Parallel Plates Internal Heat Transfer Analysis for Electronic Cooling

Alex R. Meléndez Colón
Mechanical Engineering
Bernardo Restrepo Torres, PhD
Mechanical Engineering Department
Polytechnic University of Puerto Rico

Abstract—*Two-dimensional heat transfer analysis using the Integral Equation Method between a series of parallel plates with uniform surface heating to determine the Temperature Distribution as a Thermal Model for a Supercomputer cluster arrangement at the Department of Electrical Engineering of Polytechnic University of Puerto Rico was developed. The Temperature and Velocity Profiles across the arrangement were measured and analyzed.*

KeyTerms—*Forced Convection Heat Transfer, Parallel Plates, Temperature Distribution, Turbulent Flow.*

INTRODUCTION

The Department of Electrical Engineering at the Polytechnic University of Puerto Rico has developed a supercomputer using a series of parallel clusters called LittleFE®. LittleFe® is a complete 6 node Beowulf style portable computational cluster which supports shared memory parallelism (OpenMP), distributed memory parallelism (MPI), and GPGPU parallelism (CUDA), Figure 1.

Figure 1
Supercomputer, LittleFE® Cluster Arrangement

LittleFe® began as an idea by Paul Gray (University of Northern Iowa), Dave Joiner (Kean University), Tom Murphy (Contra Costa College), and Charlie Peck (Earlham College) in 2005. While they had been teaching computational science education, they realized that their curricula depended on local computing resources that were not always present.

Higher heat density and increase heat dissipation is a major concern related to electronic cooling of this device. Also, heat dissipation techniques are of prime concern to remove the waste heat produced by electronic components to keep them within adequate operating temperature. Heat dissipation techniques include air cooling fans, heat sinks, and other forms as liquid cooling.

Also, the high thermal stresses in the solder joints of the electronic components mounted on circuit boards resulting from temperature variations are major causes of failures. Therefore, thermal control has become increasing important in the design and operation of electronic equipment.

This paper presents a thermal analysis of forced-air cooling to determine if it is adequate or not, and if not, suggest alternatives to obtain an adequate thermal environment.

FLOW MODEL

Figure 2 shows a sample of a LittleFE® cluster installed in the computer. It can be seen that most of the electronic components are so closed together i.e. the spacing between them is so small compare to its size, that an analysis with spaced heat sources is not justified.

For simplicity, a forced internal turbulent flow in a rectangular duct with a heated surface will be made.

Figure 2
Front View of a LittleFE® Cluster Board

In general, both the velocity and temperature fields develop simultaneously [1], [2]. This flow situation is shown on Figure 3.

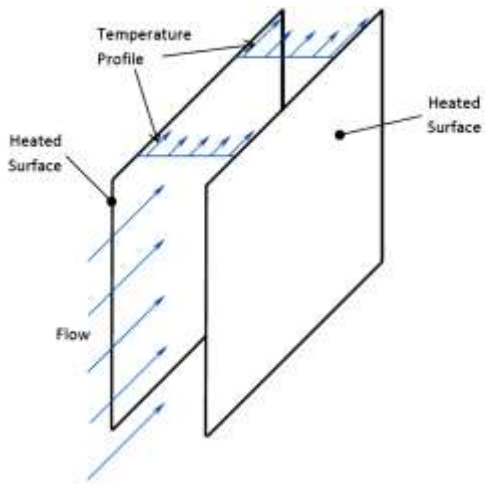

Figure 3
Two – Dimensional Flow in a Plane Duct

ASSUMPTIONS

It will be assumed that the flow enters the duct through a "shaped" unheated inlet section and that the velocity and temperature are uniform across the inlet plane as illustrated in Figure 4.

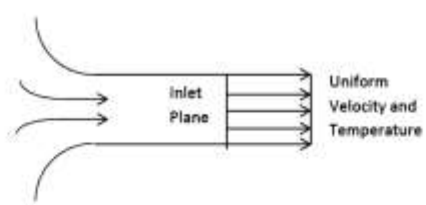

Figure 4
Assumed Inlet Plane Conditions

The air is assumed to be an ideal gas with constant properties. The flow in the development region initially consists essentially of a boundary layer on each wall with a constant velocity, uniform temperature core between these two boundary layers.

