Parametric Study of Lift-Drag Coefficients vs. Geometric Parameters of Axial Compressor/Turbine for Flow Blade Interactions

Diolinete del C. Gerena Colón
Mechanical Engineering
Bernardo Restrepo, PhD.
Department of Mechanical Engineering
Polytechnic University of Puerto Rico

Abstract—*This project is a parametric study of the Lift-Drag coefficients from blade hub to tip radius of Axial Compressor/Turbine for flow blade interactions. The study contemplates the effect of Lift and Drag forces varying the inlet blade angle and the response with an actuator Inlet Guide Vane (IGV). In the turbine section, the Lift and Drag forces are also studied varying the inlet blade angle for the rotor and the effect with different exit nozzle angles. Lift force is used to convert energy from the shaft to the fluid and vice versa while drag is the energy losses due to friction. The desired effect was found at lower inlet blade angle. The study was done using the fundamental turbomachinery equations and the compressible flow properties.*

KeyTerms—*Axial Compressor/Turbine, Drag Coefficient, Inlet Guide Vane, Lift Coefficient, Turbomachine.*

INTRODUCTION

An important goal of an aerodynamic study is to determine the forces acting on the system. The project scope is the study of the Lift and Drag coefficients response as a function to the radius discretizing from hub to tip radius, as shown in Figures 1 and 2. The first part of the study includes the effect of the IGV in the compressor Lift and Drag forces, with different rotors. The second part contemplates the response of the turbine Lift and Drag forces while the inlet relative fluid angle changes with different rotors.

The relative tangent velocity and pressure change across the rotor are the mainly components of the Lift and Drag forces that represent the system energy.

The Figure 3 represents the effect in the velocity triangle in the entrance and the exit of the compressor rotor. Figure 3 (a) is the responses while the IGV varying from 0˚ to 30˚. The thicker line shows the velocity triangle with IGV equal to cero, so the air enters to the compressor purely axial. Figure 3 (b) presents the triangle velocity variation from hub to tip radius. The absolute velocity remains constant and all lines represent the variation of relative velocity (W_1) as a function to the radius. The thicker line shows the velocity triangle sketch at tip radius. Figure 4 shows the effect in the velocity triangle in the turbine. Figure 4 (a) demonstrates the effect of changing the angle in the exit nozzle from 40˚ to 60˚. The relative inlet angle remains constant so the lines represent the change in the absolute velocity components. The thicker line represents the effect of an exit nozzle angle of 40˚. Figure 4 (b) displays the turbine velocity triangle as a function of the radius, where the thicker line represents the velocities magnitudes at the tip.

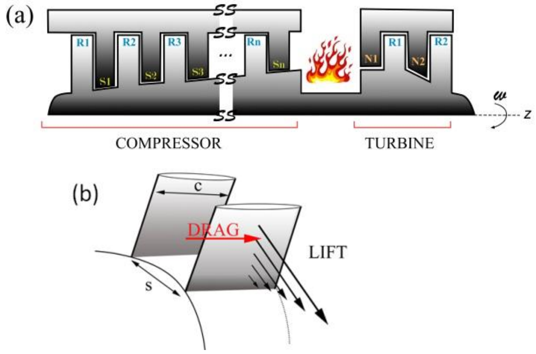


Figure 1

(a)Aircraft Engine; Compressor (Rotors and Stators) and Turbine (Nozzles and Rotors) (b) Lift and Drag Forces Acting on the Blade

