An approach in statistical control chart design

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Abstract

Several questions arise when designing statistical process control charts. Is the control chart sensitive enough to detect a particular process shift with a particular frequency? When should a process shift be considered critical and why? What should the optimum sample size be? What should the optimum sample inspection frequency be? These and some other questions are associated with a control chart "sensitivity analysis."

We will try to answer these questions through a general step by step procedure for designing statistical control charts. This procedure will not end up with a unique answer because we are dealing with design and not with analysis. Criteria for the different situations and the degree of impact of a particular defective product will govern the possibilities. Most of the time, the original control chart design will lead to particular improvement projects due to the process characterization through sensitivity analysis.

Sinopsis

Cuando se diseñan diagramas de control estadístico de procesos surgen varias preguntas: ¿Es suficientemente sensitivo el diagrama de control para detectar un cambio particular en un proceso con una frecuencia particular? ¿Cuándo se debe considerar un cambio en un proceso crítico y por qué? ¿Cuál debe ser el tamaño óptimo de la muestra? ¿Cuál debe ser la frecuencia óptima de inspección de la muestra? Estas y otras preguntas se asocian con un análisis de sensitividad de un diagrama de control.

Trataremos de contestar estas interrogantes usando un procedimiento para el diseño de diagramas de control. Este procedimiento consta de 10 pasos y no resulta en una contestación única ya que se trata de un diseño y no de un análisis. Las posibilidades las rigen la norma para las diferentes situaciones y el grado de impacto de un producto defectuoso en particular.

La mayoría de las veces, el diagrama de control original puede resultar en proyectos de mejoras debido al proceso de caracterización mediante el análisis de sensitividad.

A step by step procedure for the design of statistical control charts

The statistical process control provides a means to control random variables by using statistical methods. Since there are random variables present in manufacturing processes, statistical process control shall be required in these cases. Manufacturing processes are expected to have inherent variation. There are no two identical products. We are interested fundamentally in the control of the product variation, thus the control of the process variation.

The following is a 10-step procedure for designing control charts.

Step 1. Define the variables

For a particular part, component or product there may be one or several variables for which statistical process control is to be established. These variables must be independent from each other; they must also be random variables and should be imperative for the final product performance or reliability, or for subsequent manufacturing operations. Thus, the associated specification limits must be available for these variables. These limits may not necessarily be established on a statistical basis but functionally.

Because the primary objective of a statistical control chart is to prevent defects, whenever feasible, analysis of the possible defect causes should be performed and duplicated. If we are certain of the defect causes through duplication and if the possible causes are random variables, then we may be able to control these variables which may be process related rather than product related. It is desirable to control the root defect cause random variables when possible; thus we may end up with several variables to control.

Random variables may be discrete or continuous. Probably the most important random variable for control purposes is the continuous normal variable. Quality, measured and controlled through discrete variables, is more likely to be considered after defects or defectives have occurred. Generally, discrete variables provide less process information for reacting the out of control condition. The normal continuous variate provides more information with significantly fewer samples.

Step 2. Collect the data

Once a random variable is carefully selected as the parameter to be controlled, we need to perform some sampling in order to understand the variable behavior. Before sampling, we have to make sure the process is operating under optimum conditions: that the proper preventative maintenance is being performed; that procedures, supplies and conditions such as coolant, oil, flux, blade, temperature, relative humidity, and others, whatever they are, are the appropriate or those that the company has established as the optimum. Samples must be random; otherwise, the complete analysis will be jeopardized. Accuracy in measurements should be appropriate; for example, decimal places should be at least one more than the specification limits; otherwise, data will behave as discrete and not as continuous data. The appropriate measurement instruments, properly calibrated, must be used.

Then a particular question arises: How much random sampling? For example, for a normal variate there are techniques available to calculate the sample size required such that type I error will be expected to be committed α * 100% of the time. If this sample size is off by a particular amount (ç), then type II error will be expected to be committed ß * 100% of the time when estimating the process mean. This reasoning gives us a good idea of where to start.

This model is closely related to the process standard deviation (σ). An initial trial sample is taken in order to have a sample estimate of σ; this may be a sample of 30 to 50 units. Then, from this sample we can calculate a required sample size. Lets say that after taking 40 units, the calculated required sample size for a particular α and ß, becomes 1750. We are not required to go and complete 1750 units; what this really means is that 40 units are not enough based on the current variation or standard deviation. We may take another 40 units, for a total of 80, and recalculate the required sample size for the particular α and ß. Then several things may happen; the new required sample size may become smaller because with the larger sample size the standard deviation becomes smaller, or the new required sample size becomes larger for the opposite reason. If the data are coming from a controlled process and the units are taken randomly, the most likely thing to happen is that as more units are taken, the required sample size becomes smaller as the standard deviation gets smaller. The standard deviation will not continue decreasing forever; however, it will until it stabilizes. We may continue sampling until the calculated sample size becomes smaller than the current accumulated sample size. At this moment we may have a good

estimate of σ and be able to perform some data analyses.