These boundary layers grow until they meet on the center line of the duct, [3]. Following this, there is a region where the flow near the center line adjusts in the outer part of a boundary layer to a fully developed flow. However the changes in this second region are relative small and only the first boundary region will be considered here. The analysis presented in this paper is based on the use of the integral equation method.

TEMPERATURE DISTRIBUTION

As the boundary layers grow on the wall, the velocity near the wall is decreased and, as a consequence, the velocity in the core region, u_1, increases. Because the velocity in the inlet plane is uniform, the velocity on the inlet plane must be equal to the mean velocity, $\bar{u}_m$, in the duct. Continuity therefore requires, assuming that the density is constant, that:

$$\rho \bar{u}_m W = \rho \int_0^W \bar{u} dy \qquad (1)$$

where W is the width of the duct.

In spite of the non-symmetrical distribution of the components in the plate, it is assuming that the flow is symmetrical about the center line of the duct and that the velocity is uniform in the core region between the two boundary layers as shown in Figure 5, (1) can be written as:

$$\rho \bar{u}_m W = 2\rho \left[\int_0^\delta \bar{u} dy + \left(\frac{W}{2} - \delta\right) u_1\right] \qquad \therefore$$

$$\rho \bar{u}_m W = 2\rho \left[\frac{W}{2} u_1 - \int_0^\delta (u_1 - \bar{u}) dy\right] \qquad \therefore$$

$$\bar{u}_m W = W u_1 - 2 \int_0^\delta (u_1 - \bar{u}) dy \qquad (2)$$

Let δ_1 be the boundary layer displacement thickness and δ_2 the boundary layer momentum thickness, Figure 5, then:

$$\delta_1 = u_1 \int_0^{\delta} \left(1 - \frac{\bar{u}}{u_1}\right) dy \quad (3)$$

and:

$$\delta_2 = u_1 \int_0^{\delta} \left(1 - \frac{\bar{u}}{u_1}\right) \frac{\bar{u}}{u_1} dy \quad (4)$$

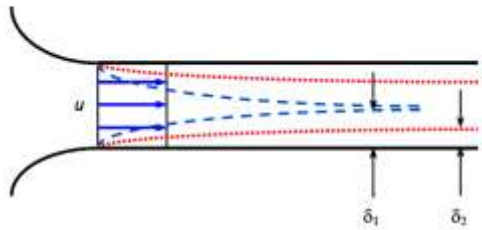

Figure 5
Boundary Layer Displacement Thickness δ_1 and Boundary Layer Momentum Thickness δ_2.

Now, (2) can be written in term of the displacement thickness as:

$$u_1(W - 2\delta_1) = \bar{u}_m W \quad (5)$$

The right-hand of (5) is a constant, therefore:

$$(W - 2\delta_1)\frac{du_1}{dz} - 2u_1\frac{d\delta_1}{dz} = 0 \therefore$$

$$\frac{1}{u_1}\frac{du_1}{dz} = \frac{2}{(W-2\delta_1)}\frac{d\delta_1}{dz} \quad (6)$$

This equation relates the gradient of the velocity in the core region to the rate of growth of the boundary layer displacement thickness. The momentum thickness, δ_2, is related to the displacement thickness by the so called form factor, H, which is defined by:

$$H = \frac{\delta_1}{\delta_2}$$

Equation (6) can therefore be written as:

$$\frac{1}{u_1}\frac{du_1}{dz} = \frac{2}{(W-2H\delta_2)}\frac{d}{dz}(H\delta_2) \therefore$$

$$\frac{1}{u_1}\frac{du_1}{dz} = \frac{2}{(W-2H\delta_2)}\left(\delta_2\frac{dH}{dz} + H\frac{d\delta_2}{dz}\right) \quad (7)$$