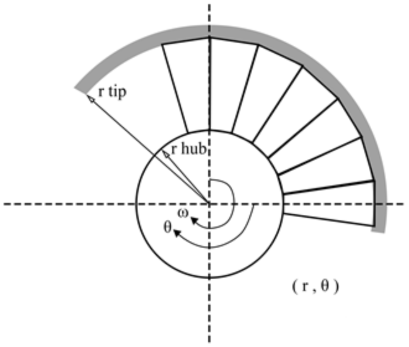


Figure 2
Sketch of a Rotor with the Hub and Tip Radius

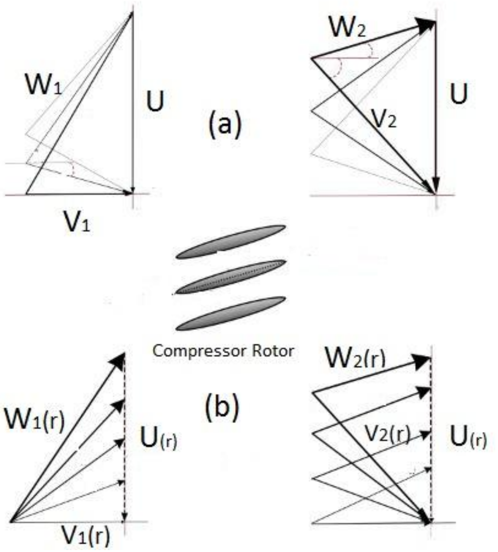


Figure 3
Diagram of Compressor Velocity Triangles, (a) while IGV Acts on the Compressor Rotor (b) as a Function of the Radius in the Compressor Section

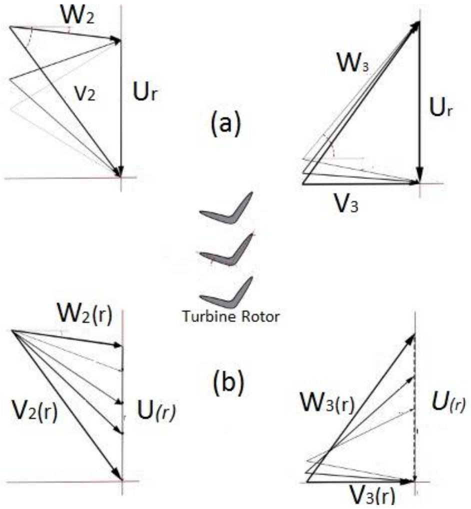


Figure4
Diagram of Turbine Velocity Triangles, (a) Varying the Exit Angle in the Turbine Nozzle (b) as a Function of the Radius.

LIFT AND DRAG FORCES

Lift force is the sum of the projections of the axial (z) and tangential blades forces in a direction normal to the average flow direction. The Drag force is the sum of the projections of the axial and tangential blade forces in a direction parallel to the average flow direction as shown in Figure 5.

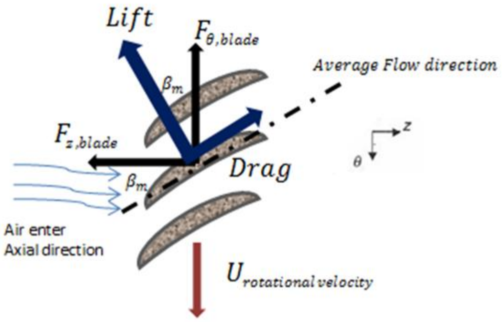


Figure 5
The Axial and Tangential Blade Forces Acting on a Compressor Blade

The axial blade force is due to the pressure change, assuming a constant axial through-flow speed. From the Newton's third law of motion the blade force is in opposite direction of the fluid force in the axial direction and the resulting Equation (1) is:

$$F_{z|fluid} = -F_{z|blade} = s\,(P_2 - P_1) \qquad (1)$$

The tangential blade force is opposite to the rotation and from conservation of tangential momentum it is (2)

$$F_{\theta|fluid} = -F_{\theta|blade} = mfr\,(W_{\theta 1} - W_{\theta 2}) \qquad (2)$$

where mfr is the mass flow rate.

Solving the axial and tangential blade forces into a Lift and Drag components [1], in force-unit/length-unit, getting Equations (3) and (4)

$$Lift = F_{\theta|blade}\cos\beta_m + F_{z|blade}\sin\beta_m \qquad (3)$$

$$Drag = F_{\theta|blade}\sin\beta_m - F_{z|blade}\cos\beta_m \qquad (4)$$

Where a mean flow direction β_m is based on the average swirl in the blade row from Equation (5),

$$\tan\beta_m \equiv W_{\theta m}/W_z \qquad (5)$$

The average swirl $W_{\theta m}$, and the average W in the rotor blade is defined by Equations (6) to (8)

$$W_{\theta m} \equiv (W_{\theta 1} + W_{\theta 2})/2 \,; \quad \text{for compressor} \qquad (6)$$

$$W_{\theta m} \equiv (W_{\theta 2} - W_{\theta 1})/2 \,; \quad \text{for turbine} \qquad (7)$$

$$W_m \equiv (W_1 + W_2)/2 \qquad (8)$$

Equations (5) to (8) are the results of the geometric analysis for the relative velocities inlet and outlet of the rotor. Figure 6 shows the relative velocity relation in the compressor section and Figure7 the relative velocity relation in the turbine. Velocities components are positive at the same direction of the rotation. The differences in (6) and (7) are explained in Figures 6 and 7.