Step 3. Confirmation of the expected distribution

It is imperative to confirm the expected probability distribution. The reason for this confirmation is that no matter which Shewart Chart is to be used, the long range behavior and event occurrence frequency must be known. One of the most common techniques used for this purpose is the Goodness of Fit Test. The objective of the Goodness of Fit Test is to compare observed data with the theoretical distribution in order to confirm the degree of fitness from the particular expected probability distribution. Obviously, there will be a difference between the behavior of these data and the theoretical distribution. However, will this difference be large enough to make us think that data may not be fitted by the particular theoretical distribution? The answer to this inquiry consists of two parts: the distribution behavior in shape and the estimation of its parameters. In the case of a normal distribution, if samples are selected randomly from a controlled process, measurements are accurate and data are homogeneous, then we could probably conclude that there will not be enough evidence to reject the hypothesis that data may be fitted by a normal distribution, but with the condition that the mean will be different from the center of the specification limits. If this is the case, we are in the right direction because we are not really interested in where the current mean is, but in the fact that the normal distribution assumption is more than reasonable, and that there is a good estimate of the true standard deviation.

For the normal variate case, there are fundamentally six reasons why the hypothesis that data may be fitted by a normal distribution might be rejected for a particular set of data, where because of its nature, it is logical to expect normality. These reasons are:

1. Measurements are not accurate enough
2. Data is not homogeneous
3. Sample is not random
4. Presence of "outliners"
5. Occurrence of type I error
6. Data may not be fitted by a normal distribution

In the first three cases, data may be collected again. In the fourth case, outliers should be identified, removed from the data set and the tests must be performed again. Developing a box plot and examining data through their natural tolerance limits might be helpful to identify "outliers". The fifth case is associated with α, the fraction of the time occurrence of type I error is expected. This means that (α * 100)% of the time that the same experiment is performed, the results will lead us to reject the hypothesis even when it is true. Tests should be performed at least once more with another set of data to confirm this type of error. In the sixth case and through the central limit theorem, we may still be able to use the control charts with fairly good results given that the variable Xbar is likely to follow a normal distribution even when the variable X is not following a normal distribution as the sample size becomes larger. However, the process capability characterization will not be true, and this fact becomes a significant limitation in the design.

Step 4. Establish the process standard parameters

We will use a normal variate to illustrate this step. The recommended technique to establish the process standard parameters is the Process Capability Analysis. The objective of the Process Capability Analysis is to evaluate the ability of the process to meet specifications or to establish the process potential. The process potential is that minimum possible fraction defective the process is capable of generating with its true standard deviation. In the case of the existence of the upper specification limit (USL) and the lower specification limit (LSL), there is no need to estimate the process mean from the data; the process standard mean should be established as the center of the specification limits. If the process standard deviation is the optimum, based on the conditions previously established in order to collect the data, with the standard process mean centralized within its specifications, then the process will provide a fraction defective which may be considered the process potential.

This author recommends a process capability ratio of at least 1.33 when feasible. If this is not the case, the control chart developed should be temporary and an engineering project may take place in order to improve that process capability ratio.

The standard process mean μ" is to be set at the center of the specifications. The standard process standard deviation is to be set as the estimated σ".

Step 5. Establishing the process critical parameters

Now that we have set the standards, what should a critical change in the process mean be? This may be speculative. The engineer has to establish a criterion which will depend on the impact of generating defects and will also depend on the type of product being manufactured. An example for this criterion may be: "A critical change in the process mean for this particular variable is that which triples the standard fraction defective or process potential."

Similarly, what should a critical change in the process standard deviation be? That may also be speculative. Let's use the same criteria for the process mean as an example: "A critical change in the process standard deviation for this particular variable is that which triples the standard fraction defective or process potential."

Simple calculations will lead to the magnitude of the process mean and process standard deviation critical values. Now we are able to draw some data analysis conclusions:

A. Data may be fitted by a normal probability distribution.

B. The process capability ratio is _____ and the capable minimum fraction defective (process potential) is approximately _____ PPM's.

C. Standard process mean:

μ'' = _____, because this is the center of the specification limits which is expected to generate the minimum fraction defective or process potential.

D. Critical change in the process mean:

μ' = _____, based on fraction defective established criteria.

E. Process standard deviation:

σ'' = _____, because this is the expected variation capability for this particular process under the optimum operating conditions.

F. Critical change in the process standard deviation:

σ' = _____, based on the fraction defective established criteria.