Very approximate relations for the variations of the wall shear stress in terms of H are given by, [4]:

$$\frac{C_f}{2} = \frac{\tau_W}{\rho u_1^2} = \frac{0.123\times 10^{-0.678H}}{\left(\frac{u_1\delta_2}{v}\right)^{0.268}} \quad (8)$$

whereC_f is the local shearing stress coefficient, andτ_w is the wall shearing stress, and:

$$\left(\frac{u_1\delta_2}{v}\right)^{\frac{1}{6}}\delta_2\frac{dH}{dz} = e^{5(H-1.4)}\left[-\left(\frac{u_1\delta_2}{v}\right)^{\frac{1}{6}}\delta_2\frac{1}{u_1}\frac{du_1}{dz} - 0.135(H-1.4)\right] \quad (9)$$

These two equations effectively constitute the turbulence model.To approximate the shear stress distribution, the integral momentum equation is used [4], written as:

$$\frac{d}{dz}\left[\int_0^{\delta}(u_1 - u)u\, dy\right] + \frac{du_1}{dz}\left[\int_0^{\delta}(u_1 - u)\, dy\right] = \frac{\tau_W}{\rho} \quad (10)$$

Equation (10) is equally applicable to laminar and turbulent flow. For the present purpose, using (3) and (4), (10) can be conveniently written as:

$$\frac{d\delta_2}{dz} + \delta_2(2 + H)\frac{1}{u_1}\frac{du_1}{dz} = \frac{C_f}{2} \quad (11)$$

Equations (8), (9), (10), and (12) can be simultaneously solved to give the variation of δ_2, H, and u_1 with z. The solutions can be used to obtain the heat transfer rate, giving:

$$\frac{T_W - T_i}{u_1} = \frac{q_W}{c_p\tau_W} \quad (12)$$

it being note that the temperature in the core region between boundary layer will be equal to the temperature on the inlet plane, i.e., T_i. Equation (12) was derived using the assumption that Pr(i.e. the Prandtl Number) is equal to 1. The solution applies until the boundary layer reaches the center line of the duct, i.e. when

$$\delta = ½W \quad (13)$$

Assuming a power law distribution [3] in the boundary layer, this is:

$$\frac{\bar{u}}{u_1} = \left(\frac{y}{\delta}\right)^{\frac{1}{n}} \quad (14)$$

where*n* is an integer constant, then using (4), it follows that:

$$\frac{\delta_2}{\delta} = \int_0^1 \left[\left(\frac{y}{\delta}\right)^{\frac{1}{n}} - \left(\frac{y}{\delta}\right)^{\frac{2}{n}}\right] d\frac{y}{\delta} \quad (15)$$

i.e., that:

$$\frac{\delta_2}{\delta} = \frac{n}{(n+1)(n+2)} \quad (16)$$

Similarly, using the definition of the displacement thickness it follows that:

$$\frac{\delta_1}{\delta} = \int_0^1 \left[1 - \left(\frac{y}{\delta}\right)^{\frac{1}{n}}\right] d\frac{y}{\delta}$$

i.e., that:

$$\frac{\delta_1}{\delta} = \frac{1}{n+1} \tag{17}$$

Dividing (17) by (16),we obtain:

$$H = \frac{n+2}{n}$$

solving for n, we obtain:

$$n = \frac{2}{H-1}$$

substituting this expression into (16) then gives:

$$\frac{\delta_2}{\delta} = \frac{H-1}{H(H+1)} \tag{18}$$

For any value of H, (18) allows δ to be found provided the value of δ_2 has been determined. Equation (13) can then be used to determine if the boundary layer has reached the center line.

It is convenient to write the above equations in dimensionless form before obtaining the solution. For this purpose, the following variables are defined:

$$U = \frac{u_1}{u_m} \tag{19}$$

$$Z = \frac{z}{W} \tag{20}$$

$$\Delta_2 = \frac{\delta_2}{W} \tag{21}$$

$$\Delta = \frac{\delta}{W} \tag{22}$$

Equations (7), (8), (9), and (11) can be written in terms of (19) – (22) as:

$$\frac{1}{U}\frac{dU}{dZ} = \frac{2}{(1-2H\Delta_2)}\left[\Delta_2\frac{dH}{dZ} + H\frac{d\delta_2}{dz}\right]$$