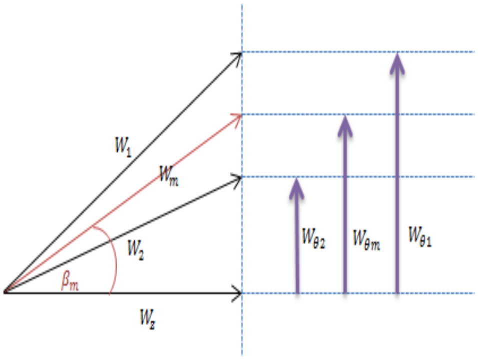


Figure 6
Compressor Relative Velocity Components

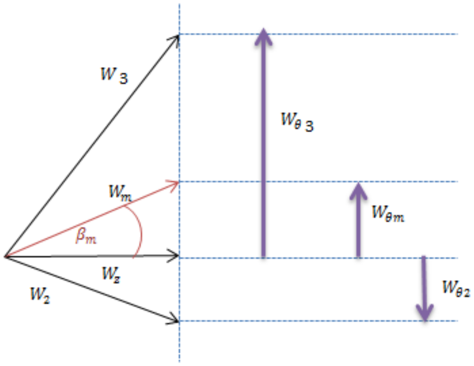


Figure 7
Turbine Relative Velocity Components

Substituting the velocity components in the Lift and Drag forces equations got (9) and (10)

$$Lift = \rho W_m s\,(W_{\theta 1} - W_{\theta 2}) - s\,(Po_1 - P_{o2})(\frac{W_{\theta m}}{W_m}) \qquad (9)$$

$$Drag = (s)\,(Po_1 - P_{o2})\,(\frac{W_z}{W_m}) \qquad (10)$$

The Lift and Drag coefficients (C_l and C_d)are non-dimensional expression equal to Equations (11) and (12)

$$C_l = {}^{Lift}\Big/_{(\rho\, c\, \frac{W_m^2}{2})} \qquad (11)$$

$$C_d = {}^{Drag}\Big/_{(\rho\, c\, \frac{W_m^2}{2})} \qquad (12)$$

And c is the minimum chord length, see Figure 1.

The turbomachine is an energy conservation device, converting mechanical energy to thermal/pressure energy or vice versa. The conversion is done through the dynamic iteration between a continuously flowing fluid and a rotating machine component. Both momentum and energy transfer are involved [2]. The Lift and Drag forces in compressor and turbine rotors are the results of that energy transfer. It is desired to get the largest Lift force because this implies that the fluid receives the same force. The Drag force is a loss to be overcome by input power [3].

METHODOLOGY

The parametric study of Lift and Drag forces are made with the assumption of homogeneous flow with steady state fluid flow. Also, the fundamental assumption in application of compressible flow is treating the flow as a calorically perfect gas. An important condition is that the axial absolute velocity component, the solidity and the rotor camber angle remain constant along the stages and along the blade radius. The study is made with the fundamentals turbomachinery equations and compute the thermodynamics properties and the dynamics parameters, based on the physical laws governing flowing fluids. The air properties are at 16 km altitude of sea level.

Compressor section Methodology

The National Advisory Commission on Aeronautics (NACA) – 65 Series Cascade Data contemplates compressor rotors inlet blade angles from 30° to

70°; to keep in this range, the study is made with inlet blades angles (β_{b1}in Figure 8) between 40 to 65 degrees, maintaining the camber angle constant along the blade. The purpose of this constrain is to get a flow angle that maintains the boundary layer attached to the blade. Camber angle is the difference of the inlet and outlet blade angle.

The study includes the effect of the IGV in the compressor. The IGV actuators could modulate the inlet flow to the compressor rotor. The inlet swirl angle (alpha 1 in Figure 8) may be zero or adjustable to positive 30° [1], so the study is in this range.

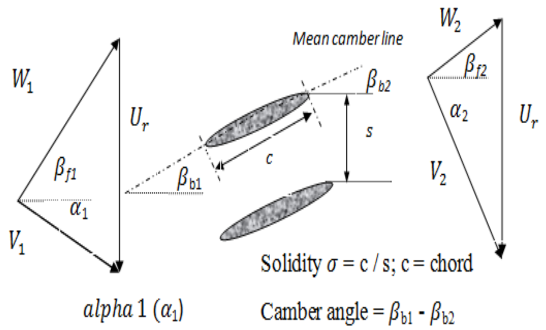


Figure 8
Velocity Triangle for a Compressor Blade

To maintain the relation between inlet and outlet velocities in the compressor section, the De Haller criterion [1] was used and assumed constant across the blade radius. The criterion sets that the relative velocities ratio must be greater or equal than 0.72.