6. Establishing the appropriate control chart type

The most common types of control charts are the following:

Table 1. The most common types of control charts

Control chart type	Distribution behavior
XBar/R Chart	Normal
XBar/S Chart	Normal
Individuals Chart	Normal
P-Chart	Binomial
NP-Chart	Binomial
C-Chart	Poisson
U-Chart	Poisson

Step 7. Establish the control chart sensitivity criteria

Once we understand the process behavior, we are able to develop the control chart. At this moment we want the process controlled with the parameters specified in step 5.

In order to develop the control chart, we need to establish the following parameters:

n = the sample size
k = the chart width
h = the samples frequency

These unknown parameters are essential to the sensitivity of the control chart.

We have established that if the process mean shifts to μ', the fraction defective is expected to increase by a particular amount which we have established as critical. Another criteria is to be established, for example, "let's say that we want a control chart that on the average will detect such a shift approximately on the third sample."

Now the Operating Characteristic Curve (OCC), the Average Run Length (ARL), α, ARL(α), ß and ARL(ß) are taken into consideration.

The OCC shows the ß(Beta) risk for various possible process changes.

The ß risk may be seen in different ways:

a. The probability of committing type II error (ß), for different process shifts.

b. The fraction of the time a process shift is expected not to be detected on the first sample after the shift occurs.

c. The fraction of the time it is expected to observe points within the control limits when in fact the process is out of control.

The ARL(ß) tells us the expected number of samples required to detect a particular shift.

$$ARL(\beta) = \frac{1}{(1-\alpha)}$$

Alpha (α), the fraction of the time it is expected to conclude that the process is out of control when in fact the process is in control (false alarms), thus, ARL(α) is the expected number of samples between false alarms.

$$ARL(\alpha) = \frac{1}{\alpha}$$

Starting with a sample size = 5 and chart width = 3, let's examine the control chart sensitivity through the OCC and ARL. What if the process mean shifts to the critical μ'?

Probably we may not be comfortable with the fraction of the time a critical shift is not detected on the first sample and the average number of samples required to detect it. In order to improve the sensitivity of the control chart, we may modify the samples size (n), the chart width (k) or both.

Decreasing the chart width (k) will decrease ß; thus increasing its ability of detection. On the other hand, α will increase, meaning that the number of false alarms will increase.

Alternatives have to be characterized in order to reach a final decision on the establishment of samples size (n) and chart width (k). Different criteria will be established based on those acceptable risks for different product types.

Samples size (n) and chart width (k) have been established.

Now, another question comes into consideration. How frequent samples of size (n) may be taken? Several criteria may be used. Let's assume that our criterion is that "when a shift of the magnitude of μ' occurs, we want to detect it on the average in EDT minutes." Using the following relationship:

$$h = \frac{EDT}{ARL(\beta)}$$

where,

h = Inspection Interval in minutes
EDT = Expected Detection Time

The inspection frequency (h) have been established.

By now, we have completed the design for controlling the process central tendency. Similarly, we may use the same approach to establish the control scheme of the process dispersion. The models required to perform this analysis are not readily available for computations. An alternative is to perform it through simulation.

Up to this point we have established a control chart that meets our criteria.

Step 8. Establish the control chart parameters

Process mean	: μ"
Process standard deviation	: σ"
Samples size	: n
Chart width	: k
Inspection frequency	: h

These are the values we want to establish as standard for the process.

Step 9. Simulate the process under different scenarios (Optional)

Computer software is available in the market for control chart simulation. Why simulation? Still there is some fine tuning to be performed. There are some process characteristics which need to be evaluated as well. For example, the frequency at which the process becomes out of control with respect to the central tendency, the frequency at which the process becomes out of control with respect to the dispersion, the average process shift with respect to the central tendency and the average process shift with respect to the dispersion. Although simulation is an optional step in the control chart design, it may be considered in order to optimize the control process.

Step 10. Establish formal control chart specifications

In this last step of the procudure you complete a form in order to establish the formal control chart specifications (fig. 1). This form will help you summarize all of the parameters and specifications established during the previous steps.

Continuous variable control chart specifications

Item number : _____ Description : _____
Characteristic : _____ Units of meas: _____
USL : _____ USL : _____
Analyst : _____ Date : / / Rev : ____

Standard values

Control chart type : _____ Samples size : _____
Process mean : _____ Std. deviation: _____
Process capability : _____ Samples frequency : ____
Fraction defective : _____ Parts per million : _____

Control chart

Central tendency chart
Chart width : _____
UCL : _____
CL : _____
LCL : _____

Dispersion chart
Chart width : _____
UCL : _____
CL : _____
LCL : _____

Type I error

Alpha (α) : _____ Alpha (α) : _____
ARL (α) : _____ ARL (α) : _____

Critical changes sensitivity

Critical change (X') : _____ Critical change (σ): ____
Fraction defective : _____ Fraction defective : ____
Parts per million : _____ Parts per million : ____
β : ___ ARL : ____ β : ___ ARL : _____

Procedure

Figure 1. A form to establish formal control chart specifications