$$\frac{d\Delta_2}{dZ} + \Delta_2(2+H)\frac{1}{U}\frac{dU}{dZ} = \frac{\tau_W}{\rho u_1^2}$$

$$\frac{\tau_W}{\rho u_1^2} = \frac{0.123 \times 10^{-0.678H}}{Re_w^{0.268}(U\Delta_2)^{0.268}}$$

$$Re_w^{1/6}(U\Delta_2)^{\frac{1}{6}}\Delta_2\frac{dH}{dZ} = e^{5(H-1.4)}\left[-Re_W^{\frac{1}{6}}(U\Delta_2)^{\frac{1}{6}}\frac{\Delta_2}{U}\frac{dU}{dZ} - 0.0135(H-1.4)\right]$$

where:

$$Re_w = \frac{u_m W}{v}$$

Equation (13) indicates that the solution must ended when

$\Delta = 0.5$

and (18) gives:

$$\Delta = \frac{H(H+1)}{H-1}\Delta_2$$

Equation (12) gives the heat transfer rate as:

$$Nu_w = \frac{\tau_w}{\rho u_1^2} U Re_w Pr^{0.4}$$

an approximate correction to account for the fact that Pr is not equal to 1 having been applied. Then,

$$Nu_W = \frac{q_W W}{k(T_W - T_i)}$$

and,

$$Re_W = \frac{u_m W}{v}$$

The mean temperature T_m, across any section in the duct is given by:

$$u_m(T_m - T_i)W = 2\int_0^{\frac{W}{2}} \bar{u}(\bar{T} - T_i)dy$$

i.e., since T is equal to T_i outside the boundary layer:

$$u_m(T_m - T_i)W = 2\int_0^{\delta} \bar{u}(\bar{T} - T_i)\,dy$$

therefore:

$$\frac{T_m - T_i}{T_W - T_i} = 2\Delta\int_0^1 U\left(\frac{\bar{T} - T_i}{T_W - T_i}\right) d\left(\frac{y}{\delta}\right) \tag{23}$$

Assuming that:

$$\frac{\bar{u}}{u_1} = \left(\frac{y}{\delta}\right)^{\frac{1}{n}}$$

and also assuming:

$$\frac{\overline{T} - T_i}{T_W - T_i} = 1 - \left(\frac{y}{\delta}\right)^{\frac{1}{n}}$$

Equation (23) gives:

$$\frac{T_m - T_i}{T_W - T_i} = \frac{2n}{(n+1)(n+2)}\Delta \quad (24)$$

Therefore, since:

$$n = \frac{2}{H-1}.$$

Equation (24) allows $(T_m – T_i)/(T_w – T_i)$ to be found, and therefore, the temperature distribution.

PROCEDURE

The velocity profile and the temperature distribution was obtained by measuring the speed and temperature at discrete points at the exit of the flow field using a uniform space grid at the ambient temperature of the room, as shown on Figure 6. The intersection of each horizontal line with a vertical line represents a nodal point where the temperatures were measured.

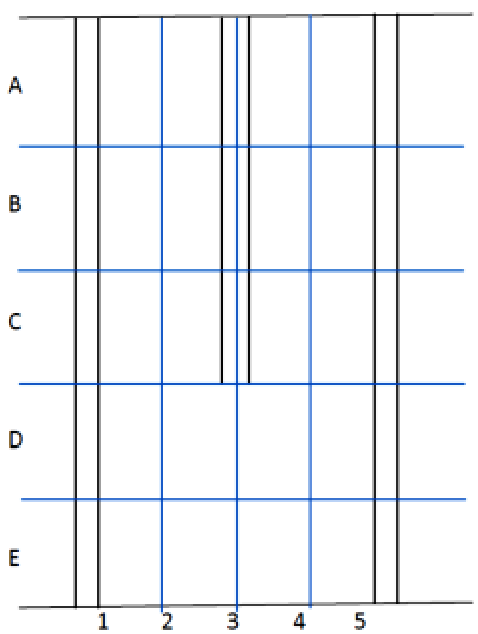

Figure 6
Two – dimensional Grid for Temperature Distribution

RESULTS AND DISCUSSION

Table 1 shows the actual Temperature Profile. As shown in Figure 2, two parallel aluminum plates have a cluster between them, i.e. a heat generating source. Also the cluster is above the bottom of the computer housing, leaving an empty space with no sources of heat, but radiation.