The exit flow angle deviates from the exit blade angle by what is called a deviation angle δ due to boundary layer buildup at the trailing edge. The following correlation is due to Carter [1].

$$\delta = \frac{\Delta\beta_f}{4\sigma^n} + 2^{\circ}\,; \quad \text{Carter's rule} \tag{13}$$

$$\beta_{f2} = \beta_{b2} + \delta \tag{14}$$

In the Equation (13), σ is the solidity and n is ½ for a compressor cascade and 1 for an inlet guide vane (or an accelerating passage such as turbines). In the Equation (14), β_{f2} is the relative fluid angle exiting the rotor and β_{b2} is the blade angle exiting the rotor. The higher exponent of solidity for a turbine (n = 1) than the compressor (n = 1/2) leads to higher exit deviation angles in a compressor, as expected. The boundary layer build up in a compressor in adverse pressure gradient is greater than its counterpart in the turbine [1]. Deviation angle is function of the blade radius because relative fluid angle inlet and outlet are function of radius. To calculate deviation angle, is necessary the value of β_{f2}, which is calculated using the relation generated by the De Haller criterion in the velocity triangle. The relative fluid angle as a function of radius$\beta_{f2}(r)$ is the exit rotor blade angle plus the deviation angle. In this way the exit angle fluid depends on the blade angle that is function of the blade radius.

Another important parameter necessary to calculate the Lift and Drag forces is the total pressure exiting the compressor rotor. This parameter is found solving stage pressure ratio Equation (15), [1] by P_{o2} where e_c is the compressor polytropic efficiency that typically is equal to 90%, γ is the ratio of specific heats, andτ_s is the stage total temperature ratio in the rotor. The inlet total pressureP_{o1}, is the atmosphere pressure and to simplify, it is assumed that the pressure loss across the IGV is insignificant and P_{o1} is considered constant across the IGV.

π_s =Stage pressure ratio for a compressor

$$\pi_s = \frac{p_{o2}}{p_{o1}} = \tau_s^{\gamma e_c/(\gamma-1)} \tag{15}$$

Turbine section Methodology

Carter's rule for deviation angle in a turbine relates the relative inlet and outlet fluid angle in the rotor. See Equation (16). It is not the most accurate relation but is adequate for preliminary design purposes [1].

$$\delta^{*} = \text{deviation angle} = \frac{m\Delta\varphi}{\sigma} \approx \frac{\Delta\varphi}{8\sigma} \approx \frac{\Delta\beta}{8\sigma} \tag{16}$$

The geometrical turbine feature is advantageous to favorable pressure gradient and thus smaller deviation angle [1]. To simplify the study, the camber angle stays constant. An important assumption is that the relative fluid angle entering the rotor (β_{f2}) is equal to the exit blade nozzle angle (β_{nozzle}) and remains constant from hub to tip radius, see Figure 9.

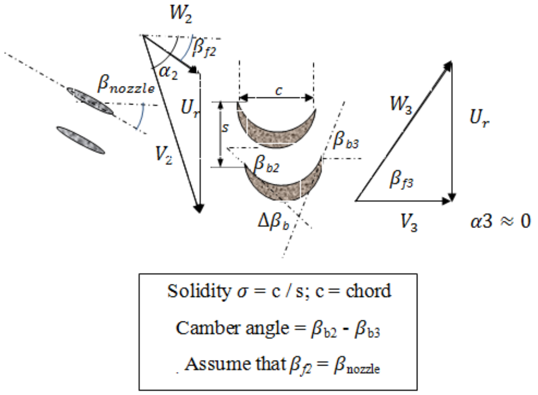


Figure 9
Velocity Triangle for a Turbine Rotor Blade
Tagged as 2 and 3

Therefore, the relative inlet velocity is also constant. The Equation (17) relates exit and entrance rotor conditions given by the exit Mach number that is assumed to increase linearly across the blade radius with the arrangement to get a Mach number value of 0.8 at the pitchline. The relation is given from the conservation of energy in the relative frame combining with the rotor exit Mach number equation in terms of axial and tangential relative velocities. But, because of the assumption of constant inlet relative angle, the analogy of the equation with absolute velocities was necessary and the resultant Equation (17) is

$$V_{\theta 3} = -1 * \sqrt{\frac{M_{3r}^2\left[a_2^2 + \frac{\gamma-1}{2} * V_{\theta 2}^2\right] - V_2^2}{1 + \frac{(\gamma-1)}{2} * M_{3r}^2}} \qquad (17)$$

