratic Model***, where table 4, represents the results summary.

Table 4
Standard Transformation

Standard Quadratic	
Std. Dev.	0.592559
Mean	734.6692
C.V. %	0.080657
PRESS	30.33728
R-Squared	0.999853522
Adj R-Squared	0.99971681
Pred R-Squared	0.999156288
Adeq Precision	331.7994614

The second transformation is the ***Square Root Model***, where table 5, represents the results summary.

Table 5
Square Root Transformation

Square Root	
Std. Dev.	0.011388968
Mean	27.09732678
C.V. %	0.042029858
PRESS	0.011206822
R-Squared	0.999839506
Adj R-Squared	0.999689712
Pred R-Squared	0.999075555
Adeq Precision	317.7159404

The third transformation is the ***Natural Log Model***, where table 6, represents the results summary.

Table 6
Natural log Transformation

Natural Log	
Std. Dev.	0.000911674
Mean	6.598322976
C.V. %	0.013816756
PRESS	7.18113E-05
R-Squared	0.999809601
Adj R-Squared	0.999631896
Pred R-Squared	0.998903303
Adeq Precision	292.3202478

The fourth transformation is the ***Base 10 Log Model***, where table 7, represents the results summary.

Table 7
Base 10 log Transformation

Base 10 Log	
Std. Dev.	0.000395935
Mean	2.865615258
C.V. %	0.013816756
PRESS	1.35445E-05
R-Squared	0.999809601
Adj R-Squared	0.999631896
Pred R-Squared	0.998903303
Adeq Precision	292.3202478

Figure 8, represents the behavior of the four different transformations in comparison with the Matlab output critical speeds.

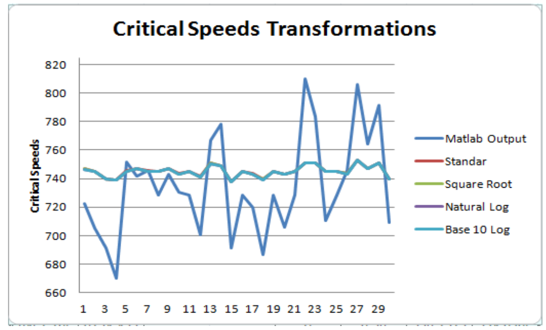

Figure 8
Transformation's Behavior

In figure 9, represents the average error of the four different transformations.

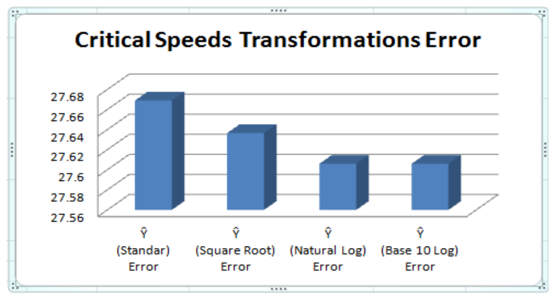

Figure 9
Transformation's Average Error

Table 8, is the selected table with the coefficients for the final equation, since it represents the lowest average % error.

Table 8
Base 10 log Transformation

Factor	Coefficient Estimate	Standard Error
Intercept	2.86223	1.616E-04
A-Disk1_od	-0.00519	8.082E-05
B-Disk2_od	-0.00694	8.082E-05
C-Disk1_len	-0.01180	8.082E-05
D-Disk2_len	0.01699	8.082E-05
AB	0.00019	9.898E-05
AC	-0.00051	9.898E-05
AD	-0.00028	9.898E-05
BC	0.00033	9.898E-05
BD	0.00029	9.898E-05
CD	-0.00084	9.898E-05