As pointed out by Incropera (2011), radiation heat transfer is negligible for polished metals because of their very low emissivity and for bodies surrounded by surface by about the same temperature. The radiation heat transfer is disregarded for two reasons. First, the forced convection heat transfer is usually much larger than that due to radiation in cooling of electronic devices. Second, the electronic components and circuit boards in convection – cooled systems are mounted so closed to each other that a component is almost entirely surrounded by other components at about the same temperature.

Table 1
Actual Temperature Profile (°C) at T_{amb} = 24°C

	1	2	3	4	5
A	27	28	28	28	27
B	27	29	29	29	27
C	27	28	28	29	27

Table 2 shows the results of a Matlab Code implementing the above procedure. The values of du_1/dz are relative small in the entrance region and results obtained, assuming H is constant, agree quite closely with those obtained allowing for variations in H.

Table 2
Theoretical Values for the Temperature Profile Variables

Mean Temperature T_m (°C)				
A	B	C	D	E
26.50	26.48	26.46	26.45	26.44
Boundary Layer Thickness (cm)				
0	0.024	0.041	0.054	0.068
Dimensionless Parameter Δ (Adimensional)				
0	0.037	0.064	0.084	0.105
Dimensionless Parameter Δ_2 (Adimensional)				
0	0.0044	0.0076	0.0099	0.0125
Reynolds Number (Adimensional)				
3099	3444	4133	5166	5166

A careful examination of the equations reveals that the independent variables are the Reynolds Number (Re) and the geometric variables. The independent geometric variables are the dimensionless parameter Δ the ratio of δ to W, the dimensionless parameter Δ_2 the ratio of δ_2 to W, and the width of the duct$W$.

In order to decide what value of n to use in (24) experiments indicates that for flow over a flat plate H = 1.4, [3]. For the air at 300 K (average temperature measured) the kinematic viscosity ν is approximately 15.68×10^{-6} m^2/s (Incropera,. 1996). The actual duct size measures 54 cm.

The Reynolds Number distribution, as expected, shows turbulent flow, i.e. Re > 2300. This turbulent flow is caused by the non-symmetrical distribution of the components in the cluster plate, which is between the two parallel plates.

The center line parameter δ indicates the limit if the boundary layer reaches the center of the duct, in this case δ = 2.70 cm. As indicated by the dimensional parameter Δ, with a maximum value of 0.105, the boundary layer never reaches the center of the duct, i.e. Δ< 0.5. The maximum calculated value of the boundary layer thickness is 0.068 cm which also confirms the boundary layer never reaches the center of the duct, i.e. δ_A< 2.70 cm.

The temperature difference ratio, i.e. (24), compares the difference between the mean temperature T_m, the temperature at the wall T_w with the temperature of the inlet planeT_i = 26.5°C. Results shows theoretical ratios are very small indicating very close temperature values. The Theoretical Temperature Profile is shown in Figure 7.

The Mean Temperature Profile is quasi-linear and for turbulent flow (i.e. Re > 2300) confirms the temperature in the core region is equal to the temperature on the inlet plane, which is the actual temperature outside the boundary layer.

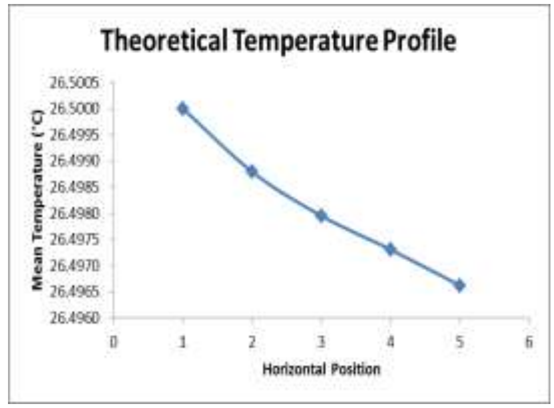

Figure 7
Theoretical Temperature Profile

Inconsistencies with the Theoretical Temperature Profile and the Actual Temperature Profiles are shown in Figures 8, 9, and 10. These inconsistencies is due to actual configuration of the system under consideration and the assumption made during the analysis, two parallel plates without a plate in between as a source of heat and turbulence.