The subscript 2 refers to the rotor entrance and subscript 3 refers to the rotor exit. It was necessary to apply an absolute value in the numerator of the equation to avoid imaginary values when the calculations are near the hub, it is because of the small Mach number and small absolute tangential velocity compared with the absolute axial velocity in this zone. The result is an abrupt deviation in all the graphs at the beginning, near the hub. With the tangential velocity component $V_{\theta 3}$, is possible to calculate relative exit angle β_{f3} with the velocity triangle relation and then calculated the deviation (16) as a function of the radius. The fluid relative exit angle as athe function of radius $\beta_{f3}(r)$ is the blade angle β_{b3} plus the deviation.

An important parameter to calculate the Lift and Drag forces is the exit total pressure; this is calculated by subtracting a percent of the dynamic pressure, (assuming that the pressure loss percent is constant across the rotor blade), to the total pressure entering the rotor. See Equation (18).

$$P_{o3} = P_{o2} - \text{ pressure loss \% } * (P_{o2} - P_2) \qquad (18)$$

The inlet rotor blade angle and the exit nozzle angle are varying from 40° to 60° . The Lift and Drag coefficients are calculated using the same equations for compressor rotor, but taken in consideration the turbine blades angles signs. It defines as positive the velocity components that projected in the rotation direction and the angles that it forms have the same sign according to the axial axis.

STUDY RESULTS AND CONCLUSIONS

A representative results sample are chosen and tabulated to visualize the tendency in Lift and Drag coefficients while the study parameters are varying, shown in Table 1. The graphs shown represent the limits of the parameters.

Hub to tip radius in meter

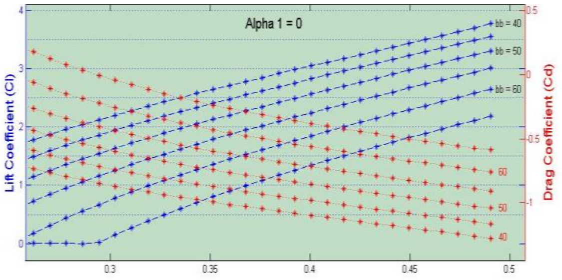
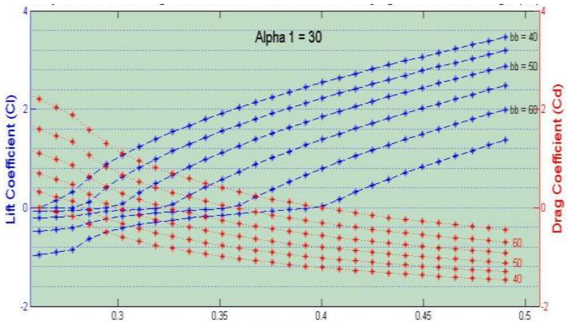
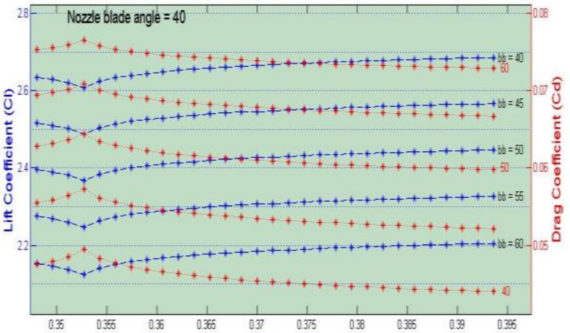


Figure10
Compressor Lift and Drag Coefficients Response without IGV and Rotor Blade Angle from 40° ~ 65°