A^2	-0.00002	7.560E-05
B^2	0.00002	7.560E-05
C^2	0.00150	7.560E-05
D^2	0.00273	7.560E-05

So, with the critical speed error chart, the final equation will be the Base 10 Log transformation Refer to "(3)";

$$\hat{y} = 10\text{^}(2.8622 - 0.005186x_1 - 0.005186x_2 - 0.01180x_3 + 0.01699x_4 - 2.4553E-05(x_1)^2 + 2.11035E-05(x_2)^2 + 0.001503(x_3)^2 + 0.00272(x_4)^2 + 0.0001874x_1x_2 - 0.0005084x_1x_3 - 0.0002805x_1x_4 + 0.000334x_2x_3 + 0.0002860x_2x_4 - 0.0008440x_3x_4 \quad (3)$$

After generating an equation for the critical speeds, The Figure 10, is the comparison between the final equation and the Matlab program for Critical the Speeds.

Variables					Critical Speeds		
	disk1_od (0.27-0.29)	disk2_od (0.34-0.36)	disk1_length (0.4-0.6)	disk2_length (0.9-1.1)	y	Ŷ (Base 10 Log)	Ŷ (Base 10 Log) Error
1	0.29	0.36	0.6	1.1	722.2340317	746.7097824	24.47575071
2	0.28	0.37	0.5	1	705.2497864	744.7716784	39.52189204
3	0.28	0.33	0.5	1	751.7959352	745.2119704	6.583964868
4	0.27	0.36	0.6	1.1	741.7897467	746.9074116	5.11766494
5	0.26	0.35	0.5	1	745.4635595	745.1862917	0.277267775
6	0.28	0.35	0.5	1	728.1712243	744.9917774	16.82055306
7	0.29	0.34	0.6	1.1	742.8321774	746.9282681	4.09609069
8	0.29	0.34	0.4	0.9	730.293937	743.2017147	12.90777764

Figure 10

% Error for Rotor-Dynamic Critical Speeds

With the final results, figure 11, is the graph for the comparison between the Base 10 log transformation and the Matlab Program for the Critical Speeds

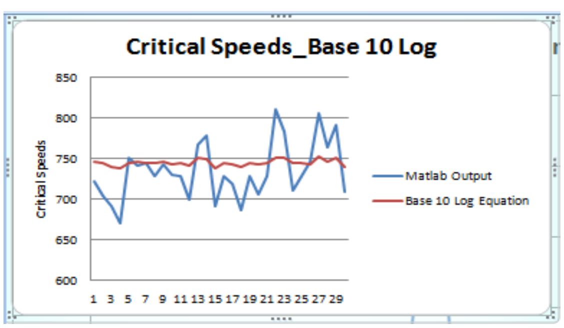

Figure 11

Graph Between Generated Equation & Matlab Program

This results show that there is a large margin of error between the generated equation and the Matlab program for the Critical Speeds, this error is generated because this analysis runs by changing four different variables. The maximum percentage of error is 27.6% and is evaluated for all the data.

CONCLUSION

The equation to calculate the critical speeds for the rotor-dynamics system changing was generated, but it wasn't a complete success, because the system is very unstable, which makes it difficult to obtained a more precise equation.

All of this was possible by using a modified Matlab program [8] and Design-Expert software [9]. Just by obtaining different critical speeds with the modified Matlab program [8] and using the Response Surface Methodology of the Design-Expert software [9], it is the best procedure to follow.

Generating an equation that is simple to use can save a lot of time in order to know the critical speeds of a rotor-dynamics system with the properties of the example presented.

Also, by working with four transformations, it can also be concluded that the Base 10 log transformation with the least error in critical speeds in comparison to the critical speeds obtained in Matlab program [8], was more useful and effective because of its % of error.

RECOMMENDATIONS

This equation generated by the Base 10 log transformation makes the work of find the critical speeds for a rotor-dynamic system a lot easier, but by elaborating another equation with other method of Response Surface with using optimization method for the rotor, can improve the response of the critical speeds of the system. That way it can be more sophisticated and obtain a better response of the system. Also by identifying different methods to analyze an unstable system, the equation can be successful, since it will obtain greater and more precise results.

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