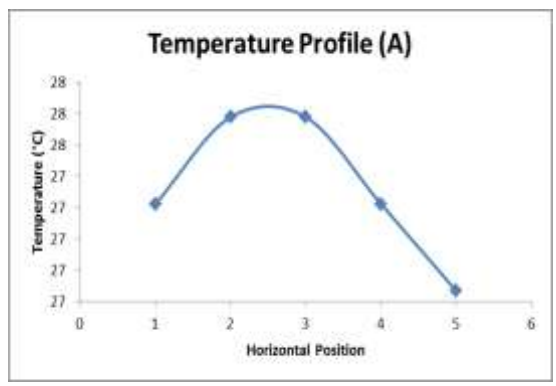

Figure 8
Temperature Distributionin Horizontal Plane A

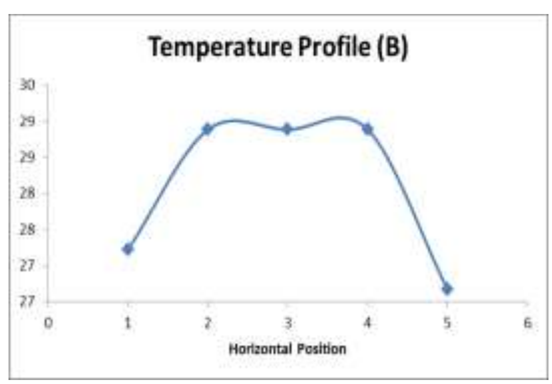

Figure 9
Temperature Distribution in Horizontal Plane B

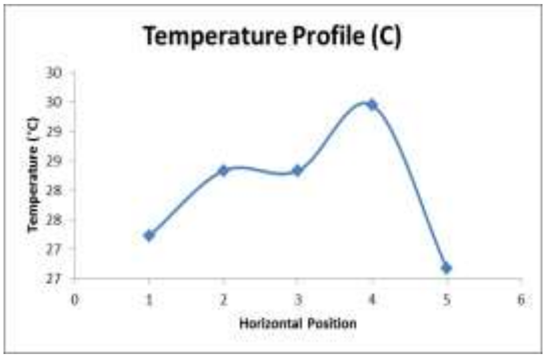

Figure 10
Temperature Distribution in Horizontal Plane C

CONCLUSION AND RECOMMENDATIONS

In spite of the inconsistencies of the theoretical and actual temperature distributions, the thermal analysis done can help determine if the thermal environment in the equipment is adequate or not.

As shown on Table 1, the maximum temperature achieved by the air is 29°C and a temperature difference between the exit and inlet air of 5C°. This is a heat transfer rate equal to 5 kJ/kg_{air}. Avram Bar-Cohen from University of Minnesota, Abhay A. Watwe and Ravi S. Prasher from Intel Corporation at Chandler, Arizona had established, based on reliability and performance considerations, a standard limit range of 65° to 85°C in commercial applications (non-military electronic systems), thus having allowable temperature rise above the ambient 45°C.

Therefore, the system is considered operating at an adequate thermal environment and no changes are recommended. A natural convection cooling environment is not recommended due to the computer housing configuration of the installed components. However, a natural convection study is recommended to confirm this.

REFERENCES

[1] Deissler, R. G., "*Turbulent Heat Transfer and Friction in the Entrance Regions of Smooth Passages*", Trans, ASME, Vol. 77, 1955, pp. 1211 – 1234

[2] Deissler, R.G., "*Analysis of Turbulent Heat Transfer and Flow in the Entrance Regions of Smooth Passages*", NACA TN, 3016, 1953.

[3] Oosthuizen, Patrick H., & Naylor, David, "*Convective Heat Transfer Analysis*", First Edition, 1999.

[4] Schlichting, H., "*Boundary Layer Theory*", 7th Edition, 1979.