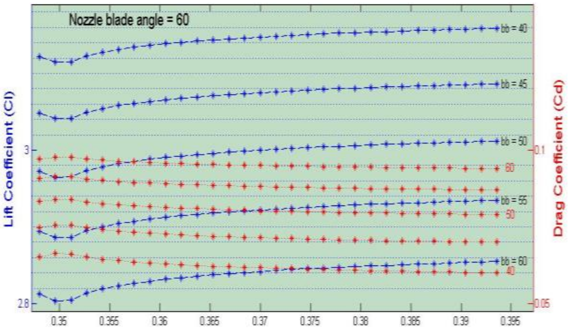
Hub to tip radius in meter

Figure 11
Compressor Lift and Drag Coefficients Response with IGV at Alpha of 30° and Rotor Blade Angle from 40° ~ 65°



Hub to tip radius in meter

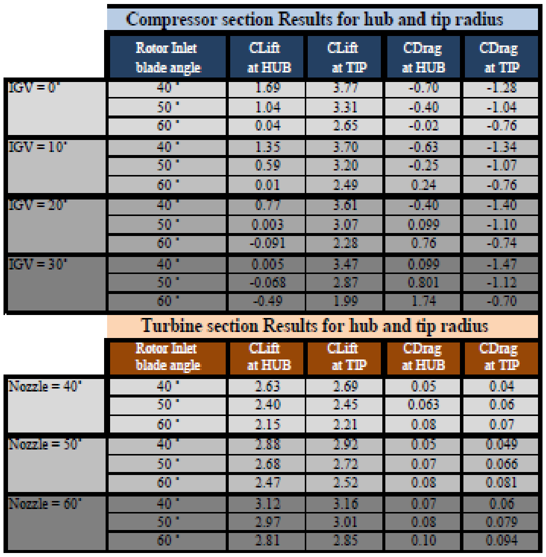
Figure 12
Turbine Lift and Drag Coefficients Response for Nozzle Exit Angle of 40° and Rotor Blade Angle from 40° ~ 60°



Hub to tip radius in meter

Figure 13
Turbine Lift and Drag Coefficients Response for Nozzle Exit Angle of 60° and Rotor Blade Angle from 40° ~ 60°

Table 1
Lift and Drag Coefficients Results for Compressor and Turbine



	Compressor section Results for hub and tip radius				
	Rotor Inlet blade angle	CLift at HUB	CLift at TIP	CDrag at HUB	CDrag at TIP
IGV = 0'	40 '	1.69	3.77	-0.70	-1.28
	50 '	1.04	3.31	-0.40	-1.04
	60 '	0.04	2.65	-0.02	-0.76
IGV = 10'	40 '	1.35	3.70	-0.63	-1.34
	50 '	0.59	3.20	-0.25	-1.07
	60 '	0.01	2.49	0.24	-0.76
IGV = 20'	40 '	0.77	3.61	-0.40	-1.40
	50 '	0.003	3.07	0.099	-1.10
	60 '	-0.091	2.28	0.76	-0.74
IGV = 30'	40 '	0.005	3.47	0.099	-1.47
	50 '	-0.068	2.87	0.801	-1.12
	60 '	-0.49	1.99	1.74	-0.70

	Turbine section Results for hub and tip radius				
	Rotor Inlet blade angle	CLift at HUB	CLift at TIP	CDrag at HUB	CDrag at TIP
Nozzle = 40'	40 '	2.63	2.69	0.05	0.04
	50 '	2.40	2.45	0.063	0.06
	60 '	2.15	2.21	0.08	0.07
Nozzle = 50'	40 '	2.88	2.92	0.05	0.049
	50 '	2.68	2.72	0.07	0.066
	60 '	2.47	2.52	0.08	0.081
Nozzle = 60'	40 '	3.12	3.16	0.07	0.06
	50 '	2.97	3.01	0.08	0.079
	60 '	2.81	2.85	0.10	0.094

Note: For Air mass flow rate of 100 kg/s, overall compressor pressure ratio of 20, turbine cooling mass fraction of 10 %, repeated stage and combustion exit at 2000 K at 1960 kPa.

Compressor Results and Conclusion

For compressor and turbine, the greatest Lift values are found at the tip radius. The goal is to obtain the maximum Lift force. As IGV acts and impart an absolute angle entering the rotor, a negative Lift coefficient appears near the hub. Compare Figures 10 and 11. This is due because the energy generated by the rotation as a function of the radius is too small near the hub. While alpha 1 increases it is more difficult to compress the fluid and the blade circulation. The blade circulation is defined as the space between blades times the change in relative tangent velocities.

Otherwise, the Drag force sign must be negative due of the desired gain pressure during rotor stage, where delta$P_o = P_{o1} - P_{o2}$. But when the entry blade angle increasesβ_{b1}, (see Figure 8) the blade does not compress the air near the hub and the results show a positive Drag, as shown in Figure 10 and in the table 1. P_{o2} is found by the stage compressor ratio Equation (15) that is a function of the total temperature. T_{o2} is computed by Euler turbine equation that is a function of the absolute tangential velocity. This component did not reach the necessary magnitude near the hub at maximum blade angles. The energy is not enough to turn the fluid in the direction of the rotation. The absolute angle alpha 2 (α_2) is negative. This effect is shown in the Figure 14. Consequently the fluid cooled and not compressed in this zone. Compressor positive Drag means that the fluid not receives the resistance to cross the rotor due to the negative pressure gradient. This effect is remarkable as IGV increases the absolute angle entering the blade; compare Figures 10 and 11.

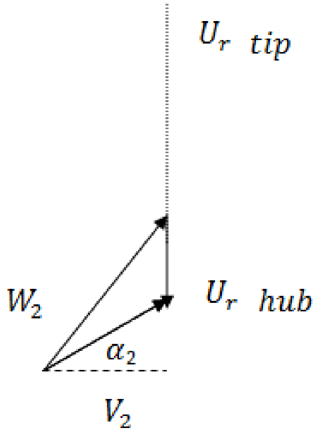


Figure 14
Compressor Velocities Triangle Diagram at the Exit Rotor with Blade Angle of 60° near the Hub

The IGV imparts an absolute tangent velocity component to the rotor entrances instead of purely axial absolute velocity. The results show that this effect of an IGV decreases the Lift force value, and studying other parameters, the pressure stage ratio also decreases and this could result in more compressor stages.

The IGV reduces the relative flow tip speed, and may also be actuated rapidly if a quick response in thrust modulation is needed [1].

Turbine Results and Conclusion

Lift force in the turbine is greater than the Lift force value in the compressor. This energy is transformed in shaft power to move the compressor.

The study sets that the fluid enters to the rotor with the relative angle β_{f2} equal to the nozzle exit angle. When nozzle exit angle increases, the Lift force increases significantly, and the Drag force also increases but slightly. Compare Figures 12 and 13. When β_{f2} increases, the relative and absolute velocities (W and V) at the rotor entrance increase significantly, but the effect in the exit velocities are not so much. This gradient makes the difference in the Lift force values. Also the coolant mass fraction decreases, and the shaft power provided by the turbine increases significantly, because of the velocity gradient.

In the study of the effect in Lift and Drag forces while changing the inlet rotor blade angles β_{b2}, the results are that while β_{b2} increases, the Lift force decreases, the coolant mass fraction increases and the generated power decreases, producing and unwanted effect. A purely axial velocity exit is achieved when $\beta_{b2} \approx 50^\circ$. The effect of varying β_{b2}, impacts mainly the exit relative velocity. Because the inlet relative velocity remains constant, the velocity gradient is the main reason for the parameters changes.

In general, a desired effect in the turbine is achieved for the largest exit nozzle and the smallest rotor inlet blade angle. But this conclusion is based only on the results of the study limited parameters. In order to get the best option an optimization study is required.

The Lift and Drag forces study in the turbine is based in the same compressor Lift and Drag equations, but takes in consideration the differences in the directions of the velocities components.

FUTURE WORKS

The study involves thermodynamics, physical parameters and constrains criterions that are important in the calculation of Lift and Drag coefficients. To obtain the best parameters selection, an optimization study is required. The physical blades parameters are not constant along the radius; this is an assumption to simplify the calculations, so as project continuity, the integration of the changes in solidity along the blade is recommended. Also other parameters as camber angle, the turbine camber line, turbine total pressure rotor loss, IGV total pressure loss are important and significant parameters in the Lift and Drag studies that should be taken in consideration in related future works. The variation of the temperature along the radius is an interesting and related topic to be considered as future work.

REFERENCES

[1] Farokhi, S., "Axial Compressor Aerodynamics", "Aerothermodynamics of Gas Turbines", *Aircraft Propulsion*, chapter 7 and 9, 2008, pp. 389-490, 527-590.

[2] Peng W.,W., "Axial Flow Compressors", "Gas Turbines", *Fundamentals of Turbomachinery*, Chapter 7 and 8,2008, pp. 191-254.

[3] Anderson J.D., "Aerodynamic Forces on a Body", *Modern Compressible Flow with Historical Perspective*, 3th edition,2003, pp. 34